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Tunable astigmatic π/2 mode converter

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***Abstract*. The scheme of a tunable astigmatic π/2 mode converter is described. The converter provides the use of input beams with the twofold variable Rayleigh length, while variations in the optical length of the converter itself do not exceed 1/6.**

Keywords: *π/2 mode converter, astigmatic optics, tuning of parameters.*

1. Introduction

It is known that the high-quality conversion of Hermite – Gaussian (HG) modes to donut Laguerre – Gaussian (LG) modes is performed with an astigmatic π/2 converter upon strict matching of the laser-beam parameters with the optical parameters of the converter [1]. If the matching condition is violated, elliptic LG beams are obtained in the best case [2], whose properties satisfy practical applications only within a limited spatial region. The quality of matching between the initial HG mode and the converter can be characterised by the beam-propagation length over which a decrease in the 'intensity barrier' for the resulting LG mode around the axis occurs by no more than some specified value [3]. A beam for which this propagation length is equal to one or several Rayleigh lengths $z_R = \pi w_0^2/\lambda$ (w_0 is the beam radius, λ is the radiation wavelength) can be considered perfect. The fulfilment of this condition is quite challenging, first of all from the practical point of view, because both precise measurements of the initial-beam parameters and precise manufacturing of optical elements, especially lenses with cylindrical surfaces, are required.

The problem is aggravated when it is necessary to change the wavelength of an incident laser beam. It is obvious that, when the wavelength of the laser beam or its size are varied, another set of optical elements is required for the π/2 converter. A disadvantage of holographic converters [4] is the necessity of using special hologram for each index l of the LG mode. The matching problem would be substantially alleviated if the parameters of required optical elements could be varied continuously rather than discretely (with the help of several available optical elements). In principle, there exist photographic objectives with a variable focal length, which can be used, in particular, to build continuously variable beam expanders (however, the problem of radiation resistance of their glued components exists in the case of high-power laser beams). A simplified version of such an objective consisting of two spherical lenses was already proposed for the use in a tunable fractional Fourier-transformer [5]. This is also possible in the case of cylindrical lenses. However, the scheme of a π/2 converter [6] and other schemes considered in Refs [7, 8] require at least three such composite lenses with a variable focus. This results in a substantial complication of the optical scheme and its adjustment. The optical length of the scheme also noticeably increases.

In this paper, the scheme of a π/2 mode converter is proposed in which only one composite doublet of the type described in Ref. [5] is used. The focal distance of the cylindrical components of the converter can be varied by rotating only some of them around the optical axis of the converter. This is achieved due to the specific optical scheme of the π/2 converter we selected.

2. Elements of a tunable π/2 mode converter

The scheme shown in Fig. 1 is proposed as the basic scheme of a tunable π/2 mode converter. The assemblies of cylindrical positive and negative lenses with 'crossed' powers $\pm 1/F$ (the generatrices of their cylindrical surfaces are orthogonal) are placed in the input and output reference planes of the converter. The distance between the assem-

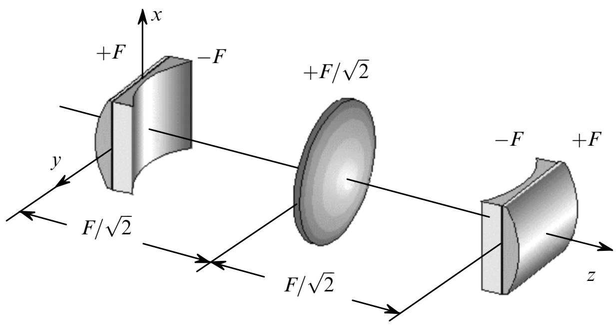

Figure 1. Optical scheme of a tunable π/2 mode converter (lens positions correspond to the Rayleigh length $z_R = F$ of the input beam).

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blies is $\sqrt{2}F$. A spherical lens with the focal distance $F/\sqrt{2}$ is placed exactly in the middle between them. This device represents a π/2 converter based on the fractional Fourier transform (FrFT) with orders 1/2 and 3/2 in mutually orthogonal planes for a beam with $z_R = F$ and a plane wave front at the converter input. The FrFT converters are described in detail in Ref. [8]. To vary the parameters of the converter shown in Fig. 1, it is necessary to change consistently the powers of all three optical elements.

2.1 Variation of the spherical-lens parameters

As mentioned in Introduction, a spherical lens with a variable focal distance can be made in the form of a doublet consisting of two identical (in the simplest case, thin) lenses L and L′. The effective focal distance of the doublet is

$$F = \frac{f^2}{2f - d}, \tag{1}$$

where f is the focal distance of an individual lens and d is the distance between the lenses (Fig. 2). The position of the front (H) and rear (H′) principal points with respect to the corresponding thin lenses is determined by the expression

$$s_H = s_{H'} = \frac{fd}{2f - d}. \tag{2}$$

The distance to the corresponding input and output cylindrical elements of the converter in Fig. 1 should be measured from these principal points. A useful property of a composite lens is the inverted arrangement of principal points, i.e., the front point is located behind the rear point along the beam propagation direction. This substantially reduces the total length of the converter.

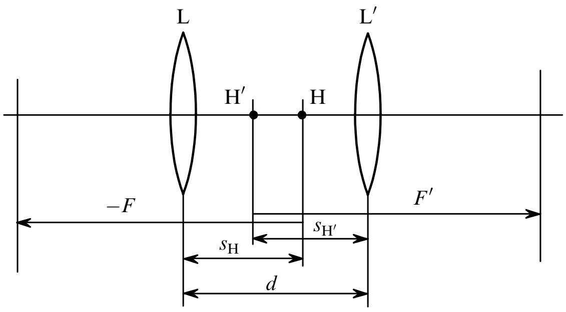

Figure 2. Scheme of the adjustment of the focal length of a spherical lens (doublet) in a π/2 converter.

In our case, according to the scheme in Fig. 1, it is necessary to set the focal distance of thin lenses equal to $f = f_{sph} = F\sqrt{2}$.

2.2 Variation of the focal distance of cylindrical elements

Consider a thin cylindrical lens with the power $1/f$ and an adjacent lens with the power of the opposite sign $-1/f$, which can be rotated around the optical axis z (Fig. 3a). It is convenient to calculate this system of lenses by the methods of tensor $ABCD$ optics [9]. In this case, we have for the first lens with the power $1/f$, which acts along the y axis, the tensor

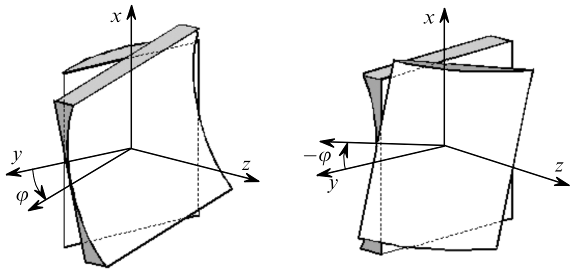

Figure 3. Components of the astigmatic device for forming a saddle-like wave front of the beam with a variable curvature. Each of the components has one immobile lens and one lens rotatable around the optical axis of a cylindrical lens. The rotatable lenses are rotated in opposite directions.

$$T_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{pmatrix}, \tag{3}$$

where the elements describing the action of the cylindrical lens in planes xz and yz are located on the major diagonal of the 2×2 submatrices A, B, C, D. Then, we can write for the second lens with the power $-1/f$ acting along the x axis and rotated through the angle φ with respect to the y axis

$$T_2 = \begin{pmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/f & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ C^2/f & CS/f & 1 & 0 \\ CS/f & S^2/f & 0 & 1 \end{pmatrix}, \tag{4}$$

where $C = \cos\varphi$ and $S = \sin\varphi$. For two cylindrical lenses in series, we obtain the tensor

$$T_{12} = T_2 T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ C^2/f & CS/f & 1 & 0 \\ CS/f & -C^2/f & 0 & 1 \end{pmatrix}. \tag{5}$$

Consider now another, similar system of cylindrical lenses (Fig. 3b), in which an immobile lens with the power $-1/f$ acts along the x axis. We will rotate a supplementary lens with the power $1/f$ in the direction opposite to the rotation of the mobile lens of the first system. For this pair of lenses, we obtain the $ABCD$ tensor

$$T'_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ C^2/f & -CS/f & 1 & 0 \\ -CS/f & -C^2/f & 0 & 1 \end{pmatrix}. \tag{6}$$

By placing both systems of lenses directly behind each other (in any order), we obtain the tensor

$$T = T_{12}T'_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2C^2/f & 0 & 1 & 0 \\ 0 & -2C^2/f & 0 & 1 \end{pmatrix}, \qquad (7)$$

which describes the device that can change the curvature of the saddle-like wave front from zero to $\pm 2/f$. In other words, we have an analogue of two cylindrical lenses with a variable focal distance $f^*(\varphi) = \pm f/(2\cos^2\varphi)$ and orthogonal powers (optical quadrupole [10]). To use the assembly of four cylindrical lenses described by tensor (7) as the input and output elements in the $\pi/2$ converter shown in Fig. 1, we should set $f = f_{cyl} = 2F$.

3. Basic parameters of a tunable $\pi/2$ mode converter

The diagram in Fig. 4 shows the positions of all the elements of a tunable $\pi/2$ mode converter. On one ordinate the Rayleigh length z_R divided by F (a constant for a given device) is plotted, and on the another – the required rotation angle φ of rotatable components in assemblies of cylindrical lenses. The abscissas of points on the solid curves (for each pair of values of z_R/F and φ) give positions of the input (B) and output (B$'$) assemblies of cylindrical lenses with respect to the centre of the device. The abscissas of points on the curves with dark triangles and circles correspond to the positions of elements L and L$'$ of the adjustable spherical lens. The curves with empty triangles and circles show the positions of the front (H) and rear (H$'$) principal points of the spherical lens.

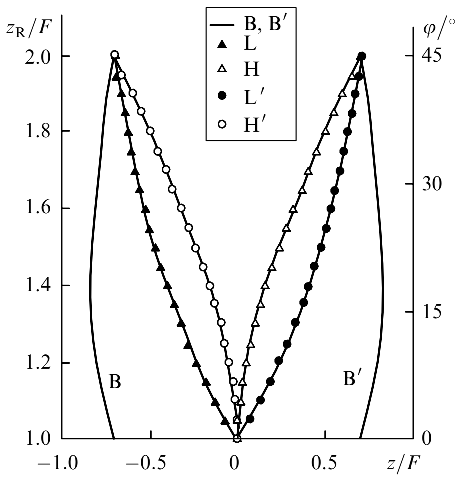

Figure 4. Diagram of positions of the elements of a tunable $\pi/2$ mode converter as functions of the Rayleigh length z_R and the rotation angle φ of cylindrical lenses (see text).

In the initial position ($\varphi = 0$), $z_R/F = 1$, and the equivalent scheme of the converter corresponds to Fig. 1. The maximum optical length of the $\pi/2$ converter achieved at $z_R/F = \sqrt{2}$ ($\varphi \approx 37.8°$) is $\sim 1.66F$. Note that for the maximum Rayleigh length $z_R = 2F$ ($\varphi = 45°$), which can be obtained upon tuning, we have the scheme of the most compact (with respect to the Rayleigh length of the input laser beam) FrFT converter. Its equivalent scheme is shown in Fig. 2b of Ref. [8].

Although the estimate of the error in the fabrication of the elements of a tunable converter is beyond the scope of this paper, note that some deviations from the conditions $f_{sph} = F\sqrt{2}$ and $f_{cyl} = 2F$ are admissible both for a spherical doublet (see section 2.1) and assemblies of cylindrical lenses. It is most important, at least for assemblies of cylindrical lenses, to minimise the scatter in the parameters of optical elements. The above conditions need not be fulfilled exactly because the required matching between the parameters of the laser beam and converter can be easily achieved either by increasing (decreasing) the distance between elements L and L$'$ or by rotating the corresponding elements in assemblies of cylindrical lenses.

4. Conclusions

The tunable $\pi/2$ mode converter proposed in this paper alleviates to a great extent the matching of the parameters of the converter and the input laser beam. The tuning range provides the change in the beam size by a factor of $\sqrt{2}$ (or the twofold change in the radiation wavelength) without replacing optical components. In this case, the change in the total relative length of the converter does not exceed 1/6. The use of the assembly of cylindrical lenses (section 2.2) in the converter described in Ref. [3] will allow one to obtain focused laser LG beams with the Rayleigh length $z_R \in [F, \infty]$.

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