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Effect of quantum fluctuations of medium polarisation on stimulated Raman scattering in a photonic crystal

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***Abstract*. Stimulated Raman scattering of femtosecond pulses in a photonic crystal consisting of alternating quarter-wave dielectric layers and layers of a nonlinear-scattering medium is calculated taking into account the quantum fluctuations of the medium polarisation. Analysis of the calculations showed that the Stokes pulse duration and intensity depend substantially on the statistic properties of polarisation. The efficient SRS conversion in a photonic crystal occurs under certain dispersion conditions providing the minimum difference between dispersions of the pump and Stokes frequencies, which are determined by the length and the number of periods of the photonic crystal.**

Keywords: *photonic crystal, femtosecond light pulses, stimulated Raman scattering, quantum fluctuations of polarisation.*

1. Introduction

Theoretical and experimental studies of photonic crystals performed during the last 15 years have shown that they are promising for applications in communication systems and devices for switching and generation of light [1, 2]. However, technological problems encountered in the fabrication of photonic crystals still prevent their practical applications in different devices. The most impressive success has been achieved in the growth and study of artificial opals and periodic semiconductor structures. At present the properties of photonic crystals and principles of the operation of devices based on them are actively studied by the methods of mathematical simulation.

The study of SRS in periodic structures [3] demonstrated the high efficiency of conversion of pump radiation to the first Stokes component compared to the efficiency of SRS excitation in a crystal of the same length. In the calculations, the system of equations taking into account the group-velocity dispersion in the second order of the dispersion theory was used. Our calculations of the second harmonic generation in a photonic crystal pumped by 100-fs pulses showed that the SHG conversion efficiency proved to be higher in the presence of the spatial rather than temporal dispersion [4, 5]. This is explained by the fact that spatial dispersion is higher than the temporal one, which allows one to increase the effective length of the photonic crystal (the number of its periods) at which the phase mismatch between the velocities of the pump and second-harmonic waves is compensated. The results of calculations performed in Ref. [5] taking into account the second derivative with respect to the spatial coordinate are quite close to data presented in Ref. [6].

In this paper, we analysed SRS in a photonic crystal using the second-order spatial dispersion approximation. The aim of the paper was to study the influence of quantum fluctuations of the medium polarisation on the SRS of a femtosecond pulse in a photonic crystal representing alternating layers of a dielectric and a nonlinear-scattering medium. We investigated the statistics of the quantum noise of polarisation at different dispersion properties of a photonic crystal, which depend on the optical lengths of the layers and the number of periods in the crystal.

2. Dispersion properties of photonic crystals

The dispersion properties of three photonic crystals with different optical lengths of layers were determined from the transmission spectra and the densities of optical modes, which were analysed in detail using expressions presented in papers [3 – 5] (see also Refs [7, 8]). The reflection and transmission spectra of restricted one-dimensional periodic structures were calculated by the method of characteristic matrix [7], and the coefficients of mode densities were found by using the calculated amplitudes and phases of the corresponding transmission coefficients. Figure 1 shows the transmission spectra $T(\omega)$ and optical mode densities $\rho(\omega)$ normalised to the speed of light for three photonic crystals with the number of periods $N_{st} = 80$ and different optical lengths of layers. The refractive indices of layers in the Nd : KGW crystal and quartz at the pump (ω_p) and Stokes (ω_s) frequencies were $n_{1p} = 1.968$, $n_{1s} = 1.977$ for Nd : KGW and $n_{2p} = 1.467$, $n_{2s} = 1.45$ for quartz.

Figure 2 shows the dependences of optical mode densities for the Stokes (ρ_s) and pump (ρ_p) waves, as well as of the length of the group mismatch of the waves on the number N_{st} of periods for three photonic crystals.

The group mismatch length is defined by the expression [9]

$$L_{gr} = \tau_p |\Delta u^{-1}|, \tag{1}$$

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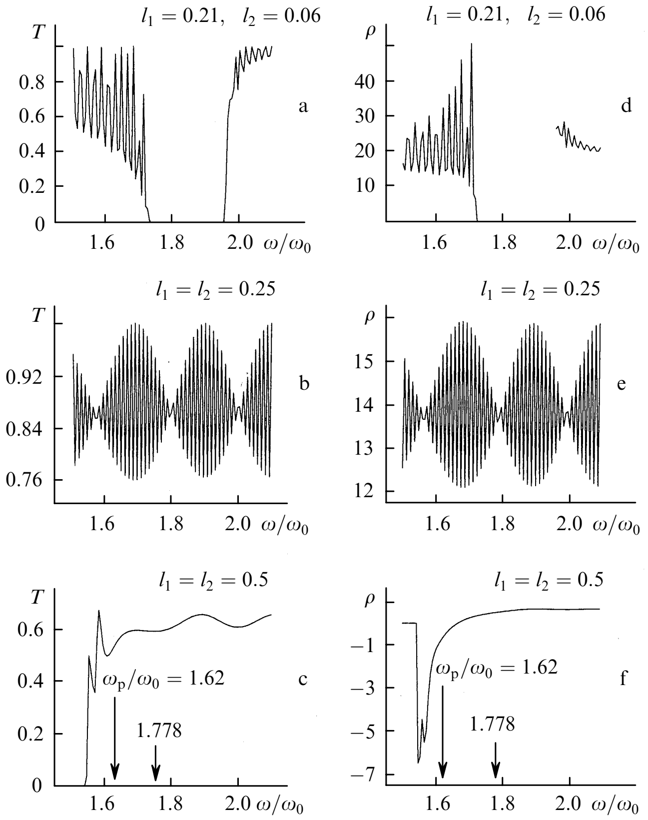

Figure 1. Transmission spectra $T(\omega)$ of photonic crystals 1–3 (a–c) and the corresponding optical mode densities $\rho(\omega)$ (d–f) for different lengths of layers of a scattering medium (l_1) and a dielectric (l_2) for $N_{st} = 80$. The layer lengths are normalised to λ_0 (the arrows indicate frequencies normalised to $\omega_0 = 10^{15}$ s^{-1}).

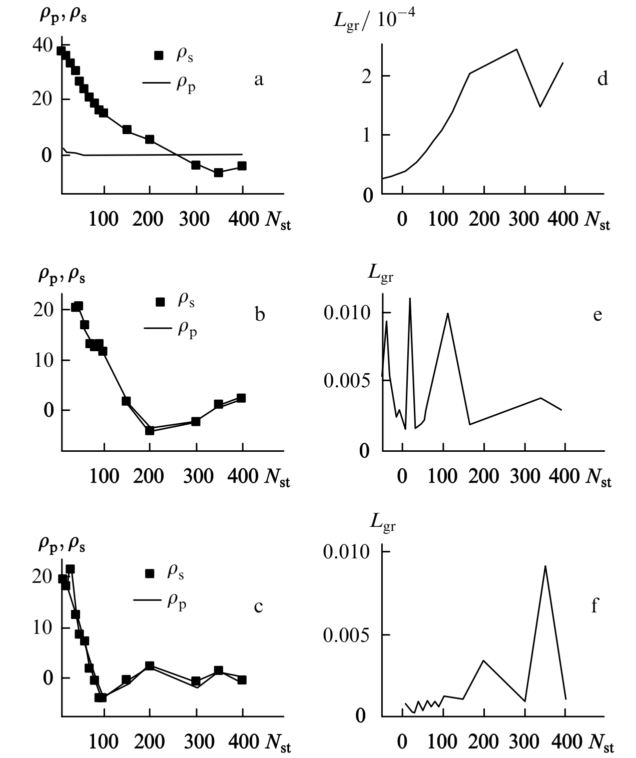

Figure 2. Dependences of the optical mode densities $\rho_{p,s}$ and the group mismatch length L_{gr} of the pump and the first Stokes waves on the number N_{st} of periods for photonic crystals 1–3 presented in Fig. 1.

where $\Delta u^{-1} = (\rho_s - \rho_p)^{-1}$ and τ_p is the pump pulse duration.

One can see from the transmission spectrum in Fig. 1a that for the smallest length of a scattering medium and a dielectric ($l_1 = 0.21$, $l_2 = 0.06$), the pump frequency falls into the 'forbidden gap' of the photonic crystal, while the first Stokes component falls into one of its side maxima. In this case, optical mode densities at these frequencies are substantially different, and their difference $\rho(\omega_s) - \rho(\omega_p)$ is 20.61. As the photonic crystal length increases, ρ_s decreases and becomes negative (Fig. 2a). The optical mode densities for the second and third photonic crystals at the pump and Stokes frequencies are close to each other and also substantially decrease with increasing the number of the crystal periods. The minimum length of the group mismatch for the 100-fs pump pulse takes place for the photonic crystal with $l_1 = 0.21$ and $l_2 = 0.06$ (Figs 2d–f).

3. Nonlinear interaction of the pump and first Stokes waves

The statistic properties of SRS in a photonic crystal were studied in the field of transmitted and reflected pump radiation by solving the system of equations with sources for polarisation of a scattering medium [3] taking into account the second derivatives with respect to the spatial coordinate. This allows us to consider the dispersion properties of both nonlinear and linear media in the second approximation of the dispersion theory.

The field in the photonic crystal is the sum of waves

$$E(z,t) = E_p^+ \exp(\mathrm{i}k_p z - \mathrm{i}\omega_p t) + E_p^- \exp(-\mathrm{i}k_p z - \mathrm{i}\omega_p t)$$

$$+ E_s^+ \exp(\mathrm{i}k_s z - \mathrm{i}\omega_s t) + E_s^- \exp(-\mathrm{i}k_s z - \mathrm{i}\omega_s t) + \text{c.c.},$$

where $E_{p,s}^{\pm}$, $\omega_{p,s}$, and $k_{p,s}$ are the slowly varying amplitudes, frequencies, and wave vectors of the pump laser wave, and the Stokes wave, respectively. Assuming that $\omega_p - \omega_s = \omega_{21}$, the photoinduced coherent oscillation in a scattering medium was written in the form

$$P_{21}(z,t) = \{q^+(z,t)\exp[\mathrm{i}(k_p - k_s)z]$$

$$+ q^-(z,t)\exp[-\mathrm{i}(k_p - k_s)z]\}\exp\{-[\mathrm{i}(\omega_p - \omega_s)t]\} + \text{c.c.},$$

where $q^{\pm}(z,t)$ are the amplitudes of polarisation of a scattering medium for counterpropagating Stokes waves. In this case, the system of equations used to analyse the temporal and energy properties of SRS has the form

$$\mathrm{i}\frac{1}{2k_p}\frac{\partial^2 E_p^+}{\partial z^2} + \frac{\partial E_p^+}{\partial z} + \frac{1}{u_p}\frac{\partial E_p^+}{\partial t} = \mathrm{i}\frac{\omega_p}{\omega_s} g_{2s}(q^+)E_s^+,$$

$$\mathrm{i}\frac{1}{2k_p}\frac{\partial^2 E_p^-}{\partial z^2} + \frac{\partial E_p^-}{\partial z} + \frac{1}{u_p}\frac{\partial E_p^+}{\partial t} = \mathrm{i}\frac{\omega_p}{\omega_s} g_{2s}(q^-)E_s^-,$$

$$\mathrm{i}\frac{1}{2k_s}\frac{\partial^2 E_s^+}{\partial z^2} + \frac{\partial E_s^+}{\partial z} + \frac{1}{u_s}\frac{\partial E_s^+}{\partial t} = \mathrm{i}g_{2s}(q^+)E_p^+, \qquad (2)$$

$$\mathrm{i}\frac{1}{2k_s}\frac{\partial^2 E_s^-}{\partial z^2} - \frac{\partial E_s^-}{\partial z} + \frac{1}{u_s}\frac{\partial E_s^-}{\partial t} = \mathrm{i}g_{2s}(q^-)E_p^-,$$

$$\frac{\partial q^+}{\partial t} = -\Gamma q^+ + \mathrm{i}[g_{1\mathrm{s}}(E_\mathrm{s}^+)E_\mathrm{p}^+] + Q^{1/2}F^+(z,t),$$

$$\frac{\partial q^-}{\partial t} = -\Gamma q^- + \mathrm{i}[g_{1\mathrm{s}}(E_\mathrm{s}^-)E_\mathrm{p}^-] + Q^{1/2}F^-(z,t),$$

where $g_{1\mathrm{s}} = \beta(\omega_\mathrm{s})/\hbar$; $g_{2\mathrm{s}} = 2\pi\hbar\omega_\mathrm{s} g_{1\mathrm{s}} N_0$; $\beta(\omega_\mathrm{s})$ is the susceptibility at the Stokes frequency; $u_i = (c/n_i)$ is the group velocities of the ith wave assumed equal to phase velocities ($i =$ p, s); N_0 is the density of scattering particles; $T_2 = \Gamma^{-1}$ is the phase relaxation time; Γ is the width of the SRS line; $F^\pm$ are the normalised statistically independent Gaussian sources of polarisation fluctuations [10–13]; $Q = 2\Gamma/SN_0$ is the intensity of polarisation fluctuations; and S is the cross section of the excited volume of the scattering medium.

The system of equations for a nonlinear medium was solved by the method of characteristics for calculating equations in the first-order partial derivatives according to the algorithm we developed earlier [3, 14, 15]. In system of equations (2), the dimensionless variables $t' = tc/\lambda_0$, and $z' = z/\lambda_0$ were used and the linear operator $\mathrm{i}(1/2k_i)(\partial^2/\partial z^2)$ in the Fourier space was replaced by the scalar factor $\exp(-\mathrm{i}K^2\Delta t/2)$, where $K = 2\pi/(N\Delta z)$ (N is the Fourier-transform parameter corresponding to the number of steps on the frequency network and Δz is the dimensional spatial step).

The initial conditions for the wave fields and polarisation $q^\pm$ were specified in the form

$$E_\mathrm{p}(z=0,t) = E_{\mathrm{p}0}\exp\left\{-2\ln 2\left[\frac{t-t_0}{\tau_\mathrm{p}}\right]^2\right\},$$

$$q^\pm(t=0) = 0,$$

where τ_p is the pump-pulse FWHM duration and t_0 is the time corresponding to the reference point. When the fast Fourier transform is used, the boundary conditions for the transmitted and reflected waves are obtained by applying it to the pump and scattered waves.

The typical characteristics of a femtosecond neodymium glass laser emitting at $\lambda_0 = 1060$ nm pulses of duration $\tau_\mathrm{p} = 100$ fs were used as radiation parameters. The parameters of a nonlinear layer of thickness l_1 (in units of λ_0) were identical to those of a Nd : KGW crystal ($T_2 = \Gamma^{-1}$ ps, $N_0 = 4.5\times10^{22}$ cm^{-3}, $\beta(\omega_\mathrm{s}) = 2\times10^{-25}$ cm ^{3} erg^{-1} [3], $\omega_\mathrm{p}/\omega_\mathrm{s} = 1.1$), and the refractive indices of a linear layer of thickness l_2 were identical to those of quartz. The total length of the nonlinear medium was $l \approx l_1\lambda_0 j$, where j is the number of layers of the active medium and $N_\mathrm{st} = 2j$ is the number of periods of the structure, which varied from 4 to 800. The intensity of the pump and Stokes waves was normalised to 1 GW cm^{-2}.

The process of frequency conversion of femtosecond pulses is nonstationary if $\tau_\mathrm{p}/T_2 \ll G$. This inequality is fulfilled for typical crystals where the transverse relaxation times are of the order of a few picoseconds. In this case, the coefficient G of nonstationary SRS conversion can be estimated from the expression [9]

$$G = \left(\frac{4g_0 l J_\mathrm{p}\tau_\mathrm{p}}{T_2}\right)^{1/2},$$

where $g_0 = 4\pi g_{1\mathrm{s}} g_{2\mathrm{s}}/\Gamma c^2$ is the SRS gain equal to 6 cm GW^{-1} and J_p is the pump intensity. The spontaneous noise level in Eqns (2) was determined by the parameter Q. The calculations were performed for a laser beam focused to a 10^{-4}-cm^2 spot on the crystal surface.

Analysis of the SRS conversion efficiency in the nonstationary regime performed in Refs [3, 14–16] showed that, to achieve the SRS regime in which the intensity of the first Stokes component exceeds that of spontaneous Raman scattering, we should have $G \geqslant 1$. Such a value of the coefficient G for a scattering medium of length l of only five wavelengths for 20 periods of the photonic crystal can be provided at $J_\mathrm{p} = 400$ GW cm^{-2} (calculations were performed for this pump intensity). Under such conditions for a nonperiodical scattering medium, the intensity of Stokes radiation exceeds the noise intensity only by several times. For this reason, the SRS conversion efficiency in the photonic crystal can be increased due to its interference properties, which substantially change the density states of the electromagnetic pump field and Stokes radiation.

4. Calculation of the SRS conversion taking into account polarisation of a scattering medium

Nonlinear system of equations (2) was calculated for three photonic crystals at a constant initial level of spontaneous scattering to find the optimal length of the SRS conversion (see Fig. 2) and to compare the results with data obtained in Ref. [3], where the second time derivative was taken into account.

Figure 3 shows the energies of transmitted and reflected pump radiation and the corresponding efficiencies of conversion to the first Stokes component for three photonic crystals. Note that the numbers of periods of photonic crystals for which the maximum SRS conversion of pump radiation is achieved substantially differ from those obtained in Ref. [3], the optimum lengths of photonic crystals being a few tens times greater than the lengths calculated earlier. Nevertheless, the qualitative agreement in the conversion efficiency in periodic structures of different types is retained. As in Ref. [3], the maximum SRS efficiency is achieved in a crystal with $l_1 = l_2 = 0.5$.

Let us compare our data with the dispersion characteristics of photonic crystals presented in Figs 1 and 2. The calculation of the optical lengths l of nonlinear crystals corresponding to the maximum SRS efficiencies for three photonic crystals gives the values 0.0034, 0.0093, and 0.02 cm. A comparison of these data with the lengths of the group mismatch of the waves (Figs 2d–f) shows that the greatest ratio $l/L_\mathrm{gr} = 30$ is obtained for the first photonic crystal, while for the second and third photonic crystals this ratio is 3.2 and 2, respectively. Therefore, the condition $l > L_\mathrm{gr}$ [9] at which the local response is nonstationary is fulfilled for all the three photonic crystals, and the stronger this inequality, the lower the SRS efficiency for femtosecond pulses. The results of the calculation of nonlinear system of equations (2) are in good agreement with this conclusion.

It is interesting to estimate which of the factors, the dispersion properties of a photonic crystal or its length, affects more strongly the SRS conversion efficiency. A comparison of the data in Figs 3d and 3f shows that the nonstationary SRS gain in the third photonic crystal is greater than that in the first crystal by a factor of 1.54. Therefore, the SRS gain in a nonlinear crystal of the same length should be 4.67, whereas the conversion efficiency in the third photonic crystal increased by a factor of 66 for $N_s = 300$. This suggests that the optical mode density in the photonic crystal is modified, and when both waves have close values of dispersion parameters, which are characterised by the length L_{gr} of the group spread of the waves, the efficiency of conversion of a pump femtosecond pulse to the Stokes pulse is maximal.

The data on the energy of the pump wave and the first Stokes component presented in Fig. 3 and results obtained in Ref. [3] differ in the effective length of maximum conversion and the conversion efficiency. As in the case of second harmonic generation, the consideration of the second-order spatial dispersion in equations leads to an increase in the effective lengths of nonlinear interaction, thereby resulting in a more adequate description of the interaction of waves with the dispersion medium. Of course, the best test of a model is the experiment. However, because the relevant experimental data are not available at present, we can verify the correctness of one or other approach by using the problem of linear properties of photonic crystals, which is based on the solution of stationary Maxwell equations. As follows from the above analysis, the use of data on the linear properties of a photonic crystal allows one to obtain good correlation with calculations of nonlinear nonstationary system of equations (2). Analysis of the interaction with shorter pulses requires the simultaneous consideration of dispersion properties of a medium and radiation parameters in the second order of the dispersion theory.

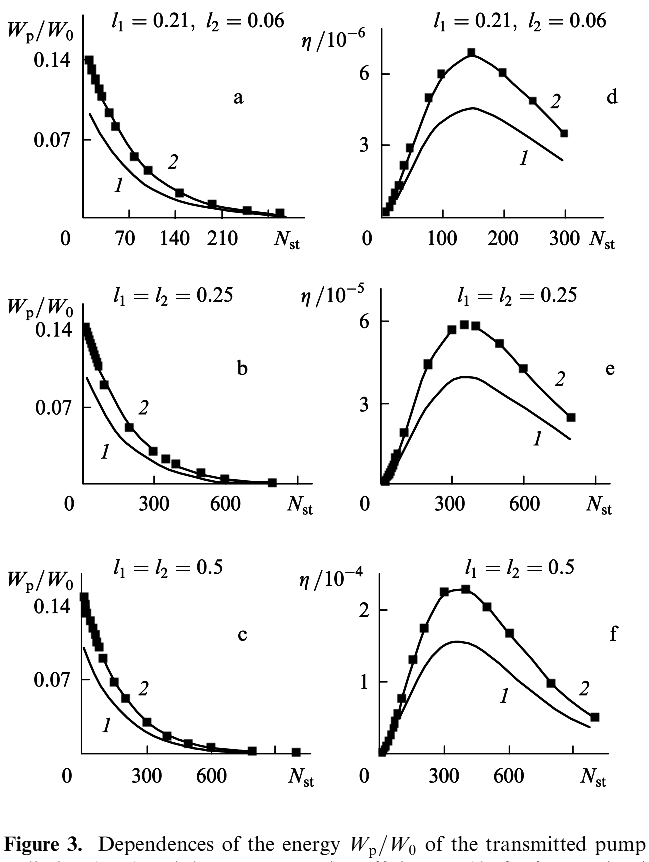

Figure 3. Dependences of the energy W_p/W_0 of the transmitted pump radiation (a–c) and the SRS conversion efficiency η (d–f) of transmitted (*1*) and reflected (*2*) waves on the number N_{st} of periods for photonic crystals presented in Fig. 1.

5. Effect of quantum fluctuations of polarisation of a nonlinear medium on the SRS parameters

Our studies [12–14] of the influence of the quantum noise of polarisation of a scattering medium on the parameters of the inverse SRS of picosecond pulses have shown that the statistic properties of polarisation are most noticeably manifested at low conversion efficiencies (fractions of per cent). An increase in the pump power leads to a loss of information on the initial properties of sources from which radiation with frequency conversion is formed. Because of small lengths of nonlinear interaction in photonic crystals, the conversion efficiency does not exceed a few per cent even when it has been substantially increased due to dispersion characteristics of a periodic structure. Therefore, quantum fluctuations of the polarisation noise should be revealed in the properties of scattered radiation.

Figure 4 shows the probability distributions for the energy $P_1^{\pm}(W_s^{\pm}/\langle W_s^{\pm}\rangle)$ and pulse duration $P_2^{\pm}(\tau_s^{\pm}/\langle\tau_s^{\pm}\rangle)$ of transmitted and reflected Stokes components obtained at different conditions of the SRS conversion in three photonic crystals ($\langle W_s^{\pm}\rangle$ and $\langle\tau_s\rangle$ are the average energy and duration of the converted waves, respectively). The results were obtained for a great number ($\sim$ 560) realisations of solutions of Eqns (2). The sources $F^{\pm}(z,t)$ were varied in calculations by means of a random number generator. The dispersions of fluctuations of these quantities $\sigma_i = (x_i - \langle x_i\rangle)^{1/2}$ and the coefficient of correlation between them

$$R_{i-j} = \frac{\langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle}{\sigma_i\sigma_j}$$

were calculated.

The results of calculations are presented in Tables 1 and 2 and Fig. 4. They correspond to three photonic crystals with different layer lengths (see Fig. 1).

It follows from Table 1 that the average duration of pulses increases with increasing energy of the transmitted and reflected waves at the Stokes frequency, the pulse duration for the reflected wave being somewhat greater

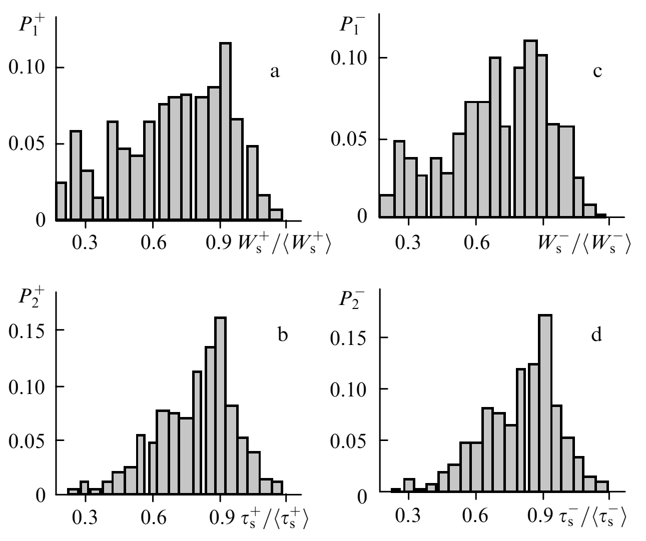

Figure 4. Histograms $P_{1,2}^{\pm}$ of the duration of transmitted (a, b) and reflected (c, d) Stokes pulses for the number of periods in photonic crystals $N_{st} = 80$ and $l_1 = 0.21$, $l_2 = 0.06$ (a–c) and $l_1 = l_2 = 0.5$ (b–d).

Table 1. Average values of the energies and durations of the Stokes waves for three photonic crystals.

Photonic crystal number	$\langle W_s^+ \rangle$	$\langle W_s^- \rangle$	σ_s^+	σ_s^-	$\langle \tau_s^+ \rangle$	$\langle \tau_s^- \rangle$	σ_i^+	σ_i^-
1	2.29×10^{-4}	3.57×10^{-4}	6.07×10^{-8}	1.08×10^{-6}	0.54	0.56	0.24	0.22
2	4.9×10^{-3}	7.02×10^{-3}	1.58×10^{-5}	2.18×10^{-5}	0.69	0.71	0.16	0.15
3	1.95×10^{-2}	2.8×10^{-2}	1.11×10^{-4}	1.58×10^{-4}	0.82	0.83	0.16	0.15

Table 2. Correlation coefficients for three photonic crystals.

Photonic crystal number	R_{1-2}	R_{1-5}	R_{1-6}	R_{5-6}	R_{3-5}	R_{3-6}	R_{2-5}	R_{2-6}	R_{4-5}	R_{4-6}
1	0.53	−0.0052	0.0212	0.51	0	0	−0.05	0.02	0	0
2	0.97	−0.011	−0.009	0.97	−0.05	−0.05	−0.02	0.01	−0.05	−0.05
3	0.95	−0.068	−0.06	0.99	0	0	−0.07	−0.06	0	0

Note. Subscripts (1 – 6) of the correlation coefficients R refer to the following parameters: W_s^+ (1), W_s^- (2), τ_p^+ (3), τ_p^- (4), τ_s^+ (5) and τ_s^- (6).

than for the transmitted wave. Analysis of the effect of dispersion properties of a photonic crystal on the conversion process performed for one realisation of noise sources and a comparison of average energies (second and third columns in Table 1) show that the conversion process is mainly determined by dispersion rather than the optical length of interaction between the pump and the first Stokes waves. The ratio of energies obtained in the first and second photonic crystals was 21.39. This ratio would be equal to three if we considered only the interaction lengths. Because the dispersion properties of the second and third photonic crystals are close, the increase in the energy of the transmitted and reflected Stokes waves is determined by the ratio of the interaction lengths and is equal to 3.97.

The correlation coefficients presented in Table 2 for three photonic crystals are also substantially different. The greatest values of R_{ij} correspond to the energies and durations of the transmitted and reflected Stokes pulses in the third photonic crystal. In the first photonic crystal, there is no correlation between the energies of transmitted pump pulses and SRS pulses and the corresponding pulse durations. Such correlations appear in SRS in the second and third photonic crystal and have different signs. This shows that, as the conversion efficiency increases, the converted radiation exerts influence on the pump.

Figure 4 shows the histograms of duration of the transmitted and reflected Stokes pulses for three photonic crystals. One can see that, depending on the dispersion properties of photonic crystals, the static properties of the quantum noise of polarisation are manifested not only in the values of average pulse durations (Table 1) but also in the type of the envelope. Thus, in the case of low SRS conversion efficiencies (Figs 4a, c), histograms are broader and have an extended leading edge, which is typical of the near-threshold SRS under normal conditions. As the SRS conversion efficiency increases (Figs 4b, d), the half-width of histograms decreases, demonstrating a decrease in the influence of quantum noise on the output characteristics of converted radiation pulses.

6. Conclusions

The SRS conversion process has been analysed using the system of equations taking into account the spatial dispersion and quantum noise in polarisation sources in a scattering medium and the dependences of the conversion efficiency on the dispersion properties of photonic crystals have been obtained. A comparison of the conversion efficiency for photonic crystals similar to those considered in Ref. [3] has shown that calculations taking into account the second spatial derivative in the basic system of equations (2) better agrees with the calculation of linear properties of photonic crystals by the matrix method. Thus, the estimates of the group length show that the optimal lengths corresponds to a substantially greater number of the structure periods than in the case of calculations taking into account the second time derivative. This analysis suggests the possibility of predicting the SRS conversion efficiency by calculating the transmission and reflection coefficients for the pump and Stokes waves and determining the lengths of dispersion spread of the waves as functions of the photonic-crystal parameters.

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