QUANTUM CALCULATIONS

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Quantum computer based on activated dielectric nanoparticles selectively interacting with short optical pulses

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***Abstract*. The operation principle of a quantum computer is proposed based on a system of dielectric nanoparticles activated with two-level atoms – cubits, in which electric dipole transitions are excited by short intense optical pulses. It is proved that the logical operation (logical operator) CNOT (controlled NOT) is performed by means of time-dependent transfer of quantum information over 'long' (of the order of 10^4 nm) distances between spherical nanoparticles owing to the delayed interaction between them in the optical radiation field. It is shown that one-cubit and two-cubit logical operators required for quantum calculations can be realised by selectively exciting dielectric particles with short optical pulses.**

Keywords: *cubit, quantum information, logical operators, resonance coherent transfer of quantum information, nanoparticles, delayed and near-field interaction between cubits.*

1. Introduction

The main problems, which should be solved to create a full-scale quantum computer, were discussed in Ref. [1]. One of these problems is the identification of cubits during quantum calculations. This paper is devoted to the solution of this problem. At present there are several proposals to realise a quantum computer [2 – 8], for example, by using electric dipole transitions in the spectrum of two-levels atoms [9 – 11]. The advantages of the latter proposal in the solution of problems indicated in Ref. [1] were considered in Refs [9 – 11]. However, the creation of a quantum computer in very small systems of size much smaller than the wavelength of light involves certain difficulties. Thus, it was shown in Ref. [9] that, to identify cubits in such systems, it is necessary to change slightly the transition frequencies of cubit atoms.

The problem of cubit identification is solved in this paper by increasing the interatomic distance without changing the transition frequency. We tackled this problem by solving the problem of resonance coherent transfer of quantum information over long distances, greatly exceeding or comparable to the wavelength of light, from one ensemble cubit to another in the field of optical pulses. An ensemble cubit is a system of two-level atoms imbedded into a dielectric, for example, glass nanoparticle. Because optical fields within such a particle weakly depend on the coordinates of observation points, cubit atoms are in-phase with each other. In this case, quantum information will be specified with the help of the Bloch states [11], i.e., with the help of observable physical quantities such as the local dipole moments of atoms and their inversions. This method for specifying quantum information, as pointed out in Ref. [11], allows one to take into account in full measure the properties of interacting cubits without loosing the coherence of their quantum states. The preparation of spherical particles doped with impurity atoms is quite possible. Such particles are used as probes in the near-field optical microscopy.

2. Basic equations of quantum computing

Consider the transfer of quantum information between two nanoparticles 1 and 2 in the optical scheme in Fig. 1 after

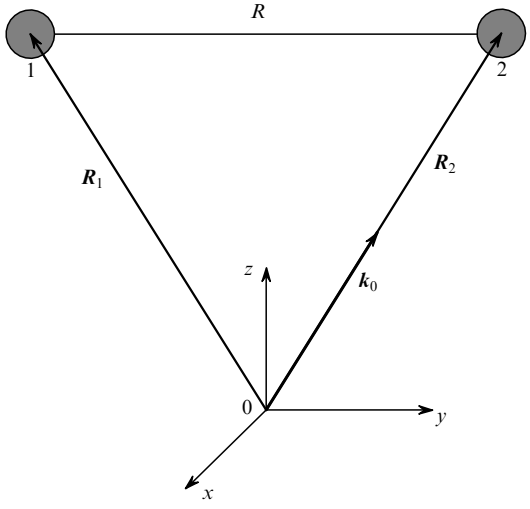

Figure 1. Optical scheme of excitation of particles by an external radiation field in the process of quantum information transfer: 1 and 2 are cubits; $\boldsymbol{k}_0$ is the external-field wave vector; $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the radius vectors of cubits.

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excitation of one of them by a short optical pulse. The field equation in this case has the form

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}_0\exp(-\mathrm{i}\omega t)=\boldsymbol{E}_{0\mathrm{in}}\exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{r})\exp(-\mathrm{i}\omega t)+\int\mathrm{rot}\,\mathrm{rot}\,N_2\frac{\boldsymbol{p}_2(t-R_2'/c)}{R_2'}\,\mathrm{d}V_2'+\int\mathrm{rot}\,\mathrm{rot}\,N_1\frac{\boldsymbol{p}_1(t-R_1'/c)}{R_1'}\,\mathrm{d}V_1', \tag{1}$$

where $\boldsymbol{E}(\boldsymbol{r},t)$ is the electric field strength of the optical wave at the observation point $\boldsymbol{r}$ at the instant t inside and outside of the nanoparticles; ω is the optical radiation frequency; $\boldsymbol{E}_{0\mathrm{in}}$ is the field amplitude of the external wave with the wave vector $\boldsymbol{k}_0$; N_1 and N_2 are the concentrations of cubit atoms in particles 1 and 2, respectively; $\boldsymbol{R}_j'=|\boldsymbol{r}-\boldsymbol{r}_j'|$ ($j=1$, 2); $\boldsymbol{r}_j'$ are the radius vectors of atoms in the jth particles; V_j' are volumes of the jth particles; differentiation is performed with respect to the coordinates of the observation point $\boldsymbol{r}$; and $\boldsymbol{p}_j=(1/2)\boldsymbol{X}_j\exp(-\mathrm{i}\omega t)$ is the induced dipole moments of atoms. The amplitudes $\boldsymbol{X}_j$ satisfy the system of equations for coupled quantum dipoles [11]

$$\dot{\boldsymbol{X}}_j=-\mathrm{i}\boldsymbol{X}_j(\omega_{0j}-\omega)-\frac{2\mathrm{i}}{\hbar}w_j|\boldsymbol{d}_{0j}|^2\boldsymbol{E}_{0j},\qquad \dot{w}_j=\frac{\mathrm{i}}{\hbar}\boldsymbol{X}_j^*\boldsymbol{E}_{0j}-\frac{\mathrm{i}}{\hbar}\boldsymbol{X}_j\boldsymbol{E}_{0j}^*, \tag{2}$$

where ω_{0j} are the transition frequencies in the spectrum of interacting two-level atoms; $\boldsymbol{d}_{0j}$ are the corresponding dipole transition moments; w_j are the inversions of atoms determining the difference of probabilities of finding atoms in the ground and excited states [12]; $\boldsymbol{E}_{0j}$ is the field acting on the jth particle without the factor $\exp(-\mathrm{i}\omega t)$. Equations of motion (2) satisfy the conservation law

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(|\boldsymbol{X}_j|^2+w_j^2|\boldsymbol{d}_{0j}|^2\right)=0, \tag{3}$$

from which it follows that

$$u_j^2+v_j^2+w_j^2=1, \tag{4}$$

where u_j and v_j enter the definition of $\boldsymbol{X}_j$ as

$$(u_j-\mathrm{i}v_j)\boldsymbol{d}_{0j}=\boldsymbol{X}_j\,. \tag{5}$$

Equations (2) do not contain relaxation terms. This means that the optical pulse duration τ_p and the characteristic transit time $t_\mathrm{ph}=R/c$ of a photon between two particles is considerably shorter than the phase and energy relaxation times, which affect the incoherent decay of the induced dipole moments and inversions of atoms.

Equation (1) for activated particles 1 and 2 should be supplemented with the corresponding volume integrals over the coordinates of nonresonance atoms in particles 1 and 2 surrounding two-level atoms. Let us represent the corresponding polarisation vector of nonresonance atoms as $\boldsymbol{P}'=N_0\alpha_0\boldsymbol{E}$, where N_0 and α_0 are the concentration and polarisability of nonresonance atoms, which are the same for particles 1 and 2 in our treatment. To simplify the following expressions, we will start from Eqn (1) and take the role of nonresonance atoms in particles into account only in final results.

Equations (2) and (1) depend on each other and allow us to describe the self-consistent interaction between particles 1 and 2 in the optical radiation field. It is necessary to consider the boundary conditions on the surfaces of particles for calculating fields inside and outside of particles. We assume that particles 1 and 2 consist of identical atoms, so that $|\boldsymbol{d}_{01}|=|\boldsymbol{d}_{02}|=d_0$, $N_1=N_2=N$, and $\omega_{01}=\omega_{02}=\omega_0$ the radii of particles are $a_1=a_2=a$. However, in this case, as will be shown below, $\boldsymbol{X}_1\neq\boldsymbol{X}_2$ and $w_1\neq w_2$ because only one of the particles is irradiated by the external field.

According to (1), expressions for the fields acting at points $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ (Fig. 1) have the form

$$\boldsymbol{E}_{01}=[\boldsymbol{X}_2]\hat{a}_{R2}N_2+[\boldsymbol{X}_1]a_{T1}N_1,\qquad \boldsymbol{E}_{02}=\boldsymbol{E}_{0\mathrm{in}}\exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{R}_2)+\boldsymbol{X}_2a_{T2}N_2+[\boldsymbol{X}_1]\hat{a}_{R1}N_1, \tag{6}$$

where square brackets mean that the corresponding quantities were determined at the delayed instants τ_1-R/c, while the rest of quantities are determined at the instant $\tau_1=t-R_1/c$. For $a/R\ll1$, we obtain for geometrical factors the expressions

$$a_{R1}^{x,z}=a_{R2}^{x,z}\approx-\frac{2\pi a}{n^2-1}\frac{1}{R}\exp(\mathrm{i}k_0R),\quad a_{R1}^{y}=a_{R2}^{y}=0,\qquad a_{T1}=a_{T2}=-\frac{4\pi}{3}, \tag{7}$$

where n is the refractive index of the activated medium inside particles. We assume for simplicity that the concentration of active atoms in particles is such that the dispersion dependence of the refractive index can be neglected. The complete expression for geometrical factors $\hat{a}_{Rj}$ and a_{Tj} of a spherical particle was obtained in Ref. [13] to describe the interaction of a probe with a sample surface in a near-field optical microscope.

In the case of exact resonance, the solution of Eqns (2) has the form

$$X_{j\beta}=-\mathrm{i}d_0w_{j0}\sin\theta_{j\beta}+X_{j\beta}^{(0)}\cos\theta_{j\beta},\qquad w_j=-\frac{\mathrm{i}}{d_0}X_{j\beta}^{(0)}\sin\theta_{j\beta}+w_{j0}\cos\theta_{j\beta}, \tag{8}$$

where $\beta=x,z$; $X_{j\beta}^{(0)}$ and w_{j0} are the initial values of quantities $X_{1\beta}$, $X_{2\beta}$ and w_j, respectively, as functions of time τ_1; and

$$\theta_{j\beta}(\tau_1)=\frac{2d_0}{\hbar}\int_{-\infty}^{\tau_1}E_{0j\beta}(\tau_1')\mathrm{d}\tau_1' \tag{9}$$

are the areas of optical pulses acting on atoms in particles 1 and 2.

3. Resonance coherent transfer of quantum information between particles upon selective excitation of one of them by an optical pulse

Consider the interaction between two identical spherical particles in the optical field. Each of the particles consists of a system of two-level atoms, for example, sodium atoms imbedded into fused silica. The distance R between the centres of particles is assumed fixed and equal to 20λ, where $\lambda=5890$ Å is the wavelength corresponding to the

$3S \to 3P$ quantum transition in a sodium atom. The radii of spherical are $a_p = 100$ nm. Then, for the concentration of atoms in particles $N = 10^{19}$ cm^{-3}, we obtain $NV \approx 4.2 \times 10^4$ atoms in each spherical particle, where V is the volume of a spherical particle. Each spherical particle containing an ensemble of cubit atoms is represented by an ensemble cubit in the process of quantum information transfer considered below. As follows from the calculation of geometrical factors $\hat{a}_{Rj}$ and a_{Tj}, the fields inside spherical nanoparticles weakly depend on the coordinates of observation points inside particles, and the local dipole moments of two-level atoms inside nanoparticles change in phase in time in the coherent radiation field.

Let us represent the action of optical radiation on particle 2 by a sequence of small-area pulses, for which solution (8) is valid for $\sin\theta_{j\beta} \approx \theta_{j\beta}$, $\cos\theta_{j\beta} \approx 1$. By using this solution, we obtain from expressions (6) the following equations

$$\dot{X}_{1\beta} = A_{1\beta}X_{2\beta} + A_{2\beta}X_{1\beta},$$
$$\dot{X}_{2\beta} = B_{1\beta}X_{2\beta} + B_{2\beta}X_{1\beta} + C_{1\beta}, \tag{10}$$

where

$$A_{1\beta} = -\mathrm{i}a_{R2}^{\beta}N_2\frac{2d_0^2}{\hbar}w_{10}, \quad A_{2\beta} = -\mathrm{i}a_{T1}N_1\frac{2d_0^2}{\hbar}w_{10},$$
$$B_{1\beta} = -\mathrm{i}a_{T2}N_2\frac{2d_0^2}{\hbar}w_{20}, \qquad B_{2\beta} = -\mathrm{i}a_{R1}^{\beta}N_1\frac{2d_0^2}{\hbar}w_{20},$$
$$C_{1\beta} = -\mathrm{i}w_{20}\frac{2d_0^2}{\hbar}E_{0\mathrm{in}\beta}\exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{R}_2).$$

In Eqns (10), the unified time τ_1 is introduced taking into account that the values of $X_{1\beta}$ and $X_{2\beta}$ weakly change during the time $2R/c$. By solving Eqns (10) for each of the small intervals $\Delta\tau_1$ from which the entire time interval of transfer of quantum information from particle 2 to particle 1 is composed, we calculate the fields and inversions of atoms at different instants τ_1. We will characterise the transfer of quantum information by the inversion of particle 1.

Consider the interaction of particle 2 with the field of a single optical pulse with the envelope in the region of particle 2 described by the expression

$$E_{0\mathrm{in}\beta}^{(2)}(\tau_1) = A_{\mathrm{in}\beta}\exp(\mathrm{i}\gamma_p\tau_1), \tag{11}$$

where $\gamma_p = \pi/\tau_p$ and τ_p is the pulse duration. For the $\pi/100$-pulse, we obtain $A_{\mathrm{in}\beta} = -[\pi\hbar\mathrm{i}\gamma_p/(4d_0)] \times 10^{-2}$ CGS units. Then, the general solution of Eqns (10) is

$$X_{1\beta}(\tau_1) = D_1[\exp(\lambda_1\tau_1) + \exp(\lambda_2\tau_1)] + F_1\exp(\mathrm{i}\gamma_p\tau_1),$$
$$X_{2\beta}(\tau_1) = D_2[\exp(\lambda_1\tau_1) + \exp(\lambda_2\tau_1)] + F_2\exp(\mathrm{i}\gamma_p\tau_1), \tag{12}$$

where

$$D_1 = \frac{1}{2}[X_{1\beta}(0) - F_1]; \quad D_2 = \frac{1}{2}[X_{2\beta}(0) - F_2];$$

$$F_1 = -\frac{s_{0\beta}A_{1\beta}}{(\mathrm{i}\gamma_p - A_{2\beta})(B_{1\beta} - \mathrm{i}\gamma_p) + B_{2\beta}A_{1\beta}};$$

$$F_2 = -\frac{s_{0\beta}(\mathrm{i}\gamma_p - A_{2\beta})}{(\mathrm{i}\gamma_p - A_{2\beta})(B_{1\beta} - \mathrm{i}\gamma_p) + B_{2\beta}A_{1\beta}};$$

$$s_{0\beta} = -\mathrm{i}w_{20}\frac{2d_0^2}{\hbar}\exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{R}_2)A_{\mathrm{in}\beta};$$

$$\lambda_{1,2} = \frac{1}{2}(A_{2\beta}+B_{1\beta}) \pm \left[\frac{1}{4}(A_{2\beta}+B_{1\beta})^2 - A_{2\beta}B_{1\beta} + A_{1\beta}B_{2\beta}\right]^{1/2}.$$

By substituting quantities (12) into solution (8), we find the acting fields E_{01} and E_{02} and inversions w_1 and w_2 as functions of time τ_1. Let us study the properties of solution (12).

Case A. Let the initial inversions of atoms in particles 1 and 2 be the same, i.e., $w_{10} = w_{20}$. Then, $\lambda_{1,2} = -\mathrm{i}\nu_{1,2}$, where the oscillation frequencies ν_1 and ν_2 of dipole moments, which are low compared to ω, have the form

$$\nu_1 = \nu_T - \nu_R, \quad \nu_2 = \nu_T + \nu_R,$$
$$\nu_T = \frac{2d_0^2}{\hbar}Nw_{10}a_T, \quad \nu_R = \frac{2d_0^2}{\hbar}Nw_{10}a_R^{\beta}. \tag{13}$$

The frequency ν_T determines the oscillations of quantities $X_{1\beta}$ and $X_{2\beta}$, which are caused by the near-field interaction between atoms within one particle, while ν_R is the characteristic frequency of quantum information transfer, which is caused by the interaction between atoms in different particles in the wave zone.

Case B. The initial inversions are different, i.e., $w_{10} \neq w_{20}$. In this case, $\lambda_{1,2} = -\mathrm{i}\nu'_{1,2}$, where

$$\nu'_1 = \nu'_T - \nu'_R, \quad \nu'_2 = \nu'_T + \nu'_R,$$
$$\nu'_T = \frac{d_0^2}{\hbar}N(w_{10} + w_{20})a_T, \tag{14}$$
$$\nu'_R = \nu'_T\left\{1 + 4\left[\left(\frac{a_R^{\beta}}{a_T}\right)^2 - 1\right]\frac{w_{10}w_{20}}{(w_{10}+w_{20})^2}\right\}^{1/2}.$$

By using solution (12) and conservation law (3) for $w_{10} = w_{20}$ and $F_1 = F_2 = 0$, we obtain the equalities

$$|X_{1\beta}(\tau_1)|^2 = \frac{1}{2}|X_{1\beta}(0)|^2[1 + \cos(\nu_R\tau_1)],$$
$$w_1^2(\tau_1) = 1 - \frac{1}{2}\frac{|X_{1\beta}(0)|^2}{d_0^2}[1 + \cos(\nu_R\tau_1)]. \tag{15}$$

Here, we took into account that oscillations at frequencies ν_1 and ν_2 interfere during the evolution of quantum system 1 interacting with quantum system 2 if they are in the quantum states specified by quantities w_{10}, $X_{1\beta}(0)$ and w_{20}, $X_{2\beta}(0)$, respectively.

Let us assume that the duration of an optical pulse incident on particle 2 is $\tau_p = 10^{-14}$ s, the concentration N of atoms in particles 1 and 2 is 10^{19} cm^{-3}, the radius of particles is $a_p = 100$ nm, and the distance between particles is $R = 20\lambda$. For the external $\pi/100$-pulse, we have $A_{\mathrm{in}\beta} = -0.39\mathrm{i} \times 10^3$ CGS units, $F_1 \approx 0.99\mathrm{i} \times 10^{-23}w_{10}w_{20}$ CGS units, and $F_2 \approx -0.96 \times 10^{-19}w_{20}$ CGS units. Then, the characteristic frequency ν_R of quantum information

transfer from particle 2 to particle 1 is $-1.94\times10^{10}w_{10}$ s^{-1}. The characteristic time of quantum information transfer is $T_{\rm p}=|\pi/(2v_R)|=(0.8\times10^{-10}/|w_{10}|)$ s, which is considerably longer than the time $t_{\rm ph}=0.33\times10^{-13}$ s and is considerably shorter than the relaxation times $T_1=10^{-7}$ s and $T_2'=3\times10^{-9}$ s. For comparison, the characteristic frequency $v_{\rm a}$ of quantum information transfer from one atom to another over the distance $R=20\lambda$ is 0.44×10^6 s^{-1}. The transfer of quantum information also occurs when the external pulse is switched off. At the instant $\tau_1=T_{\rm p}$, the inversion of the first particle achieved its maximum value $w_1\approx1$ if the initial inversions of atoms in particles 1 and 2 are the same: $|w_{10}|=|w_{20}|\approx1/10$. The initial inversions can be produced by optical pumping in the scheme in Fig. 1.

The influence of nonresonance atoms in particles related to the role of additional volume integrals in Eqn (1), as follows from calculations, results in the entanglement of the near and delayed fields upon the interaction of activated particles in the optical field. For this reason, the characteristic time $T_{\rm p}$ of quantum information transfer decreases approximately by a factor of a_T/a_R and is equal to $\sim10^{-12}$ s.

4. One-cubit and two-cubit logical operators in quantum calculations

Quantum calculations in a quantum computer can be performed using one-cubit and two-cubit unitary transformations representing logical operators NOT and CNOT (controlled NOT), respectively, which are represented in the explicit form below. In quantum computing experiments [6], these operators were realised with the help of the corresponding sequence of radio-frequency $\pi/2$-pulses exciting spin quantum transitions. Optical quantum transitions differ substantially from magnetic spin transitions. Therefore, we should determine the operators NOT and CNOT that will be used in an optical quantum computer based on activated nanoparticles.

We will represent the solution of Eqns (2), which has the form (8) in the case of exact resonance, in the matrix form

$$\begin{bmatrix} X(\tau)\\ d_0w(\tau)\end{bmatrix}=\hat{M}^{(1)}(\tau,\tau_0)\begin{bmatrix} X(\tau_0)\\ d_0w(\tau_0)\end{bmatrix}, \tag{16}$$

where the unitary operator is

$$\hat{M}^{(1)}=\begin{bmatrix}\cos\theta_{\rm in} & -{\rm i}\sin\theta_{\rm in}\\ -{\rm i}\sin\theta_{\rm in} & \cos\theta_{\rm in}\end{bmatrix}. \tag{17}$$

Here, the indices j and β numerating cubits and indicating the projection of induced dipole moments and the local time index τ_1 are omitted. The index at the area of the optical pulse $\theta_{\rm in}$ indicates that unitary transformation (16) occurs under the action of external optical radiation. In this case, the operator $\hat{M}^{(1)}$ is the NOT operator for mutually independent cubits in a quantum computer containing 10^3 cubits.

The Hadamard transformations [1] can be realised under the action of $\pi/2$-pulses. Indeed, let us represent the wave functions of a cubit in the form

$$\Psi=|a||0\rangle+|b|\exp({\rm i}\Delta\varphi)|1\rangle,$$

where $\Delta\varphi$ is the difference between phases of the complex coefficients a and b of the quantum superposition; and $|0\rangle$ and $|1\rangle$ are the ground and excited states of the cubit with energies W_0 and W_1, respectively. In this case, $W_1-W_0=\hbar\omega_0$, and the states

$$|0\rangle=\begin{pmatrix}0\\1\end{pmatrix},\quad |1\rangle=\begin{pmatrix}1\\0\end{pmatrix}$$

are the eigenfunctions of the effective spin operator [12]. Then, the quantum transition from the $|1\rangle$ state with the inversion $w_0=+1$ corresponds to the unitary transformation

$$\begin{pmatrix}0 & -{\rm i}\\ -{\rm i} & 0\end{pmatrix}\begin{pmatrix}0\\ d_0\end{pmatrix}=\begin{pmatrix}-{\rm i}d_0\\ 0\end{pmatrix}.$$

Taking into account the relation

$$\frac{1}{2}X=d_0a^*b$$

between the local dipole moment and coefficients of the quantum superposition, we have for the quantum transition under study

$$\exp({\rm i}\Delta\varphi)=-\frac{\rm i}{2|a||b|}. \tag{18}$$

For $|a|=|b|=1/\sqrt{2}$, we obtain the transformation $H|1\rangle=(1/\sqrt{2})(|0\rangle-|1\rangle)$. In a similar way, we obtain the transformation $H|0\rangle=(1/\sqrt{2})\times(|0\rangle+|1\rangle)$ under the action of the $\pi/2$-pulse taking into account that the unitary transformations

$$\begin{pmatrix}0 & -{\rm i}\\ -{\rm i} & 0\end{pmatrix}\begin{pmatrix}0\\ -d_0\end{pmatrix}=\begin{pmatrix}{\rm i}d_0\\ 0\end{pmatrix}$$

are performed upon the transition of the cubit from the state with the inversion $w_0=-1$ to the superposition state $(1/\sqrt{2})(|0\rangle+|1\rangle)$. As follows from (18), the transformation $H|1\rangle$ occurs when the phase difference of the quantum states is $\Delta\varphi=-\pi/2$, while the transformation $H|0\rangle$ occurs when $\Delta\varphi=\pi/2$.

Let us show that the action of operator (17) on a cubit depends on the polarisation of external optical radiation. We represent the electric field strength of an external wave at the cubit location in the form

$$\boldsymbol{E}_{\rm in}=\sqrt{2}E_{0\rm in}[\boldsymbol{x}_0\cos(\omega\tau_1)+\boldsymbol{y}_0\sin(\omega\tau_1)], \tag{19}$$

where $E_{0\rm in}$ is the real amplitude, and $\boldsymbol{x}_0$ and $\boldsymbol{y}_0$ are the unit vectors along the coordinate axes x and y. Equations (2) were derived in Ref. [11] taking into account the condition $\boldsymbol{d}_0\times\boldsymbol{E}=0$, where $\boldsymbol{d}_0=\boldsymbol{d}_0'-{\rm i}\boldsymbol{d}_0''$ and $\boldsymbol{E}$ is the acting field. The quantities $\boldsymbol{d}_0'$ and $\boldsymbol{d}_0''$ have the form [12]

$$\boldsymbol{d}_0'=d_0\frac{\boldsymbol{x}_0}{\sqrt{2}},\quad \boldsymbol{d}_0''=d_0\frac{\boldsymbol{y}_0}{\sqrt{2}}.$$

Let us separate in (19) the negative-frequency part, which is proportional to $\exp(-{\rm i}\omega t)$. Then, the corresponding induced dipole moment of the cubit under study will be determined by the quantity

$$\boldsymbol{X} = \frac{d_0}{\sqrt{2}}(\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0)(u - \mathrm{i}v), \tag{20}$$

while the pulse area can be found from expression (9), where the direction of $\boldsymbol{E}_{0\mathrm{in}}$ is determined by the vector $\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0$. A change in the direction of circular polarisation of the field (19) is equivalent to a change in the sign at $\boldsymbol{y}_0$. This leads to the corresponding change in the sign at $\boldsymbol{y}_0$ in (20). Therefore, by changing the direction of circular polarisation of the exciting external field, we also change the direction of the vector $\boldsymbol{X}$ in the complex plane.

The simultaneous action of the Hadamard transformation on two cubits transfers them to the entangled states. Indeed, we have for two cubits in the ground state

$$H_1 \otimes H_2|0\rangle_1|0\rangle_2 = \frac{1}{2}\big(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\big). \tag{21}$$

By calculating the average value $\hat{\boldsymbol{d}}_1 + \hat{\boldsymbol{d}}_2$ of the dipole moment operator with the help of wave function (21), we obtain

$$\langle \hat{\boldsymbol{d}}_1 + \hat{\boldsymbol{d}}_2 \rangle = \frac{1}{2}\frac{d_0}{\sqrt{2}}[(\boldsymbol{x}_0 + \mathrm{i}\boldsymbol{y}_0)\exp(\mathrm{i}\omega_0\tau_1)\exp(-\mathrm{i}k_0R_1) + (\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0)\exp(-\mathrm{i}\omega_0\tau_1)\exp(\mathrm{i}k_0R_1) + (\boldsymbol{x}_0 + \mathrm{i}\boldsymbol{y}_0)\exp(\mathrm{i}\omega_0\tau_1)\exp(-\mathrm{i}k_0R_2) + (\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0)\exp(-\mathrm{i}\omega_0\tau_1)\exp(\mathrm{i}k_0R_2)] = \boldsymbol{p}_1 + \boldsymbol{p}_2.$$

If one of the cubits, for example, cubit 2 is initially in the excited state, then the Hadamard transformation transfers cubits 1 and 2 to the next state:

$$H_1 \otimes H_2|0\rangle_1|1\rangle_2 = \frac{1}{2}\big(|0\rangle_1|0\rangle_2 - |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 - |1\rangle_1|1\rangle_2\big). \tag{22}$$

The calculation of the average value $\hat{\boldsymbol{d}}_1 + \hat{\boldsymbol{d}}_2$ of the operator with the help of wave function (22) gives

$$\langle \hat{\boldsymbol{d}}_1 + \hat{\boldsymbol{d}}_2 \rangle = \frac{1}{2}\frac{d_0}{\sqrt{2}}[(\boldsymbol{x}_0 + \mathrm{i}\boldsymbol{y}_0)\exp(\mathrm{i}\omega_0\tau_1)\exp(-\mathrm{i}k_0R_1) + (\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0)\exp(-\mathrm{i}\omega_0\tau_1)\exp(\mathrm{i}k_0R_1) - (\boldsymbol{x}_0 + \mathrm{i}\boldsymbol{y}_0)\exp(\mathrm{i}\omega_0\tau_1)\exp(-\mathrm{i}k_0R_2) - (\boldsymbol{x}_0 - \mathrm{i}\boldsymbol{y}_0)\exp(-\mathrm{i}\omega_0\tau_1)\exp(\mathrm{i}k_0R_2)] = \boldsymbol{p}_1 - \boldsymbol{p}_2.$$

Therefore, using π- and $\pi/2$-pulses acting simultaneously on any pair of cubits, we can generate entangled states of the type (21), (22), which contain the basis Bell states [14].

Consider the logical operator CNOT as a sequence of the following transformations: the $\pi/2$ pulse acts on one of the cubits, for example, cubit 2, then cubits oscillate freely during the time T_p of quantum information transfer between them, and cubit 2 is again subjected to the action of the $\pi/2$-pulse. We show that such a CNOT operator in the system of electric dipoles substantially differs from the CNOT operator in the system of magnetic moments [1], where magnetisation changes in time proportionally to the vector product of magnetisation and the magnetic field strength.

Let cubit 1 controls the behaviour of cubit 2. According the conservation law, we have for these cubits

$$|X_1|^2 + d_0^2 w_1^2 = |X_2|^2 + d_0^2 w_2^2.$$

Let us assume that cubit 1 is in the $|0\rangle_1$ ground state, i.e., $w_1 = -1$. This means that $|X_1|^2 = 0$, so that the state of cubit 1 remains invariable. Let us irradiated cubit 2 with the $\pi/2$-pulse. As a result, the inversion becomes $w_2 = 0$ and $|X_2|$ takes the maximum value equal to d_0. After switching off the $\pi/2$-pulse, cubit 2 freely oscillates due to coherent resonance transfer of quantum information in the scheme considered in section 3 (case B) at frequencies $v'_{1,2}$ (14) during the time T_p. Let us find frequencies $v'_{1,2}$ for a specific case, when glass nanoparticles with the refractive index $n_0 = 1.5$ are doped with sodium atoms with the transition dipole moment $d_0 = 6.24 \times 10^{-18}$ CGS units. In this case, $N_0\alpha_0 \approx 0.07$, and for $w_{20} \approx w_2 = 0$, we obtain

$$v'_1 \approx 0, \quad v'_2 \approx -\frac{4\pi}{3}\frac{d_0^2}{\hbar}Nw_1 > 0.$$

Within the time $T_\mathrm{p} = \pi/v'_2$ after switching off the first $\pi/2$-pulse, the second $\pi/2$ pulse is switched on, which transfers cubit 2 to the initial state with $w_2 = -1$. In the case, $|X_2| = 0$ and the action of the CNOT operator is terminated.

5. On the physical realisation of a quantum computer based on activated particles selectively interacting with short optical pulses

As pointed out in Ref. [1], a full-scale quantum computer should contain approximately 10^3 cubits. Consider a system consisting of a such number of ensemble cubits representing spherical particles made, for example, of glass activated with two-level sodium atoms at the $3S \rightarrow 3P$ transition (the yellow line of sodium at $\lambda = 5890$ Å). The radius of particles is $a_\mathrm{p} = 100$ nm, the distance between them is $R = 20\lambda$, the concentration of sodium atoms in particles is $N = 10^{19}$ cm^{-3}. The area occupied by 10^3 cubits is approximately 10^{-3} cm^2. Each cubit can be coupled with a microfibre through which optical pulses are delivered. Dipole radiation signals from cubits, corresponding to the process of quantum calculations, can be detected in the wave zone with respects to cubits on a special screen. Let us show how such a design of an optical quantum computer makes it possible to solve the basic physical problems pointed out in Ref. [1].

(i) It is necessary to create a system of cubits in the ground state before the beginning of quantum calculations. It is obvious that spin systems in crystals require the use of low temperatures and extremely strong magnetic fields. Ions in vacuum traps can be cooled down to very low temperatures by laser methods. However, it is necessary to use ultrahigh vacuum in this case. The initial register was created in experiment [6] by using a special procedure for selecting nuclear spins in the ground state to realise a simplest algorithm of quantum calculations. For more complicated problems, the selection of cubits in the ground state can be impossible. For optical transitions in a system of activated nanoparticles, the problem of selection of cubits

in the ground state is solved automatically even at room temperatures.

(ii) A quantum computer requires the use of selective excitation of any cubit by optical pulses. In experiment [6], cubits in a two-cubit quantum computer were nuclear spins of ^{1}H and ^{13}C with frequencies $\omega_1/2\pi \approx 500$ MHz and $\omega_2/2\pi \approx 125$ MHz, respectively. This allowed the authors [6] to solve the problem of cubit identification over the exciting pulse frequency, but the location of cubits among 10^{20} molecules in a liquid trichloroethylene solution remained uncertain. The problem of cubit identification in a quantum computer based on activated nanoparticles can be completely solved. Indeed, the assumed cubit size is ~ 100 nm and the distance between them is $\sim 20\lambda$. Therefore, each cubit can be connected to a separate fibre of micrometre diameter through which optical pulses from a common radiation source will be delivered at a fixed frequency coincident with the transition frequency in the spectrum of two-level impurity atoms.

(iii) It is known that it is sufficient to use one- and two-cubit blocks to obtain the required set of transformations in quantum calculations [1]. One-cubit NOT transformations and the Hadamard transformation can be physically realised under the action of external optical pulses. To perform the NOT operation, it is necessary to take into account the interaction of cubits in the radiation field. In principle, it is necessary that the CNOT operation could be performed for any cubit pairs in a quantum computer. In the case of nuclear and electron spins, the interaction between cubits is short-range, which leads to significant problems in the physical realisation of this operator. In this paper, we proposed to realise the CNOT operator by using the long-range, proportional to $1/R$, delayed interaction of atoms in the optical radiation field. In this case, each cubit is irradiated by the resulting field representing the superposition of an external field and the self-consistent field of dipoles. To enhance the transfer of quantum information with increasing the distance between cubits up to tens of micrometres and to realise the logical CNOT operator, the concentration of two-level atoms in nanoparticles should be increased up to 10^{19} cm^{-3}.

(iv) During quantum calculations, relaxation processes take place in computer cubits, which results in random variations in the amplitudes and phases of the cubit state vectors. This loss of coherence of the quantum states in this process prevents quantum calculations and should be minimised. The characteristic time of this process is determined by the phase (T_2') and energy (T_1) relaxation times. In the case of nuclear magnetic resonance in liquids, the relaxation times T_2' and T_1 are $1-10$ s. For the optical transitions of ions in traps, the coherence loss time is determined by the decay time of spontaneous emission and the time of collisions with residual atoms. Therefore, the studies of quantum coding and correction of errors considered in Ref. [1] are of interest. In our paper, the relaxation times for sodium atoms in glass are $T_1 = 10^{-7}$ s and $T_2' = 3 \times 10^{-9}$ s. The phase relaxation time T_2^*, which is caused by the inhomogeneous broadening of impurity atoms, can be minimised by increasing the homogeneity of the distribution of impurity atoms in the glass.

The use of metastable quantum states with the lifetime $\sim 10^{-3}$ s is undesirable, in our opinion, because this will result in a significant increase in the time of quantum information transfer between cubits. It seems that the use of short ($\sim 10^{-14}$ s) optical pulses, allowing the realisation of about 10^5 unitary one-cubit transformations without a noticeable loss in the coherence of quantum states, is sufficient for the implementation of quite complicated algorithms for quantum calculations.

Let us make some energy estimates. To excite a sodium atom in a dielectric particle by the $\pi/2$-pulse, the electric field strength should be $E_0 = \pi\hbar/(4d_0\tau_p)$, where τ_p is the optical pulse duration. For $\tau_p = 10^{-14}$ s, we obtain $E_0 = 1.26 \times 10^{-4}$ CGS units. To excite $NV = 4.2 \times 10^4$ atoms in a dielectric particle by a single optical pulse, the specific power of an external source should be $E_0^2 NV/(4\pi\tau_p) = 0.529 \times 10^3$ W cm^{-3}. We can assume in this case that the strengths of the acting and external electric fields are comparable. The power absorbed by a dielectric particle upon excitation of half the impurity atoms is $(1/2)NV\hbar\omega_0/T_1 \approx 0.8$ W. The thermal energy released inside one particle is $(1/2)(NV\hbar\omega_0/T_1)T_1$. For $T_1 = 3 \times 10^{-9}$ s, the temperature of the particle is approximately 300 K. Therefore, the temperature of a dielectric particle excited by an optical pulse of duration $\tau_p = 10^{-14}$ s increases by $\Delta T = T(\tau_p/T_1) = 3 \times 10^{-3}$ K.

(v) The states of cubits after the termination of calculations should be measured. At present the technology of such measurements is absent [1]. It seems that the methods of near-field optical microscopy [13] will play an important role in the realisation of quantum computers based on activated nanoparticles and the measurements of cubit states.

Thus, we have described in this paper the operation principle of an optical quantum computer based on activated spherical particles selectively interacting with short optical pulses. The physical realisation of unitary transformations, which can be used in quantum calculations in a quantum computer, have been theoretically substantiated.

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