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On self-focusing and defocusing of few-cycle pulses in hydrogen-containing ferroelectrics

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***Abstract*. The role of diffraction in the propagation of few-cycle pulses is studied in hydrogen-containing ferroelectrics in the case of the active role of optical electronic and tunnelling transitions. It is shown that, unlike optical electronic transitions, tunnelling proton transitions can produce the defocusing effect if they are overlapped by the spectral components of the pulse. The parameters of a medium and the pulse are estimated at which the defocusing effect dominates over self-focusing. It is shown that nonresonance quasi-monochromatic pulses in the optical range are subjected to self-focusing in such media.**

Keywords: *few-cycle pulse, tunnelling transition, self-focusing, defocusing.*

1. Introduction

Self-focusing of optical pulses is one of the main effects preventing the use of solid homogeneous dielectrics in optical fibres for fibreoptic communication systems. This effect can be compensated for, as a rule, with the help of dopants distributed inhomogeneously and axially symmetric in the cross-sectional area of a glass fibre [1 – 3]. Self-focusing can change to defocusing in transparent solvents after the addition of absorbing dyes into them [4].

The optics of few-cycle pulses (FCPs) of duration down to one cycle of electromagnetic oscillations, for which the concept of a carrier frequency is no longer applicable, has been extensively developed in the last years [5 – 7]. The properties of the final stage of self-focusing of FCPs were studied in Ref. [8], where dumbbell-like structures appearing in the positive-dispersion region were analysed in detail and the self-division effect accompanying self-focusing of FCPs was investigated.

The spectral width $\delta\omega$ of the FCP is comparable with its central frequency and for the pulse duration $\tau_p \sim 10$ fs achieves the value of $\sim 1/\tau_p \sim 10^{14}$ s^{-1}. Therefore, the spectrum of FCPs well overlaps the frequencies of normal vibrational and vibronic modes [9], as well as of tunnelling transitions in hydrogen-containing ferroelectrics [10]. It was shown in Ref. [10] that high-power broadband electromagnetic FCPs can efficiently propagate in a soliton regime in the system of strongly interacting tunnelling transitions at the temperature close to the temperature of the ferroelectric phase transition, where the absorption of weak monochromatic waves is usually strong. It was also shown in Ref. [10] that these pulses are stable with respect to self-focusing. At the same time, the FCP spectrum lies, as a rule, in the transparency region for electronic transitions. The nonlinear refractive index n_2 of solid dielectrics in this region is usually positive, which results in strong self-focusing.

The question arises of whether tunnelling transitions in ferroelectrics interacting with FCPs can compensate for self-focusing caused by optical electronic transitions occurring in the ferroelectric. A positive answer to this question can substantially simplify the manufacturing technology of fibres for fibreoptic communication systems using FCPs for the data transfer. In this case, there would be no need to produce artificially the inhomogeneity across the fibre cross section with the help of dopants, and the tunnelling quantum transitions between the proton states in a two-well potential, which are inherent in crystals of the KDP type, can play the role of transitions in absorbing dyes. This paper is devoted to the study of this question.

2. Formulation of the model and basic equations

Consider a ferroelectric of the KDP type, in which tunnelling proton transitions can occur between the minima of two-well crystal potentials stimulating the displacement of heavy potassium ions along the ferroelectric axis [11]. The tunnelling of protons occurs in a perpendicular plane. In addition, the ferroelectric has the electron-optical susceptibility with distinct nonlinear properties in the transparency region.

Consider a pulse propagating along the optical axis, which coincides with the ferroelectric axis. In this case, first, tunnelling transitions will occur and, second, the quadratic electron-optical nonlinearity will be absent. Consider the case when the FCP spectrum lies in the transparency region for electronic transitions.

The Maxwell equation for a FCP propagating in such a medium has the form

$$\Delta E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}, \quad (1)$$

where c is the speed of light; E is the pulse-field strength;

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$$P = P_t + P_e \tag{2}$$

is the macroscopic polarisation; P_t is the contribution to polarisation from tunnelling proton transitions; and P_e is the electronic polarisation.

Under the condition

$$\omega_0 \tau_p \ll 1 \tag{3}$$

(ω_0 is the quantum tunnelling frequency, τ_p is the input pulse duration), P_t satisfies the equation [10, 12–14]

$$\frac{\partial P_t}{\partial t} = N d \omega_0 W \sin\theta, \tag{4}$$

where N is the concentration of tunnelling protons; d is the tunnelling transition dipole moment; W is the initial difference of populations of the symmetric and asymmetric proton states; and

$$\theta = \frac{d}{\hbar}\int_{-\infty}^{t} E \mathrm{d}t. \tag{5}$$

Because electronic transitions with the characteristic frequency ω_e lie in the transparency region, i.e., [9]

$$\omega_e \tau_p \gg 1, \tag{6}$$

the expansion

$$P_e = \chi E + \chi_3 E^3 - \kappa \frac{\partial^2 E}{\partial t^2}, \tag{7}$$

is valid for the electronic polarisation [9, 12–16], where χ and χ_3 are the linear and cubic nonlinear inertialess susceptibilities and $\kappa \equiv 0.5(\partial^2\chi/\partial\omega^2)|_{\omega=0}$ is the parameter of the electronic group dispersion in the low-frequency region. Because electronic transitions in the transparency region have the normal dispersion and facilitate self-focusing, we have $\chi_3 > 0$ and $\kappa > 0$.

By substituting (2)–(7) into (1) and using the approximation of quasi-unidirectional propagation along the optical axis [17], we obtain the nonlinear wave equation describing the FCP dynamics

$$\frac{\partial^2\theta}{\partial z \partial \tau} + a\sin\theta + b\frac{\partial}{\partial\tau}\left(\frac{\partial\theta}{\partial\tau}\right)^3 - g\frac{\partial^4\theta}{\partial\tau^4} = \frac{c}{2n}\Delta_\perp\theta. \tag{8}$$

Here, $\tau = t - nz/c$; $n = (1 + 4\pi\chi)^{1/2}$ is the linear refractive index of the medium; $a = 2\pi d^2 N\omega_0 W/(\hbar c n)$; $b = 2\pi\hbar^2\chi_3 \times (ncd^2)^{-1}$; $g = 2\pi\kappa/(nc)$; and $\Delta_\perp$ is the transverse Laplacian. We have obtained (8) by neglecting the dispersion of the electronic nonlinearity as the effect of a higher order of smallness in approximation (6).

In the absence of tunnelling transitions and diffraction, expression (8) transforms to the modified Korteweg–de Vries equation for E, which has no soliton and breather equations in our case ($b > 0$, $g > 0$). Because $\chi_3 > 0$, electronic transitions promote self-focusing of pulses. In the absence of electronic transitions, equation (8) transforms to the sine-Gordon equation, whose solitons can be stable with respect to transverse perturbations [10]. Thus, equation (8) contains contributions from electronic and tunnelling transitions, which produce for their solitons the focusing and defocusing effects, respectively. Therefore, the study of dynamics of FCPs using equation (8) can shed light on the problem of diffraction of such pulses in the nonlinear regime.

3. Approximate analysis of the dynamics of a soliton-like pulse in the eikonal approximation

We will find the approximate solution of equation (8) by the method of the averaged Ritz–Whitham Lagrangian [18, 19]. To equation (8), the Lagrangian density

$$L = \frac{1}{2}\frac{\partial\theta}{\partial z}\frac{\partial\theta}{\partial\tau} - a(1-\cos\theta) + \frac{b}{4}\left(\frac{\partial\theta}{\partial\tau}\right)^4 + \frac{g}{2}\left(\frac{\partial^2\theta}{\partial\tau^2}\right)^2 - \frac{c}{4n}(\nabla_\perp\theta)^2 \tag{9}$$

corresponds. We will take a trial solution of equation (8) in the form of a solitary FCP based on the following reasoning. In the case of quasi-soliton propagation of the pulse, the states of the medium in front of the pulse and behind it are identical. Due to condition (3), the FCP spectrum overlaps tunnelling transitions (i.e., it contains Fourier components that are resonant with tunnelling transitions). Therefore, during the FCP propagation, a complete population inversion should occur in the tunnelling subsystem accompanied by its returning to the initial state. For the electronic transitions, the situation is opposite because of (6), and, hence, a change in the population of electronic levels is comparatively weak. These conditions can be satisfied by taking a trail solution in the form [20, 21]

$$\theta = 4\arctan\{\exp[\rho(\tau - \Phi)]\}, \tag{10}$$

where $\rho(z, r_\perp)$ and $\Phi(z, r_\perp)$ are the sought-for functions of coordinates. The function Φ is a soliton eikonal [15, 16], while ρ has the meaning of the inverse duration proportional to the pulse amplitude, which becomes clear from the expression for the FCP electric field

$$d\frac{E}{\hbar} = \frac{\partial\theta}{\partial\tau} = 2\rho\,\mathrm{sech}[\rho(\tau - \Phi)]. \tag{11}$$

We will assume below that the FCP profile at the entrance to a medium (for $z = 0$) has the form (11), where $\rho = \rho_0 = 1/\tau_p$.

By substituting (10) into (9) and integrating the expression obtained over the 'fast' variable τ, we find the 'averaged Lagrangian'

$$\Lambda = \frac{1}{4}\int_{-\infty}^{\infty} L\mathrm{d}\tau = -\rho\frac{\partial\Phi}{\partial z} - \frac{a}{\rho} + \frac{b}{3}\rho^3 + \frac{g}{3}\rho^3 - \frac{1}{2}\frac{c}{n}\rho(\nabla_\perp\Phi)^2 - \frac{\pi^2 c}{24n}\frac{\nabla_\perp\rho}{\rho^3}. \tag{12}$$

By varying Λ over Φ and ρ, we write the corresponding Euler–Lagrange equations in the form

$$\frac{\partial\Phi}{\partial z} + \frac{V_\perp^2}{2} - \frac{ca}{n\rho^2} - \frac{c}{n}(g + 4b)\rho^2 =$$

$$= \frac{\pi^2}{12\rho^3}\frac{c^2}{n^2}\left[\Delta_\perp\rho - \frac{3}{2\rho}(\nabla_\perp\rho)^2\right], \tag{13}$$

$$\frac{\partial\rho}{\partial z} + \nabla_\perp(\rho \boldsymbol{V}_\perp) = 0, \tag{14}$$

where $\boldsymbol{V}_\perp = \nabla_\perp \Phi$.

In the one-dimensional approximation ($\Delta_\perp = 0$), we find from (13) and (14) that $\rho = \rho_0 = 1/\tau_p = \text{const}$, $v_\perp = 0$, and $\Phi = z(1/v - n/c)$, where the propagation velocity v in the laboratory system is determined by the relation

$$\frac{1}{v} = \frac{n}{c} + \frac{c}{n}\left[a\tau_p^2 + \frac{g+4b}{\tau_p^2}\right]. \tag{15}$$

In the absence of an electron subsystem ($g = b = 0$), solution (10), (11), (15) coincides, as expected, with the exact solution of the sine-Gordon equation. The electronic transitions further slow down the pulse propagation.

The first part of equation (13) describes diffraction in the transverse dynamics of the pulse. The neglect of the right-hand side of (13) corresponds to the eikonal approximation (geometrical optics approximation) for soliton-like pulses [19]. In this case, the system (13), (14) takes the form of the hydrodynamic equations for an ideal liquid [where (13) is the Cauchy integral and (14) is the continuity equation].

Consider this case in more detail. By comparing (13) with the hydrodynamic Cauchy integral [22], we obtain the equality

$$\int\frac{\mathrm{d}p}{\rho} = -\frac{c}{n\rho^2} - \frac{c}{n}(g+4b)\rho^2,$$

where p is the 'pressure' and ρ is the 'density' of a hypothetical ideal liquid. By differentiating the latter relation with respect to ρ, we obtain

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{2c}{n}\left[\frac{a}{\rho^2} - (g+4b)\rho^2\right].$$

It is obvious that the stability criterion for the FCP (11) corresponds to the condition $\mathrm{d}p/\mathrm{d}\rho > 0$ of the liquid flowing [10, 20, 21, 23, 24]. Then, the input value $\rho = \rho_0$ should satisfy the inequality

$$\rho_0 \equiv \frac{1}{\tau_p} < \rho_c^e \equiv \left(\frac{a}{g+4b}\right)^{1/4}, \tag{16}$$

where ρ_c^e is the critical value of the inverse pulse duration, which divides the self-focusing and defocusing regimes neglecting diffraction. The critical pulse duration is defined as $\tau_c = 1/\rho_c^e$. For $\tau_p < \tau_c$, defocusing occurs, and in the opposite case – self-focusing.

Therefore, the eikonal approximation restricts the pulse amplitude and duration. We will present the corresponding numerical estimates taking diffraction into account. Condition (16) is necessary but not sufficient (see below) for the FCP not to experience self-focusing.

One can see from (15) that, when condition (16) is fulfilled, the velocity v monotonically increases with the FCP amplitude. For this reason, the parts of the pulse front with a greater amplitude, corresponding to the centre of the FCP cross section, leave behind during the pulse propagation the parts where the amplitude is smaller. Due to this defocusing effect, the pulse can take the form of an 'electromagnetic shell' or a 'light bullet' [25]. If condition (16) is not fulfilled, we arrive at the opposite situation, when the FCP self-focusing finally occurs.

4. Consideration of diffraction

Consider now the influence of diffraction [the right-hand side of (13)] on the FCP dynamics. System (13), (14) slightly differs from the corresponding system describing the transverse dynamics of a continuous light beam [26]. We assume that the transverse structure of the FCP is axially symmetric. By writing (13) and (14) in the cylindrical coordinate system z, r, we will seek the approximate solution for ρ in the self-similar form [26]

$$\rho(z,r) = \rho_0\frac{R_0^2}{R^2(z)}\exp\left[-\frac{r^2}{R^2(z)}\right], \tag{17}$$

where R_0 is a constant equal to the FCP input radius and $R(z)$ is the current radius.

Below, we are interested in the axial dynamics ($r^2/R^2 \ll 1$) of the pulse. Therefore, we will write the solution for Φ in the form of the expansion

$$\Phi(z,r) = f_1(z) + {}^1/_2 f_2(z) r^2 + \ldots. \tag{18}$$

By substituting (17) and (18) into (13) and (14) and retaining the terms of the order of $\sim r^2/R^2$, we obtain, by equating expressions with the same powers of r,

$$f_2 = \frac{R'}{R}, \tag{19}$$

$$f_2' + f_2^2 = 4AR^2 - \frac{4B}{R^6} - 5D, \tag{20}$$

$$f_1' = AR^4 + \frac{B}{R^4} - DR^2, \tag{21}$$

where $A = ac/(n\rho_0^2 R_0^4)$; $B = (c/n)(g+4b)\rho_0^2 R_0^4$; $D = \pi^2 c^2 \times (3n^2\rho_0^2 R_0^4)^{-1}$; the prime denotes the derivative with respect to z; the last terms in (20) and (21) take into account the deviation from the geometrical optics approximation.

By substituting (19) into (20), we obtain the equation

$$R'' = -\frac{\partial U}{\partial R} \tag{22}$$

for the FCP radius, which formally coincides with the equation of motion for a Newtonian unit-mass particle in an external field with the potential energy

$$U(R) = -AR^4 - \frac{B}{R^4} + \frac{5D}{2}R^2. \tag{23}$$

The first integral in (22) has the form

$$\frac{(R')^2}{2} + U(R) = \Sigma, \tag{24}$$

where the constant $\Sigma = -AR_0^4 - b/R_0^4 + 5DR_0^2/2$ is determined from the input condition $R'(0) = 0$ [see (19); $f_2(0) = 0$ because the input pulse has a plane front].

By integrating (24), we can represent the solution $R(z)$ in quadratures. On the other hand, by analysing the curve $U(R)$, we can make qualitative conclusions about the behaviour of the radius of the FCP propagating in a medium.

The last term in the right-hand side of (23) takes into account diffraction effects in the transverse dynamics of the FCP. This part of $U(R)$ is similar to the potential energy of a harmonic oscillator, the difference being that here $R \geqslant 0$. Therefore, unlike the case of continuous beams with a distinct carrier frequency, diffraction enhances self-focusing of FCPs. This difference can be explained in the following way. The dimensionless parameter is $\varepsilon = \lambda/R$, where λ is the characteristic wavelength determining the degree of influence of the wave properties of the pulse on its dynamics. For $\varepsilon \ll 1$, the eikonal approximation is fulfilled. If $\varepsilon \sim 1$, the wave properties (diffraction) should be taken into account. During self-focusing of a monochromatic beam, its wavelength λ almost does not change, whereas $R \to 0$, which results in an increase in ε and in the role of diffraction. Therefore, due to diffraction, self-focusing of the beam can change to its divergence, if the beam intensity is lower than a threshold value [26]. The FCP has no carrier frequency, and its characteristic size along the propagation direction plays the role of $\lambda \sim \upsilon\tau = \upsilon/\rho$, where υ is the pulse velocity, which only slightly differs from c/n in the quasi-unidirectional approximation adopted here. Then, using (17), we obtain $\varepsilon \sim cR/(n\rho_0 R_0^2)$ at the centre of the FCP cross section. One can see from here that $\varepsilon \to 0$ upon self-focusing, and the eikonal approximation is fulfilled even better, while the relative role of diffraction decreases.

Before analysing equation (24) in the general form, we consider the case when the contribution of electronic transitions can be neglected. Then, $B = 0$, and the FCP dynamics is described by the sine-Gordon equation [see (10) for $b = g = 0$). This case was analysed in Ref. [10] using the eikonal approximation.

One can see from (22) that defocusing is possible when $(\partial U/\partial R)_{R=R_0} < 0$. This and (22) give

$$R_0 > R_c = \pi\left(\frac{5c}{12na}\right)^{1/2}, \tag{25}$$

where R_c is the critical value of the input FCP radius separating the defocusing ($R > R_c$) and self-focusing ($R < R_c$) regimes. In the opposite case, a soliton in the sine-Gordon equation experiences self-focusing. Note that the eikonal approximation used in Ref. [10] gives only the latter effect. Therefore, the conclusion of paper [10] about the stability of the soliton with respect to self-focusing is valid for pulses with a sufficiently large cross section. Let us estimate the numerical value of R_c. By taking $d \sim 10^{-18}$ cgs units, $N \sim 10^{21}$ cm^{-3}, $\omega_0 \sim 10^{13}$ s^{-1}, $n \approx 1$, we obtain $a \sim 10^{15}$ s^{-1}cm^{-1}. Then, we find from (25) $R_c \sim 0.1$ mm.

For $R_0 > R_c$, the solution of equation (24) can be written in the form

$$R = \begin{cases} \dfrac{R_0}{\operatorname{sn}[K(q_i) - z/l_1, q_1]}, & R_c < R_0 \leqslant \sqrt{2}R_c, \\ \dfrac{R_0}{\operatorname{cn}(z/l_2, q_i)}, & R_0 \geqslant \sqrt{2}R_c, \end{cases} \tag{26}$$

where $i = 1, 2$; $K(q_{1,2})$ is the total elliptic integral of the first kind; sn and cn are elliptic sine and cosine, respectively; $l_1 = nR_cR_0/[\pi(10)^{1/2}c\tau_p]$; $l_2 = nR_cR_0/\{2\pi c\tau_p[5(1 - R_c^2 \times R_0^2)]^{1/2}\}$; $q_1 = 2R_0^2/R_c^2 - 1$; $q_2 = 0.5(1 - 2R_c^2/R_0^2)/(1 - R_c^2/R_0^2)$.

Solution (26) is singular, i.e., R tends to infinity at a finite value $z = l_{df} = K(q_{1,2})\, l_{1,2}$. The FCP defocusing occurs as if explosively. By taking $R_0 \sim R_c \sim 0.1$ mm, $\tau_p \sim 10^{-14}$ s, and $n \sim 1$, we find $l_{df} \sim 1$ cm. Note, however, that for $z \to l_{df}$, the approximation of quasi-unidirectional propagation used above is no longer valid. Indeed, one can see from (26) and (17)–(20) that $\rho \to 0$ and $f_1', f_2' \to \infty$ for $R \to \infty$ $(z \to l_{df})$.

Then, the FCP propagation velocity is $\upsilon = (n/c + \partial\Phi/\partial z)^{-1} \to 0$ at all the points in space. However, the approximation of quasi-unidirectional propagation is valid if the pulse velocity is close to c/n. Therefore, this approximation is invalid for $z \to l_{df}$ and it is necessary to find other methods. For $R_0 < R_c$, solution (24) has the form

$$R(z) = R_0 \operatorname{sn}\left[K(q_3) + \frac{z}{l_3}, q_3\right], \tag{27}$$

where

$$l_3 = \left(\frac{6}{5}\right)^{1/2} \frac{qn}{\pi} \frac{R_0 R_c}{c\tau_p}; \quad q_3 = \left[2\left(\frac{R_c}{R_0}\right)^2 - 1\right]^{-1/2}.$$

Because the radius R is positive, the coordinate z in (27) can lie in the interval $0 < z < l_f$, where the focusing lens $l_f = l_2 K(q)$. For the parameters of the medium and pulse adopted above, we find $l_f \sim 1$ cm.

It follows from (17) and (26) that $\rho \to \infty$ for $z \to l_f$, which corresponds to a strong self-compression of the pulse and its peak amplification. One can see from (18) and (20) that for $z \to l_f$ the velocity of axial propagation of the FCP is

$$\upsilon = \left(\frac{n}{c} + \frac{\partial\Phi}{\partial z}\right)^{-1} \to \frac{c}{n},$$

i.e., increases. These conclusions do not contradict to the general properties of solitons, according to which a decrease in the duration is accompanied by the increase in the amplitude and velocity. On the other hand, it is clear that electronic transitions should be taken into account at the final stage of self-focusing, when the amplitude of a soliton strongly increases.

Consider the FCP dynamics taking electronic transitions into account. It follows from (22) and (23) that defocusing $[(\partial U/\partial R)_{R=R_0,\rho=\rho_0} < 0]$ can be achieved when the input-pulse parameters satisfy the condition

$$\rho_0 = \frac{1}{\tau_p} < \rho_c^e \left(1 - \frac{R_c^2}{R_0^2}\right)^{1/4}. \tag{28}$$

Therefore, the consideration of the electron subsystem along with tunnelling transitions leads to two conditions for a stable propagation of the FCP: for its input radius (25) and amplitude (duration) (28). In the eikonal approximation, ($R_0 \gg R_c$), condition (28) transforms to (16). In addition, note that criterion (25) follows naturally from (28) as the requirement for the integrand to be nonnegative.

Let us estimate the parameters of the pulse corresponding to its stable propagation. By taking $\chi_3 \sim 10^{-13} - 10^{-14}$

cgs units [27], $\kappa \sim \chi/\omega_e^2$, $\chi \sim 0.1$, $\omega_e \sim 10^{15}$ s^{-1}, and the values of n, a, and d used above, we obtain $\rho_c^e \sim 10^{14}$ s^{-1}. Therefore, the input-pulse duration should exceed 10 fs. The pulse intensity corresponding to ρ_c^e is estimated as $I_c \sim cE^2/4\pi \sim c\hbar^2(\rho_c^e)^2/(4\pi d^2) \sim 10^{12} - 10^{13}$ W cm^{-2}. This estimate corresponds to the maximum FCP intensity.

Note that the known nonlinear Schrödinger equation can be obtained under natural nonresonance conditions $dE/\hbar \ll \omega \ll \omega_e$ from (8) in the limit of quasi-monochromatic pulses with the carrier frequency ω. Under these conditions, tunnelling transitions are only weakly excited, and, therefore, we can assume that $\sin\theta \approx \theta - \theta^3/6$. Assuming that

$$d\frac{E}{\hbar} = \psi \exp[\mathrm{i}(\omega\tau - kz)] + \mathrm{c.c.},$$

where $k = -a/\omega + g\omega^3$ is the wave number in the co-moving coordinate system, using the expansion [1, 5, 15, 16]

$$\int_{-\infty}^{\tau} \psi \exp[\mathrm{i}(\omega\tau' - qz)]\mathrm{d}\tau' \approx \left(-\mathrm{i}\,\frac{\psi}{\omega} + \frac{1}{\omega^2}\frac{\partial\psi}{\partial\tau} + \frac{\mathrm{i}}{\omega^3}\frac{\partial^2\psi}{\partial\tau^2} \right) \exp[\mathrm{i}(\omega\tau - qz)] + \mathrm{c.c.}$$

and neglecting comparatively rapidly oscillating terms, we obtain from (8), taking dispersion and nonlinearity into account in a minimal degree, the equation

$$\mathrm{i}\,\frac{\partial\psi}{\partial z} - \alpha\,\frac{\partial^2\psi}{\partial T^2} - \beta|\psi|^2\psi = \frac{c}{2n\omega}\,\Delta_\perp\psi, \qquad (29)$$

where $T = t - z/v_g$; $v_g = (n/c + a/\omega^2 + 3g\omega^2)^{-1}$ is the group velocity; $\alpha = 3g\omega(\omega_c^4/\omega^4 - 1)$; $\omega_c = (a/3g)^{1/4}$ [5]; and $\beta = a/(2\omega^3) + 3b\omega$. Because a, $b > 0$, we have $\beta > 0$, and equation (29) has one-dimensional soliton solutions in the region of the anomalous group dispersion corresponding to $\alpha > 0$ or $\omega < \omega_c$. On the other hand, β is proportional to $\chi_3 > 0$ [1]. Therefore, nonresonance quasi-monochromatic solitons of the envelope should experience self-focusing, which usually occurs in the transparency region of solids.

5. Conclusions

We have found that tunnelling transition play a fundamental role in the transverse dynamics of FCPs. Under conditions (25) and (28) for the input parameters of a unipolar FCP, the pulse defocusing can be observed in a homogeneous medium. As mentioned above, equation (8) has no exact analytic solutions in the form of solitary pulses. The exclusion is the case $g = -2b$ corresponding to the integrated equation Konno – Kamiyama – Sanuki [28]. Therefore, the choice of a trial solution in the form (10) is somewhat arbitrary. At the same time, we believe that the physical arguments presented in section 3 in the choice of the trail solution are quite convincing. Therefore, we have every reason to believe that trial solution (10) corresponds at least qualitatively to a real physical process of the pulse propagation in a nonlinear medium. In any case, the pulse spectrum should contain Fourier components, which are resonant with tunnelling transitions that can provide the defocusing effect. This conclusion shows that there exist fundamentally new possibilities for overcoming self-focusing in the case of FCPs compared to conventional methods for producing the transverse inhomogeneity in optical fibres applied in the optics of quasi-monochromatic pulses. It is important that tunnelling transitions are inherent in crystals of the KDP type, and therefore no artificial methods are required for doping resonance impurities, which are used to obtain defocusing of pulses with a distinct carrier frequency.

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