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Influence of the Talbot effect on the loss permutations of Fabry – Perot resonator modes

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***Abstract*.** **The loss permutations of Fabry – Perot resonator modes caused by the harmonic spatial perturbation of the radiation phase on one of the mirrors are studied numerically. The periods and amplitudes of perturbations are found at which the second or third mode in the eigenvalue modulus becomes the first mode. It is shown that in the case of perturbations with the period l_0, at which the Talbot length is equal to the double resonator length, the permutations are caused by an increase in the losses of the fundamental mode. It is also shown that the perturbation amplitudes with the period l_0, which equalise losses of the modes, depend linearly on the inverse Fresnel number F^{-1}.**

Keywords: *Fabry – Perot resonator, mode composition, perturbations.*

1. Introduction

The influence of perturbations of the radiation phase on mode characteristics has attracted the attention of researchers from the time of the first studies on the mode composition of open optical resonators. Many papers devoted to this problem published before 1990 are cited in monograph [1], where the influence of phase perturbations was analysed in stable, plane, and unstable resonators. For resonators of a special type, changes in the mode parameters caused by deviations of the shape of resonator mirrors from a perfect shape were also considered [2]. The most important types of perturbations existing in optical resonators are well known at present. Most of them (displacements, tilts and sagging of mirrors, and thermal lenses) can be classified as large-scale perturbations [1]. Another type of perturbations is small-scale random variations in the refractive index of an intracavity medium [1]. Until recently, the mode characteristics of optical resonators with perturbations of the intermediate scale were not studied.

However, the effect of perturbations of the radiation phase on the mode characteristics of wide-aperture semiconductor amplifiers and lasers with resonators produced due to reflection of radiation from the crystal chips has been adequately studied. The main process preventing the enhancement of the output power of these devices is the filamentation of radiation resulting finally in the damage of reflecting facets. The theoretical and experimental study of filamentation showed that it is controlled by the Talbot effect [3]. This is explained by the fact that after the round trip of radiation in the resonator, the field perturbations, which appeared near one of the mirrors and have the scale l_0 for which the Talbot length is twice the resonator length, are reproduced. As a result, a periodic modulation of the carrier concentration and refractive index appears near the corresponding mirror.

The calculations of the established state of the field and the nonlinear active medium of semiconductor amplifiers and lasers give a strongly irregular asymmetric [4] or symmetric [5] distribution. The mode composition of a perturbed resonator was not determined in the numerical studies of filamentation probably because of the difficulty of this problem.

In Ref. [6], a simpler problem was solved, namely, the losses and spatial characteristics of the lowest modes of the Fabry – Perot resonator were studied in the case of weak harmonic spatial perturbations (HSPs) of the radiation phase on one of the mirrors. Such an approach cannot replace the solution of problems for different perturbations of the resonator such as misalignments, deviations from the parallel arrangement of mirrors, weak sagging of mirrors, etc. However, it allows one to estimate the influence of perturbations of any scale on the losses and spatial characteristics of the lowest modes in the Fabry – Perot resonator.

The loss permutations of the Fabry – Perot resonator modes caused by periodic phase perturbations have not been studied so far. This study is of interest for controlling the mode composition of lasers and is important for the understanding of processes proceeding in gas lasers with turbulence or acoustic disturbance in the active medium. One can also assume that variations in the mode composition of wide-aperture Fabry – Perot resonators caused by periodic perturbations of the radiation phase on one of the mirrors correspond qualitatively to variations in the mode composition of semiconductor amplifiers and lasers mentioned above. In this case, the study of the loss permutations of the Fabry – Perot resonator modes is important for the substantiation of some assumptions about the development of filamentation. In addition, this study is also of importance for the theory.

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In this paper, we studied the loss permutations of the Fabry–Perot resonator modes for HSP frequencies near l_0^{-1}. It is at these perturbation frequencies that the loss permutations occur at minimal perturbation amplitudes [6].

2. Investigation procedure

We calculated the mode characteristics of an optical resonator from a set of field distributions on an unperturbed mirror obtained after successive round trips of radiation in the resonator [7]. Analytic expressions for the eigenfunctions [1] were used both for the construction of the field distribution from which iterations began and for obtaining a system of algebraic equations, whose solutions were employed for determining the eigenvalues and eigenfunctions of the perturbed resonator. To control the accuracy of the algorithm, we determined the difference of the moduli of the eigenvalues, which were found numerically and analytically for the unperturbed resonator. In all cases studied, the maximum variation in the eigenvalue moduli caused by HSPs exceeded this difference by more than an order of magnitude.

For each perturbation $A\sin(2\pi x/l)$ at the scale l, we calculated a set of complex eigenvalues and eigenfunctions [$\gamma_j(\varepsilon)$, $U_j(x;\varepsilon)$, where $j = 0, 1, ..., M-1$ is the mode number; M is the number of modes; x is the transverse coordinate; and ε shows that the eigenvalues and eigenfunctions are perturbed]. The moduli of complex spatial distributions $U_j(x;\varepsilon)$ were always normalised:

$$|U_{nj}(x;\varepsilon)| = \frac{|U_j(x;\varepsilon)|}{\max\limits_x |U_j(x;\varepsilon)|}.$$

Because large-amplitude HSPs cause the loss permutations of the modes, we used the reordering of the first modes over the moduli of eigenfunctions $|U_{nj}(x)|$ ($j = 0, 1, 2, 3$). The degree of deviation of the perturbed distribution from the unperturbed one was described by the expression

$$D_{ij} = \max_x \| U_{ni}(x;\varepsilon)| - |U_{nj}(x;0)\|.$$

We assumed that D_{ij} is greater for $i \neq j$ than for $i = j$. After the reordering, the mode close in shape to the fundamental mode ($j = 0$) came to the first place, the mode close in shape to the first mode ($j = 1$) was at the second place, etc. The mode losses were permuted correspondingly. If the algorithm operated correctly, the losses of different modes were a continuous function of the perturbation period. An incorrect reordering resulted in jumps of mode losses. Such jumps appeared when perturbation amplitudes were approximately twice as large as those in paper [6]. The replacement of the eigenfunctions of an unperturbed Fabry–Perot resonator found analytically by the eigenfunctions calculated numerically did not improve the algorithm operation: the perturbation amplitude at which the jumps of losses appeared remained the same.

3. Results of calculations

Consider the influence of the spatial frequencies of a perturbation from the region near l_0^{-1} on the mode characteristics of the Fabry–Perot resonator. Figure 1 shows the dependences of losses $L_j(\varepsilon) = 1 - |\gamma_j(\varepsilon)|^2$ of the first modes on the dimensionless frequency $w_u = l_0/l$ of the perturbation (l_0 and l are the phase perturbation periods on the resonator mirror). The value of l_0 was determined from the equality of the double length of the resonator to the Talbot length: $2L = kl_0^2/\pi$, where k is the wave number. The radius a of mirrors was chosen so that the Fresnel number $F = (a/l_0)^2$ was equal to 16, i.e., four perturbation periods l_0 fitted in the radius a. One can see from Fig. 1 that the increase in the HSP amplitude for $w_u = 1$ results in a substantial increase in the fundamental-mode losses, so that they become first equal to the first-mode losses (Fig. 1a), then to the second-mode (Fig. 1b), and the third-mode (Fig. 1c) losses. In this case, the distribution over the losses of the 1–3 modes is not violated. Such a situation also takes place in some frequency region near $w_u = 1$.

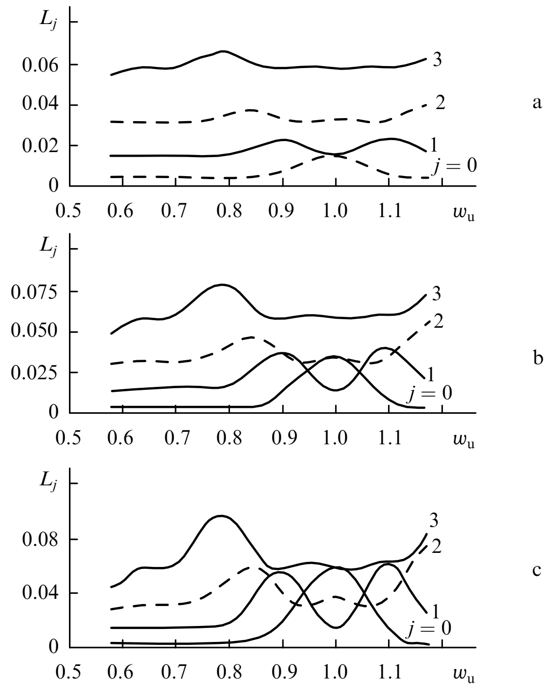

Figure 1. Dependences of losses $L_j(\varepsilon)$ of the first ($j = 0, 1, 2, 3$) modes of the Fabry–Perot resonator on the dimensionless frequency w_u of sinusoidal spatial perturbations of the perturbation amplitude $A = 0.092$ (a), 0.146 (b), and 0.197 (c). The Fresnel number is $F = 16$.

We can assert that the influence of the Talbot effect on the fundamental-mode losses increases with increasing perturbation amplitude in the frequency region near $w_u = 1$. However, beyond this region, the loss permutations for the first modes take place. Thus, for $w_u = 0.87 - 0.95$ and $w_u = 1.05 - 1.13$, the second-mode losses are smaller than the first-mode losses (Fig. 1c). It follows from a comparison of Figs 1c and 1b that the appearance of the region where the second and first modes are permuted is explained by the increase in the losses for the fundamental and first modes in the corresponding perturbation range. The lowest losses in the perturbation range studied ($w_u = 0.6 - 1.15$) correspond to the fundamental ($w_u = 0.6 - 0.93$), second ($w_u = 0.93 - 0.95$), first ($w_u = 0.95 - 1.05$), second ($w_u = 1.05 - 1.07$), and again the fundamental mode ($w_u = 1.07 - 1.15$) (Fig. 1c). Figure 2 shows the distributions $|U_{nj}(x)|$ for modes with identical losses for $w_u = 1$. A noticeable asymmetry of the distributions with respect to the resonator axis is caused by the use of a sinusoidal perturbation. One can see that the equating of the fundamental-mode losses with losses of higher-order modes is caused by a greater modulation depth of the spatial distribution of the fundamental mode. The profiles of the first modes ($j = 1$, 2, 3) are less distorted in this case.

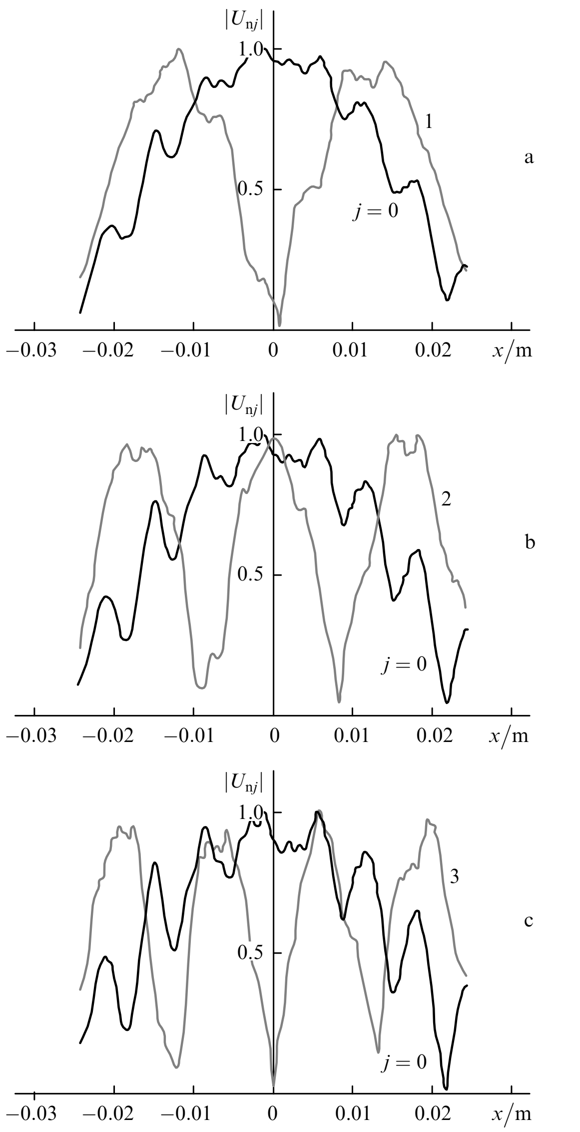

Figure 2. Distributions of the eigenfunction modulus $|U_{nj}(x)|$ for modes with identical losses for perturbations at the scale $l = l_0$ for $L_0(\varepsilon) = L_1(\varepsilon)$, $A = 0.092$ (a), $L_0(\varepsilon) = L_2(\varepsilon)$, $A = 0.146$ (b), and $L_0(\varepsilon) = L_3(\varepsilon)$, $A = 0.197$ (c) and $w_u = 1$. The Fresnel number is $F = 16$.

Consider the influence of the amplitude of perturbation at the spatial frequency $w_u = 1$ on the losses of the Fabry–Perot resonator modes. Figure 3 shows the dependences of the fundamental-mode losses and losses for the three higher-order modes on the HSP amplitude for different Fresnel numbers F of the resonator. One can see that the fundamental-mode losses are most sensitive to HSPs. Losses for the three higher-order modes weakly depend on HSPs. Their changes are most noticeable at small Fresnel numbers (Fig. 3c). An increase in F noticeably reduces the influence of HSPs on the losses of the first modes (Fig. 3a). One can also see from Fig. 3 that the degeneracy points [intersection

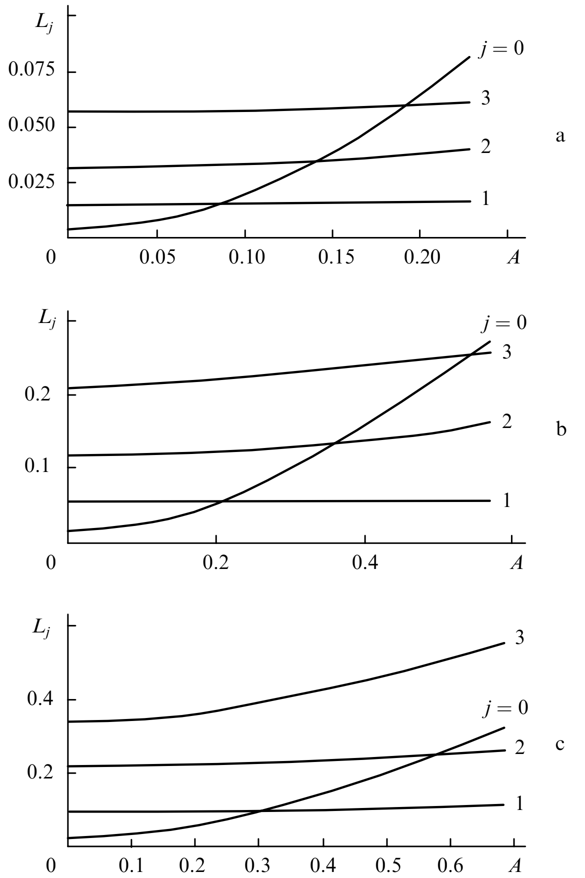

Figure 3. Dependences of losses L_j of the first modes ($j = 0$, 1, 2, 3) on the HSP amplitude for the Fresnel numbers $F = 16$ (a), 6 (b), 4 (c) and $w_u = 1$.

points of the curves $L_0(\varepsilon)$, $L_1(\varepsilon)$, $L_2(\varepsilon)$, and $L_0(\varepsilon)$, $L_1(\varepsilon)$, $L_2(\varepsilon)$] draw together with increasing F and are displaced to the lower perturbation amplitudes. This can be explained by an increase in the sensitivity of the fundamental mode to HSPs and a decrease in the difference between the moduli of the eigenvalues.

Figure 4 shows the dependences of the amplitudes of perturbations leading to the equating of the losses of the fundamental and first [curve (*1*)], fundamental and second [curve (*2*)], fundamental and third [curve (*3*)] modes on F^{-1} for $w_u = 1$. One can see that these dependences are close to linear, and as the number of the mode with losses equal to the fundamental-mode losses increases, the straight

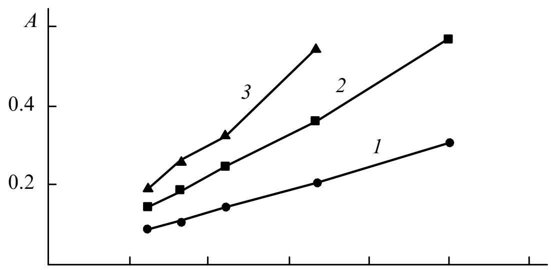

Figure 4. Dependences of the perturbation amplitudes A for the degeneracy points on F^{-1} for $L_0(\varepsilon) = L_1(\varepsilon)$ (*1*), $L_0(\varepsilon) = L_2(\varepsilon)$ (*2*), $L_0(\varepsilon) = L_3(\varepsilon)$ (*3*) and $w_u = 1$.

lines are not only displaced upward but their slope also increases.

4. Conclusions

Our calculations have shown that the method for reordering the first modes proposed in the paper allowed us to extend the range of the HSP amplitudes at which the mode composition of a Fabry – Perot resonator can be studied. However, the fundamental mode retains its basic properties at maximal perturbation amplitudes used in the method despite the distribution irregularity (Fig. 2c). Thus, this approach did not allow greater perturbation amplitudes to be obtained, and to study stronger perturbations (turbulence, acoustic disturbances), it should be further developed.

The use of a phase corrector in the Fabry – Perot resonator at frequencies close to l_0^{-1} allows one to make the losses of the first or second mode (or of both these modes simultaneously, if the perturbation period and amplitude correspond to a degeneracy point) the lowest ones among all of the resonator modes. However, this is accompanied by the distortions of the spatial distributions of the modes. The question about mode permutations and a simultaneous minimisation of the profile perturbation (for example, due to the action at several frequencies) remains open.

Analytic studies of filamentation in wide-aperture semiconductor amplifiers and lasers are performed at present by the perturbation method [3, 8]. In this case, the transverse size of the lasing region is assumed infinite. As a result, the mode structure of the resonator disappears, the transversely homogeneous distributions of the light field and carrier concentration prove to be the unperturbed state, which are subjected to small periodic perturbations. This means that filamentation is considered as a perturbation of the fundamental mode. At the same time, the estimates performed in the paper have shown that in some cases the permutations of the fundamental mode with higher-order modes begin earlier than the noticeable deformations of its distribution. Due to the mode permutations and equating of their losses, lasing can occur at a few higher-order modes. This assumption does not contradict to the data reported in Ref. [1] (see p. 165) and calculations [4] because asymmetric distributions obtained in Ref. [4] can be readily explained by a superposition of the fields of several modes. Further studies of multimode lasing by the selective method of determination of the field configuration [7] can explain the nature of filamentation.

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