

INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA

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Deformation of the signal envelope due to a strong dispersion of the refractive index in the gain region

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***Abstract*. It is shown that, when a narrow-band signal with a non-Gaussian envelope propagates in a medium with a strong dispersion, the signal envelope being distorted due to the dispersion of the gain (or) absorption in the medium. Nevertheless, the amplitude-modulated signal having a symmetric (even) intensity envelope remains the same. The propagation velocity of the 'middle point' (i.e., the symmetry centre) of the signal coincides with the real group velocity of the wave with a given carrier frequency in the medium and can be subluminal, superluminal or negative. Therefore, the concept of a real group velocity (including the superluminal or negative velocity) is also applicable in media with a strong dispersion of the gain (or absorption).**

Keywords: *superluminal group velocity, signal distortion during its propagation, dispersion.*

1. The study of the propagation of a wave packet in a high-dispersion medium has aroused considerable recent interest (see, for example, Refs [1 – 3] and references therein). This concerns first of all the propagation of the wave-packet maximum at the superluminal group velocity (so that the signal delay time proves to be shorter than the 'light' delay time) and at a negative group velocity (so that the signal delay proves to be negative)[1]. The 'self-regeneration' of a partially transmitted signal (both for the negative and superluminal group velocity of the signal) is also of interest.[2]

To realise these effects in practice, the dispersion of a medium should be high [1], i.e., the refractive index of the medium should strongly depend on the radiation frequency. At the same time, a strong dependence of the refractive index of the medium on the radiation frequency should be accompanied by the same strong frequency dependence of the absorption coefficient (or gain) of the medium (according to the Kramers – Kronig relations [4]). Therefore, strictly speaking, a high-dispersion medium always selectively absorbs or amplifies light, and a study of the propagation of a wave packet in it should take into account not only the dispersion of the real refractive index but also the dispersion of the absorption coefficient or gain of the medium.

This can be done most simply by using the complex delay time of a signal [2, 3, 5][3]. The consideration of the real and imaginary parts of the time delay during the signal propagation in a high-dispersion medium is inevitable in the general case when both the real and imaginary parts of the complex refractive index of the medium strongly depend on the wave frequency. In this case, the real and imaginary parts of the signal delay time prove to be of the same order of magnitude, and we arrive at the following alternative. Either they both are small compared to the signal duration and, therefore, can be neglected or they are comparable with the signal duration and should be taken into account.

It should be emphasised that the signal becomes inevitably strongly distorted when the imaginary delay time is comparable with the signal duration[4]. The exception from this rule is a signal with a Gaussian envelope, whose complex envelope is independent of the imaginary delay time [2, 3, 5]. It is for this reason that no distortions were observed in the calculations of a Gaussian signal propagat-

[1]There is no qualitative difference between the first and second effects. In any case, we are dealing with the 'prediction' of a signal by a medium (see, for example, Refs [2, 3]), which arrives at the observation point with the 'light' delay time. The choice between these effects depends on the relation between the 'light' delay time (which is determined only by the geometrical thickness of a substance layer) and the duration of the 'prediction' interval (which also depends on dispersion).

[2]This effect is also a side result of the 'prediction': the regeneration of a non-transmitted part of a signal can be treated as an inevitable error (it is impossible to 'predict' a sudden termination of the signal transmission as long as the transmitted part of the signal does not contain information on this termination)].

[3]Review [5] contains not only a number of original results on the superluminal and negative group velocities of a signal in a dispersion medium but also a detailed analysis of the meaning of the complex group velocity (and, hence, of the complex delay time of a signal). Of course, all quantities measured by real instruments are real. Here, as in Ref. [5], we bear in mind that the real amplitude and phase of a signal being received coincide approximately with the real modulus and argument of a complex function, which differs from the initial time envelope only by the complex shift of the argument. Therefore, it is convenient to use the complex delay time of a signal for studying distortions of its shape.

[4]Of course, the signal distortion can be caused not only by the imaginary part of the delay time (appearing in the first order of the classical dispersion theory) but also by the amplitude diffusion (appearing in the second order [5, 6]) and higher-order corrections. Nevertheless, the first order of the dispersion theory (i.e., the complex delay time approximation) has a special status compared to higher-order corrections. It differs qualitatively from the higher-order corrections in that the first-order signal distortion (i.e., its complex delay time) cannot be eliminated by narrowing the signal spectrum (i.e., by increasing its duration) because the complex delay time of the signal does not tend to zero when the signal spectrum is narrowed.

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ing in a frequency-selective amplifying medium despite the complex delay time of the signal [2].

This paper makes up for this deficiency and is devoted to the analysis of distortions of a non-Gaussian signal propagating in an amplifying medium. The approach used here is completely similar to that proposed in Refs [2, 3]. The only difference is that now we consider not Gaussian but three-peak signals (which consist, however, of three Gaussians). Even this modification proved to be quite sufficient to demonstrate the type of signal distortions in an amplifying dispersion medium.

2. Consider the propagation of a narrow-band signal

$$E(z,t) = A(z,t)\exp(-\mathrm{i}\omega_1 t) + A^*(z,t)\exp(\mathrm{i}\omega_1 t) \qquad (1)$$

with the carrier frequency ω_1 and the complex envelope $A(z,t)$ in a homogeneous isotropic medium along the z axis. Let the wave packet be propagating in a medium with the refractive index $n(\omega) = n_0 + \Delta n(\omega)$, where $\Delta n(\omega)$ is the complex addition to the refractive index of the medium, which is caused by the gain line centred at the frequency ω_0 and n_0 is the background (non-resonance) refractive index of the medium, which weakly depends on the radiation frequency near ω_0.

In Ref. [2], the expression

$$A(z,t) = \exp[\mathrm{i}k_0 n(\omega_0)z + \xi g(\Omega_0)]A^{(0)}[t-\tau(z)] \qquad (2)$$

was obtained in the complex delay time approximation for the complex envelope of a wave packet of an arbitrary shape propagated through a dispersion medium layer of thickness z. Here, $k_0 = \omega_0/c$ is the wave number; $\xi = \alpha_0 z$ is the optical thickness of a medium layer; $\alpha_0 = \mathrm{i}k_0\Delta n(\omega_0)$ is the amplitude gain at the centre ω_0 of the spectral line; $g(\Omega) = \mathrm{i}k\alpha_0^{-1}\Delta n(\omega_0+\Omega)$ is the complex form-factor of the line; $\Omega = \omega - \omega_0$ is the frequency detuning of the wave from ω_0; $\Omega_0 = \omega_1 - \omega_0$ is the carrier frequency shift; $A^{(0)}(t) \equiv A(0,t)$ is the signal shape on the medium layer boundary (i.e., for $z=0$); and the complex delay time $\tau(z)$ of the signal is determined by relations

$$\tau(z) \equiv \tau_0 + \tau_r + \mathrm{i}\tau_i, \quad \tau_0 \equiv \frac{z}{v_{ph}}, \quad v_{ph} \equiv \frac{c}{n_0},$$
$$\tau_r \equiv \xi\frac{\partial\,\mathrm{Im}\,g(\Omega_0)}{\partial\Omega_0}, \quad \tau_i \equiv -\xi\frac{\partial\,\mathrm{Re}\,g(\Omega_0)}{\partial\Omega_0}, \qquad (3)$$

where the parameter τ_0 is the vacuum time of the signal delay, which is always positive. The parameter τ_r is the additional (with respect to the vacuum time) real delay time of the signal, which can be both positive and negative[5]. If $\tau_r > 0$, then the signal propagates at a subluminal group velocity, if $\tau_r < 0$, but $\tau_0 + \tau_r > 0$, the signal propagates at a positive superluminal velocity, and if $\tau_0 + \tau_r < 0$, it propagates at a negative group velocity. The parameter τ_i is the imaginary part of the signal delay time. We will show below that it describes the signal distortion due to the dispersion of the absorption coefficient (or gain) of the medium.

3. Let us formulate some general features of the distortion of the signal envelope $A(t)$ due to a purely imaginary time delay, when the function $A(t)$ is replaced by the function $A_1(t) \equiv A(t - \mathrm{i}\Delta)$, which differs from $A(t)$ only by a purely imaginary time delay[6] $\mathrm{i}\Delta$.

Consider the case when the initial signal has a modulated amplitude [i.e., $A(t)$ is an analytic function having real values for the real argument t]. We can claim the following:

(i) The function $A_1(t)$ is not necessarily real for the real argument t. This means that the amplitude-modulated signal, which has experienced an imaginary shift, acquires in the general case the phase modulation as well.

(ii) If the function $A(t)$ has no singularities in the band of the complex plane between the straight lines $\mathrm{Im}\,t = 0$ and $\mathrm{Im}\,t = -\Delta$ and also rapidly decreases at $t \to \infty$ within this band, then

$$\int_{-\infty}^{+\infty} A(t)\mathrm{d}t = \int_{-\infty}^{+\infty} A_1(t)\mathrm{d}t,$$

i.e., the area under the signal envelope is preserved[7].

(iii) $A(t+\mathrm{i}\Delta) = A^*(t-\mathrm{i}\Delta)$, i.e., a change in the sign of the imaginary shift results in the replacement of the envelope by the complex conjugate envelope and does not lead to a change in the signal intensity envelope: $|A(t-\mathrm{i}\Delta)|^2 = |A(t+\mathrm{i}\Delta)|^2$.

(iv) $|A(t-\mathrm{i}\Delta)|^2 = |A(t)|^2 + O(\Delta^2)$, i.e., in the case of a small imaginary shift, the distortion of the time dependence of the amplitude-modulated signal intensity has a higher order of smallness. This means that the distortion of the intensity envelope due to the imaginary shift is first unnoticeable, but then drastically increases.

(v) If $|A(t)|^2 = |A(-t)|^2$, i.e., if the function $A(t)$ is even or odd with respect to $t = 0$, then $|A_1(t)|^2 = |A_1(-t)|^2$, i.e., the 'shifted' function has the same properties. This means that in the approximation of the complex delay time of the signal, the imaginary part of the delay time characterises only the deformation of the signal envelope (the imaginary shift only distorts the signal, without the shift or vanishing of its centre of symmetry, if the initial signal had this centre).

4. Consider, as in Ref. [2], the gain line with a Lorentzian shape of width $\Delta\Omega_{1/2}$ and the coherence time $\tau_{coh} \equiv 2/\Delta\Omega_{1/2}$. In this case,

$$g(\Omega) = (1 - \mathrm{i}2\Omega/\Delta\Omega_{1/2})^{-1}. \qquad (4)$$

Consider (in contrast to Ref. [2]) the propagation of the three-peak signal

$$A^{(0)}(t) = \frac{5}{4}\exp\left[-\frac{(t-T)^2}{T^2}\right] - \frac{7}{4}\exp\left(-\frac{t^2}{T^2}\right) + \frac{5}{4}\exp\left[-\frac{(t+T)^2}{T^2}\right] \qquad (5)$$

with the characteristic duration T. This signal represents a 'trident' composed of three Gaussians (Fig. 1a).

Let us use the same parameters of the signal and the path length as in Ref. [2][8]: the signal duration $T = 10\tau_{coh}$ and the

[5]It is this time that can be treated as the dispersion delay (when τ_r is positive) or advance (when τ_r is negative). The real part of the delay time is related to the real group velocity ($\tau_0 + \tau_r = z/v_{gr}$) by the known expression $[v_{gr} = (\partial\mathrm{Re}\,k(\omega)/\partial\omega)^{-1}]$.

[6]In the general case the delay time is a complex quantity. Because the meaning of its real part is obvious, we consider here only its imaginary part.

[7]But not under the time dependence of the radiation intensity. Even if the radiation intensity is an analytic function, because the absolute value is calculated nonanalytically, the complex shift of the complex envelope of the signal leads to the results that differ from those obtained due to the complex shift of the signal intensity envelope.

[8]Note that no significant distortion in the shape of the Gaussian signal was observed at these values of parameters in Ref. [2].

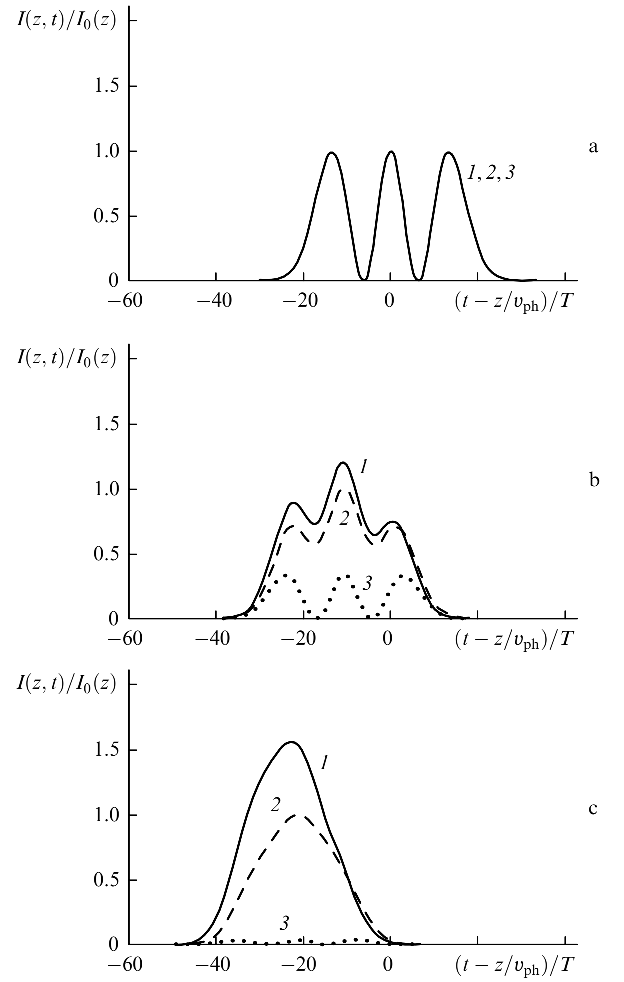

Figure 1. Distortion of the initial three-peak signal (5) ($T = 10\tau_{coh}$, $x_0 = \Omega_0\tau_{coh} = 5$) propagating in an amplifying medium for $\xi = 0$, $\tau = 0$ (a); $\xi = 300$, $\tau = (-10.651{+}4.438i)\tau_{coh}$ (b); and $\xi = 600$, $\tau = (-21.302 + 8.876i)\tau_{coh}$ (c). Curves (*1*) are numerical calculations, curves (*2*) are calculated from expressions (2) and (3), curves (*3*) are calculated from expression (2) for $\tau_i = 0$.

shift of the carrier frequency with respect to the spectral line centre is $x_0 = \Omega_0\tau_{coh} = 5$. The time dependences of the signal intensity $I(z,t) \equiv |A(z,t)|^2$ calculated for different path lengths ξ (and, hence, for different complex delay times τ) are shown in Fig. 1. The calculations were performed numerically and also in the complex delay time approximation [i.e., using analytic expressions (2) and (3)], and in the real group velocity approximation [i.e., using expression (2) with the additional condition $\tau_i = 0$). To compare the results obtained at different optical densities of layers, they were normalised to the intensity at the signal centre calculated in the complex delay time approximation[9].

Figure 1 (and its comparison with data [2]) shows that the initial time dependence of a signal propagating in a medium with the gain dispersion is indeed distorted. This distortion is well described in the approximation of the complex delay time of the signal. An attempt to ignore the imaginary part of the delay time leads to the results that are unreal not only for the amplitude (which is not most important), but for the time dependence of the signal as well (which is much worse[10]). The signal distortion proves to be considerable when the imaginary part of the delay time is comparable to the signal duration. For example, in our case, the distortion of the signal propagated through a substance layer with the optical thickness $\xi = 300$ ($\tau_i = 0.44T$) is substantial, although the initial 'trident' can be still recognised in the signal shape. For $\xi = 600$ ($\tau_i = 0.88T$), no traces of the 'trident' are remained, and the signal shape reminds of a Gaussian.

Although the signal shape is distorted already in the first order of the dispersion theory, its time dependence retains its initial symmetric shape (of course, if there is what to retain, as in our case). In this case, the middle point of the signal travels in space at the real group velocity. When the real group velocity is negative, the middle point of a non-Gaussian signal still appears (as for a Gaussian signal) behind a substance layer earlier than in front of it, whereas in the case of a superluminal group velocity, the delay time is still shorter than the 'light' delay time.

Therefore, in the general case (in the presence of dispersion of the absorption coefficient at the frequency used), the shape of a signal propagating in a high-dispersion medium is strongly distorted due to the complex delay (or advance) time of the signal. Such a distortion is absent when the carrier frequency of the signal is specially chosen (for example, at the centre of the absorption line [1, 3] or in the middle between two identical gain lines) and also when the signal has a special shape (for example, a Gaussian[11]).

Figure 1 can make an impression that the imaginary part of the delay time produces the distortion of the signal envelope, which is qualitatively similar to the amplitude diffusion in the second order of the classical dispersion theory (i.e., the blurring of inhomogeneities). However, this is not the case, because the imaginary delay time of the signal can not only smooth off inhomogeneities that are present but also reveal hidden inhomogeneities. The latter is demonstrated in Fig. 2, where the calculated shapes of the signal

$$A^{(0)}(t) = \frac{5}{4}\exp\left[-\frac{(t-T)^2}{T^2}\right] + \exp\left(-\frac{t^2}{T^2}\right) + \frac{5}{4}\exp\left[-\frac{(t+T)^2}{T^2}\right] \quad (6)$$

propagated in a medium under the same conditions as in Fig. 1 are presented. One can see that the situation shown in Fig. 2 is completely opposite to that presented in Fig. 1: the initially bell-shaped signal becomes U-shaped during its propagation.

Moreover, the signal distortion in the complex delay time approximation is reciprocally periodic for signals of some type, i.e., the signal shape periodically returns to the initial shape with increasing the path length. This situation

[9]Optical densities used here correspond to extremely large signal gains. We assume in this paper that this problem is solved somewhat, for example, by introducing frequency-nonselective or weakly selective absorption (see more detailed discussion in [2]).

[10]Or better, which depends on the point of view. Indeed, the signal distortion (in the complex delay time approximation) reduces to the shift of its complex envelope in the complex plane. Therefore, the observation of the signal distorted this way can be considered as the experimental realisation of the analytic continuation of the function from the real axis to the complex plane. Such a possibility is of interest for data processing systems.

[11]Taking this circumstance into account, a Gaussian signal and the centre of the absorption line considered simultaneously in [1] both provide the stability of the shape of a signal propagating in a high-dispersion medium.

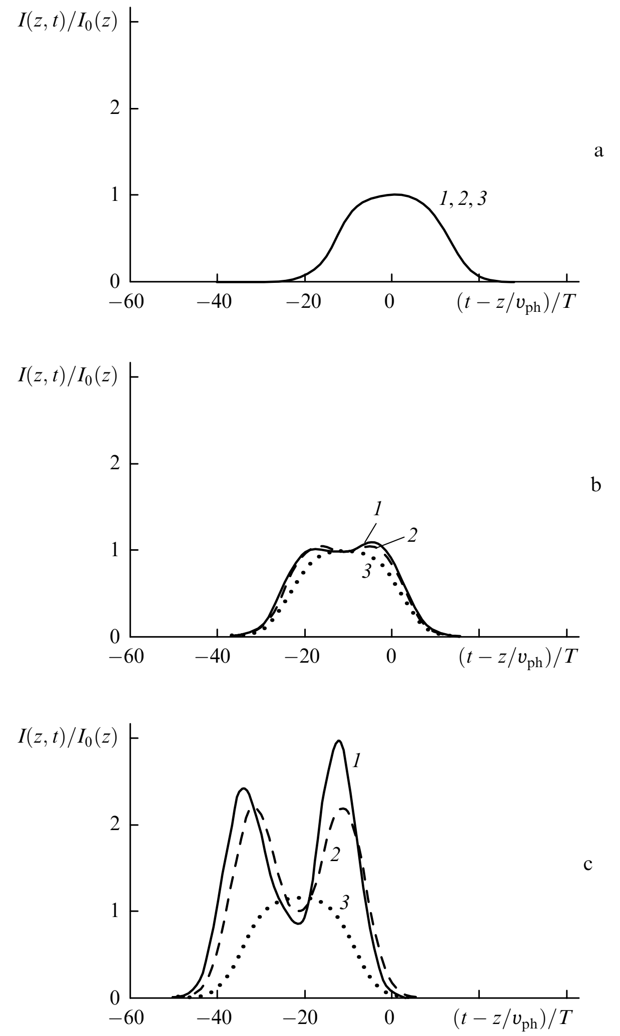

Figure 2. Distortion of the initial bell-shaped signal (6) ($T = 10\tau_{coh}$, $x_0 = \Omega_0\tau_{coh} = 5$) propagating in an amplifying medium for $\xi = 0$, $\tau = 0$ (a); $\xi = 300$, $\tau = (-10.651 + 4.438i)\tau_{coh}$ (b); and $\xi = 600$, $\tau = (-21.302 + 8.876i)\tau_{coh}$ (c). The notation of the curves is as in Fig. 1.

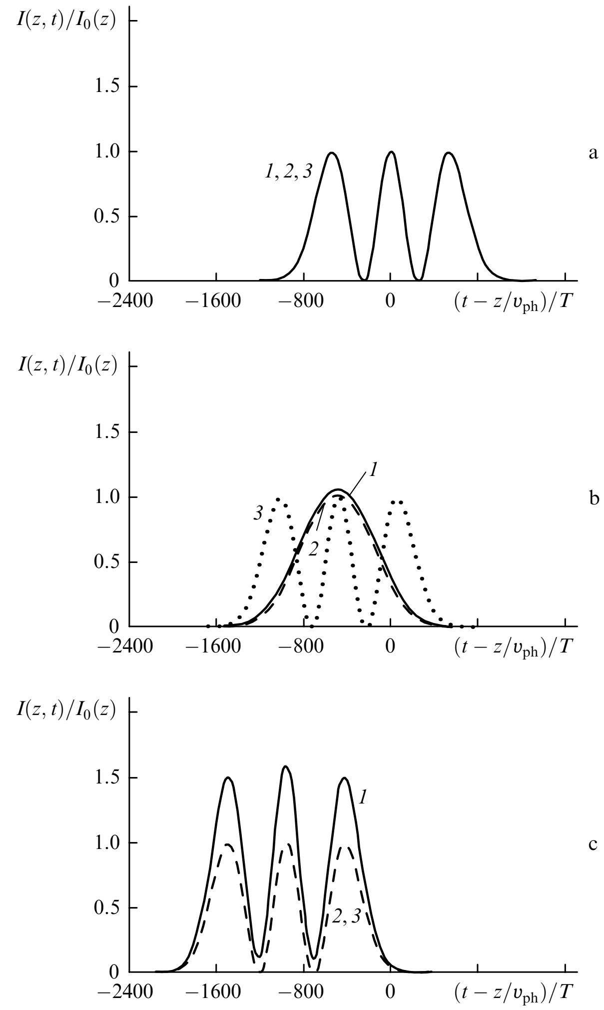

Figure 3. Distortion of the initial three-peak signal (5) ($T = 400\tau_{coh}$, $x_0 = \Omega_0\tau_{coh} = 2$) propagating in an amplifying medium for $\xi = 0$, $\tau = 0$ (a); $\xi = 4000$, $\tau = (-460 + 640i)\tau_{coh}$ (b); and $\xi = 8000$, $\tau = (960 + 1280i)\tau_{coh}$ (c). The notation of the curves is as in Fig. 1. Curve (*3*) is plotted at a different scale compared to curves (*1*) and (*2*).

is illustrated in Fig. 3 for signal (5). The notation in this figure is the same as in Figs 1 and 2. Note that curve (*3*) in Fig. 3, which was calculated in the real group velocity approximation, is plotted at a different scale[12] (for the convenience of comparing the signal shapes).

One can see that the initial three-peak signal (see Fig. 3a) is virtually completely 'normalised' over the path length $\xi = 4000$, which does not prevent it to acquire again the three-peak shape when $\xi = 8000$. It is clear [see (9)] that such a behaviour of the signal is related to the periodicity of an exponential as a function of the imaginary part of its argument.

5. Let us summarise the results of the study:

(i) A narrow-band non-Gaussian signal is strongly distorted during its propagation in a medium with a high dispersion of the gain or absorption. This distortion can be easily taken into account in the complex delay time approximation. The real part of the delay time characterises the delay (or advance) of the signal, and the imaginary part determines its distortion.

(ii) The manifestation of the same imaginary part of the signal delay time depends substantially on the shape of its envelope. In particular, the amplitude-modulated signal, whose complex envelope has the centre of symmetry, is distorted, but remains symmetric and its centre propagates at the real group velocity. Therefore, the real group velocity (subluminal, superluminal or negative) in the amplifying or absorbing dispersion medium retains the meaning of the propagation velocity of the centre of symmetry of the signal, which is distorted during its propagation remaining symmetrical.

(iii) In the complex delay time approximation, a Gaussian wave packet (unlike a non-Gaussian packet) propagates without a change in the envelope shape and duration. Therefore, its propagation along a path without distortions does not warrant at all the undistorted transmission (along the same path) of a signal of different shape. In other words, the propagation velocity of a Gaussian

[12] All the curves in Figs 1 and 2 are normalised to the intensity at the signal centre, which was calculated in the complex delay time approximation. In Fig. 3, only the dependences calculated numerically or in the complex delay time approximation were normalised in this way. The time dependence of the signal calculated in the real group velocity approximation was normalised to its own intensity at the signal centre. Otherwise (see Figs 1 and 2), it would be difficult to compare it with the dependences calculated numerically and in the complex delay time approximation because of a too great difference in their amplitudes.

signal (including the superluminal and negative velocities) has its own meaning consisting in the fact that this velocity can exist even when the concept of the group velocity of a signal with an arbitrary shape of the envelope cannot be used.

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