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INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA

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Collision frequency shift of a short electromagnetic pulse

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***Abstract*. The frequency shift of a short electromagnetic pulse interaction with a plasma-like medium is discovered and studied. The shift is caused by elastic collisions of free electrons with ions or neutral particles.**

Keywords: *ionisation, femtosecond laser pulse, frequency shift*.

1. Introduction

The interaction of ultrashort electromagnetic pulses (EMPs) at different frequencies with matter is the object of extensive studies (already performed or planned). This interaction is nonstationary due to the short pulse duration. The studies of the emission spectrum of the matter in the case of such interaction is of great interest both from the fundamental point of view and for possible applications (see, for example, Ref. [1]). It has been shown in Refs [2 – 4] devoted to the study of the emission spectrum induced by collision processes in a plasma that EMP harmonics can be efficiently generated in a sufficiently dense plasma due to electron – ion collisions.

In this paper, the frequency shift of an EMP caused by collision processes is studied. The study is based on the three-dimensional approach [5, 6], which allows one to determine the frequency shift of the EMP during nonstationary processes proceeding upon the interaction of the EMP with matter. An attractive feature of this approach is that it determines the emission spectrum emerging from matter in the wave zone with respect to the interaction region. It is shown that nonstationary collisions in the EMP field cause the frequency shift of the pulse, and the dependence of the frequency shift on the parameters of the EMP and matter are determined. In the case of a weakly ionised matter, the EMP spectrum caused by collisions of electrons with neutral particles shifts to the red, whereas in a stripped plasma the spectrum shifts to the blue. The dependence of the EMP frequency shift in the stripped plasma on the EMP and plasma parameters is found.

The influence of ionisation processes on the EMP frequency shift was not considered because the EMP intensity was too low to produce ionisation (see section 3) or a gas was completely ionised. Note that the ionisation shift of the EMP and the combined action of stimulated Raman scattering (SRS) and ionisation were investigated in Ref. [5].

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2. Basic relations

We will find the EMP frequency shift using the approach developed in papers [5, 6]. Let us differentiate two Maxwell's equations

$$\mathrm{rot}\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \quad \mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{1}$$

n times with respect to t. The electric current density $\boldsymbol{J}$ in (1) is determined self-consistently in terms of the electric field strength $\boldsymbol{E}$ and the magnetic induction $\boldsymbol{B}$. By multiplying scalarwise the obtained equations by $\partial^n\boldsymbol{E}/\partial t^n$ and $\partial^n\boldsymbol{B}/\partial t^n$, respectively, and subtracting the results, we find

$$\frac{\partial}{\partial t}\left\{\frac{1}{8\pi}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2 + \left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\right\} + \mathrm{div}\left\{\frac{c}{4\pi}\left[\frac{\partial^n\boldsymbol{E}}{\partial t^n}, \frac{\partial^n\boldsymbol{B}}{\partial t^n}\right]\right\} = -\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}. \tag{2}$$

By integrating (2) with respect to t from $-\infty$ to $+\infty$, we obtain

$$\mathrm{div}\left\{\frac{c}{4\pi}\int_{-\infty}^{+\infty}\left[\frac{\partial^n\boldsymbol{E}}{\partial t^n}, \frac{\partial^n\boldsymbol{B}}{\partial t^n}\right]\mathrm{d}t\right\} + \frac{1}{8\pi}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2 + \left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\Bigg|_{t=-\infty}^{t=+\infty} = -\int_{-\infty}^{+\infty}\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}\mathrm{d}t, \tag{3}$$

where $f\big|_{t=-\infty}^{t=+\infty} \equiv f(t=+\infty) - f(t=-\infty)$. By integrating (3) over a volume V_0 with the surface S, we find

$$\frac{c}{4\pi}\int_S\int_{-\infty}^{+\infty}\left[\frac{\partial^n\boldsymbol{E}}{\partial t^n}, \frac{\partial^n\boldsymbol{B}}{\partial t^n}\right]\mathrm{d}t\mathrm{d}\boldsymbol{s} = -\int_{V_0}\int_{-\infty}^{+\infty}\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}\mathrm{d}t\mathrm{d}^3\boldsymbol{r} - \frac{1}{8\pi}\int_{V_0}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2 + \left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\Bigg|_{t=-\infty}^{t=+\infty}\mathrm{d}^3\boldsymbol{r}, \tag{4}$$

where $\mathrm{d}\boldsymbol{s} = \boldsymbol{e}_s\mathrm{d}s$ is the vector element of the surface S with the external normal $\boldsymbol{e}_s$.

Let us determine the position of the surface S_{in} [a part of the surface S in (4)] thorough which electromagnetic radiation enters the gas, and also of the surface S_{out} ($S =$

$S_{\rm in}+S_{\rm out}$) through which radiation leaves the gas volume V_0. Radiation can also leave the volume V_0 through the surface $S_{\rm in}$. We assume that surfaces $S_{\rm in}$ and $S_{\rm out}$ are located at a sufficiently large distance from the plasma of volume V, i.e., in the wave zone with respect to the volume V and, in addition, $\boldsymbol{B}=\boldsymbol{H}$ on these surfaces, where $\boldsymbol{H}$ is the magnetic field strength.

The fields $\boldsymbol{E}$ and $\boldsymbol{H}$ in the wave zone satisfy the relation $\boldsymbol{E}=[\boldsymbol{H},\boldsymbol{k}]$, where $\boldsymbol{k}=[\boldsymbol{E},\boldsymbol{H}]/|[\boldsymbol{E},\boldsymbol{H}]|$ is the direction of the energy flux at a fixed point of the surface S (see, for example, Ref. [7]). By substituting this relation into (4), we obtain

$$\frac{c}{4\pi}\int_S\int_{-\infty}^{+\infty}\left(\frac{\partial^n\boldsymbol{H}}{\partial t^n}\right)^2\mathrm{d}t\boldsymbol{k}\mathrm{d}\boldsymbol{s}=-\int_{V_0}\int_{-\infty}^{+\infty}\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}\,\mathrm{d}t\mathrm{d}^3\boldsymbol{r}$$
$$-\frac{1}{8\pi}\int_{V_0}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2+\left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\Bigg|_{t=-\infty}^{t=+\infty}\mathrm{d}^3\boldsymbol{r},$$

or for the spectral energy density per unit surface, $I(\omega,\boldsymbol{r})=[c/(4\pi^2)]|\boldsymbol{H}(\omega,\boldsymbol{r})|^2$, where [7]

$$\boldsymbol{H}(\omega,\boldsymbol{r})=\int_{-\infty}^{+\infty}\boldsymbol{H}(t,\boldsymbol{r})\exp(\mathrm{i}\omega t)\mathrm{d}t,$$

we have

$$\int_S\int_0^{\infty}\omega^{2n}I(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}=-\int_V\int_{-\infty}^{+\infty}\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}\,\mathrm{d}t\mathrm{d}^3\boldsymbol{r}$$
$$-\frac{1}{8\pi}\int_{V_0}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2+\left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\Bigg|_{t=-\infty}^{t=+\infty}\mathrm{d}^3\boldsymbol{r}. \quad (5)$$

Here, we took into account that the electric current density $\boldsymbol{J}$ is concentrated in the plasma volume V. Note that relation (5) is also valid for $n=0$:

$$\int_S\int_0^{\infty}I(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}=-\int_V\int_{-\infty}^{+\infty}\boldsymbol{EJ}\mathrm{d}t\mathrm{d}^3\boldsymbol{r}$$
$$-\frac{1}{8\pi}\int_{V_0}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right)\Big|_{t=-\infty}^{t=+\infty}\mathrm{d}^3\boldsymbol{r}, \quad (6)$$

which expresses the energy conservation law. We rewrite equality (6) taking into account the direction of propagation of radiation through surfaces $S_{\rm in}$ and $S_{\rm out}$:

$$\varepsilon_{\rm out}+\varepsilon_{\rm back}-\varepsilon_{\rm in}=-\int_V\int_{-\infty}^{+\infty}\boldsymbol{EJ}\mathrm{d}t\mathrm{d}^3\boldsymbol{r}$$
$$-\frac{1}{8\pi}\int_{V_0}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right)\Big|_{t=-\infty}^{t=+\infty}\mathrm{d}^3\boldsymbol{r}, \quad (7)$$

where

$$\varepsilon_{\rm in}=-\int_{S_{\rm in}}\int_0^{\infty}I_{\rm in}(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}$$

is the energy of electromagnetic radiation entering the volume V_0;

$$\varepsilon_{\rm out}=\int_{S_{\rm out}}\int_0^{\infty}I_{\rm out}(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s},$$

$$\varepsilon_{\rm back}=\int_{S_{\rm back}}\int_0^{\infty}I_{\rm back}(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}$$

is the radiation energy leaving the volume V through surfaces $S_{\rm out}$ and $S_{\rm back}$ ($S_{\rm back}=S_{\rm in}$, so that radiation emerges back with respect to entering electromagnetic radiation), respectively.

By introducing the $2n$ power of frequencies averaged over the corresponding emission spectra

$$\langle\omega^{2n}\rangle_{\alpha}=\int_{S_\alpha}\int_0^{\infty}\omega^{2n}I_\alpha(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}$$
$$\times\left(\int_{S_\alpha}\int_0^{\infty}I_\alpha(\omega,\boldsymbol{r})\mathrm{d}\omega\boldsymbol{k}\mathrm{d}\boldsymbol{s}\right)^{-1},\ \ \alpha=\mathrm{in},\mathrm{out},\mathrm{back}, \quad (8)$$

and the backward reflection coefficient $R_{\rm back}=\varepsilon_{\rm back}/\varepsilon_{\rm in}$, we write relation (5) in the form

$$\langle\omega^{2n}\rangle_{\rm out}=\frac{\langle\omega^{2n}\rangle_{\rm in}-\langle\omega^{2n}\rangle_{\rm back}R_{\rm back}-\varepsilon_{\rm in}^{-1}(e_n+h_n)}{1-R_{\rm back}-\varepsilon_{\rm in}^{-1}(e_0+h_0)},\quad n=0,1,\ldots, \quad (9)$$

where

$$h_n=\frac{1}{8\pi}\int_{V_0}\mathrm{d}^3\boldsymbol{r}\left[\left(\frac{\partial^n\boldsymbol{E}}{\partial t^n}\right)^2+\left(\frac{\partial^n\boldsymbol{B}}{\partial t^n}\right)^2\right]\Bigg|_{t=-\infty}^{t=+\infty};$$

$$e_n=\int_V\int_{-\infty}^{+\infty}\frac{\partial^n\boldsymbol{E}}{\partial t^n}\frac{\partial^n\boldsymbol{J}}{\partial t^n}\,\mathrm{d}t\mathrm{d}^3\boldsymbol{r},\quad n=0,1,\ldots.$$

Expression (9) determines the $2n$ power of the frequency averaged over the spectrum of radiation emerging through the surface $S_{\rm out}$. For definiteness, we will analyse below the spectral characteristics of this radiation. Equality (9) was derived taking relation (7) into account. Expression (9) is valid for any EMP intensity and an arbitrary irradiation geometry, and determines the value of $\langle\omega^{2n}\rangle_{\rm out}$ in the wave zone with respect to a plasma produced upon ionisation, where, as a rule, the spectrum of emission emerged from the plasma is detected.

When collisions are taken into account, the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are absent for $t=\pm\infty$, therefore, terms h_n entering (9) are zero for all values of n. For this reason, the shift of the square of frequency $\langle\omega^2\rangle_{\rm out}$ averaged over the spectrum of radiation propagated through matter with respect to the square of frequency $\langle\omega^2\rangle_{\rm in}=\omega_0^2$ averaged over the spectrum of the incident pulse, neglecting backward reflection, has the form

$$\langle\omega^2\rangle_{\rm out}-\omega_0^2=\left(-e_1+\omega_0^2e_0\right)(\varepsilon_{\rm in}-e_0)^{-1}, \quad (10)$$

where

$$e_0=\int_V\mathrm{d}^3\boldsymbol{r}\int_{-\infty}^{+\infty}\mathrm{d}t\boldsymbol{EJ};\quad e_1=\int_V\mathrm{d}^3\boldsymbol{r}\int_{-\infty}^{+\infty}\mathrm{d}t\,\frac{\partial\boldsymbol{E}}{\partial t}\frac{\partial\boldsymbol{J}}{\partial t}. \quad (11)$$

It is assumed that the spectra of radiation incident on matter and emerging from it are detected in the wave zone with respect to the volume V. According to the energy conservation law (7), we obtain $\varepsilon_{\rm in}-e_0=\varepsilon_{\rm out}$.

Taking the short pulse duration into account, we assume that ions and neutral particles are at rest during the interaction of the pulse with matter. Therefore, the electric current density $\boldsymbol{J}$ entering expressions (11) is the electron current density. In addition, already at low ionisation degrees the interaction of matter with electromagnetic radiation is determined by free electrons [8]. This allows us to neglect the bound electron current and assume that the electric current density $\boldsymbol{J}$ is the free-electron current density $\boldsymbol{j}$.

The free-electron current excited by nonrelativistic EMPs in an isotropic medium is described by the equation

$$\frac{\partial \boldsymbol{j}}{\partial t} = \frac{e^2 n_e}{m_e}\boldsymbol{E} + \frac{e}{m_e}\boldsymbol{F}_{fr}, \tag{12}$$

$$\boldsymbol{F}_{fr} = -\int d^3\boldsymbol{v} m_e \boldsymbol{v}\Big[\sum_i \nu_{ei}(v) + \sum_a \nu_{ea}(v)\Big] f_e(\boldsymbol{v} - \boldsymbol{V}_e)$$
$$= -m_e n_e \boldsymbol{V}_e \nu(V_e) = -\frac{m_e}{e}\boldsymbol{j}\nu(V_e), \tag{13}$$

where the summation is performed over all ions and neutral particles; $\boldsymbol{F}_{fr}$ is the volume density of the frictional force caused by collisions of electrons with ions with charges e_i (the collision frequency is ν_{ei}) and also of electrons with neutral particles of the type a (the collision frequency is ν_{ea}); f_e, e, m_e, n_e, and ν are the electron velocity distribution function, charge, mass, concentration, and the efficient frequency of collisions of electrons with heavy particles, respectively; and $\boldsymbol{V}_e = n_e^{-1}\int \boldsymbol{v} f_e d^3\boldsymbol{v}$ is the hydrodynamic velocity of the electron motion. Note that Eqn (12) describes, in particular, the dynamics of the current of free electrons produced upon ionisation of a medium by a short intense EMP [9]. In this case, the electron concentration n_e is substantially nonstationary. The exact values of the frequency shift and broadening of the pulse in the case of nonstationary ionisation were determined in Refs [5, 6]. We will study the collision effect assuming that n_e is independent of time, bearing in mind that the generalisation to the case of ionisation produced by the pulse can be performed within the framework of the general approach [5, 6] by neglecting the current of bound electrons. By substituting (12) into (11) and eliminating the derivative $\partial \boldsymbol{J}/\partial t$, we find

$$e_0 = \frac{m_e}{e}\int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt\,\nu \boldsymbol{j}\boldsymbol{u}_E,$$
$$e_1 = \int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt\left[\frac{e^2 n_e}{m_e}\nu \boldsymbol{E}^2 + \left(\frac{\partial \nu}{\partial t} - \nu^2\right)\boldsymbol{j}\boldsymbol{E}\right], \tag{14}$$

where $\boldsymbol{u}_E = (e/m_e)\int_{-\infty}^{t}\boldsymbol{E}dt'$. The current density $\boldsymbol{j}$ is the solution of Eqn (12) under the condition $\boldsymbol{j}(t=-\infty)=0$ and has the form

$$\boldsymbol{j} = en_e\left[\boldsymbol{u}_E - \exp\left(-\int_{-\infty}^{t}\nu dt'\right)\int_{-\infty}^{t}\nu \boldsymbol{u}_E \exp\left(\int_{-\infty}^{t'}\nu dt''\right)dt'\right]. \tag{15}$$

When the EMP duration τ exceeds the period of fast oscillations of the electromagnetic field ($\tau\omega_0 \gg 1$) and the collision frequency ν is low compared to ω_0, we can calculate e_0 and e_1 using the approximation of slowly varying amplitudes. In this approximation,

$$\boldsymbol{E} = \frac{1}{2}[\boldsymbol{E}_0 \exp(-i\omega_0 t) + c.c.], \tag{16}$$

where $\boldsymbol{E}_0$ is the amplitude slowly varying over the period $2\pi/\omega_0$. Then, with an accuracy to $(\tau\omega_0)^{-1}$ and ν/ω_0, we obtain from (15)

$$\boldsymbol{j} = en_e\boldsymbol{u}_E,\quad \boldsymbol{u}_E = \frac{i}{2}[\boldsymbol{V}_E\exp(-i\omega_0 t) - c.c.],$$
$$\boldsymbol{V}_E = \frac{e\boldsymbol{E}_0}{m_e\omega_0}. \tag{17}$$

By using (16) and (17) and the fact that $\nu = \nu(u_E^2)$ in an isotropic medium, we can write expressions (14) for e_0 and e_1, with an accuracy to $(\tau\omega_0)^{-1}$ and ν/ω_0, in the form

$$e_0 = \int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt n_e m_e \overline{\nu u_E^2},$$
$$e_1 = \omega_0^2\int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt n_e m_e\left[\overline{\nu u_E^2} + 2\overline{\frac{\partial\nu}{\partial u_E^2}\frac{e^2(\boldsymbol{E}\boldsymbol{u}_E)^2}{m_e^2\omega_0^2}}\right], \tag{18}$$

where the bar means averaging over the electromagnetic field period $2\pi/\omega_0$. By substituting (18) into (10), we obtain

$$\frac{\langle\omega^2\rangle_{out} - \omega_0^2}{\omega_0^2} = \Delta$$
$$= -\frac{2}{\varepsilon_{out}}\int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt n_e \overline{\frac{\partial\nu}{\partial u_E^2}\frac{e^2(\boldsymbol{E}\boldsymbol{u}_E)2}{m_e^2\omega_0^2}}. \tag{19}$$

Note that the terms proportional to ν in (19) are cancelled. Taking (16) and (17) into account, we find

$$\frac{e\boldsymbol{E}\boldsymbol{u}_E}{m_e\omega_0} = \frac{i}{4}[\boldsymbol{V}_E^2\exp(-2i\omega_0 t) - c.c.].$$

The value of $\boldsymbol{V}_E^2$ substantially depends on the EMP polarisation. By introducing the degree η of circular polarisation ($\eta = 0$ and ± 1 for linear and circular polarisations, respectively [7]), the expression for $|\boldsymbol{V}_E^2|$ can be written in the form

$$|\boldsymbol{V}_E^2| = |\boldsymbol{V}_E|^2 P(\eta),\quad P(\eta) = (1-\eta^2)(1+\eta^2)^{-1}. \tag{20}$$

Therefore, in the case of a circularly polarised pulse, $P = 0$ and the frequency shift of the pulse, within the framework of approximations made above, is zero.

We will assume below that the velocity distribution of electrons is Maxwellian, with the temperature T_e.

3. Low degree of ionisation

Consider (19) at comparatively low EMP intensities when the degree of ionisation is low and collisions of electrons with neutral particles are important (in this case, $m_e|\boldsymbol{V}_E|^2 \ll 4T_e$). Then, we obtain from relation (13) determining ν the expression

$$\frac{\partial\nu}{\partial u_E^2} = \frac{1}{3(2\pi)^{1/2}}\left(\frac{m_e}{T_e}\right)^{7/2}\int_0^{\infty} v^4\left(\frac{m_e v^2}{5T_e} - 1\right)$$
$$\times\Big[\sum_i \nu_{ei}(v) + \sum_a \nu_{ea}(v)\Big]\exp\left(-\frac{m_e v^2}{2T_e}\right)dv, \tag{21}$$

which is independent of u_E. The form of the function $\nu_{ea}(v)$ substantially depends on the electronic structure of neutral particles of the type a. When v is comparatively small, the frequency $\nu_{ea}(v)$ can be approximated by the power function (see, for example, [10]): $\nu_{ea} = n_a\sigma_a v_a(v/v_a)^{\beta_a}$, where $\beta_a \geqslant 0$; n_a is the concentration of neutral particles of the type a; and σ_a and v_a are their characteristic collision cross section and velocity, respectively. The frequency of electron–ion collisions is $\nu_{ei} = 4\pi e^2 e_i^2 n_i \Lambda m_e^{-2} v^{-3}$, where Λ is the Coulomb logarithm and n_i is the concentration of ions with charges e_i [10]. Then,

$$\Delta = -\frac{3}{10}\frac{n_e T_e P^2(\eta)}{\varepsilon_{out}}\left[\sum_a \frac{4\beta_a}{9\sqrt{\pi}}\Gamma\left(\frac{5+\beta_a}{2}\right)\left(\frac{2T_e}{m_e v_a^2}\right)^{\beta_a/2} \times n_a\sigma_a v_a - \nu_C(T_e)\right]\int_V d^3\boldsymbol{r}\int_{-\infty}^{+\infty} dt\left(\frac{m_e|\boldsymbol{V}_E|^2}{2T_e}\right)^2, \quad (22)$$

$$\nu_C(T_e) = \sum_i \frac{4(2\pi)^{1/2}}{3}\frac{e^2 e_i^2 n_i \Lambda}{m_e^{1/2} T_e^{3/2}}, \quad (23)$$

where Γ is the Euler gamma function. It follows from relations (22) and (23) that, when the degree of ionisation is low, the shift Δ of the squares of frequencies proves to be negative. Therefore, in the case of a low degree of ionisation, the spectrum of a pulse propagated through matter is shifted to the red. As the degree of ionisation increases, the frequency of electron–ion collisions dominates. As a result, the spectrum of the pulse shifts in the opposite direction: when the matter is almost fully ionised, the spectrum of the pulse shifts to the blue.

4. Stripped plasma

Consider an EMP of arbitrary intensity, lower than the relativistic intensity for the frequency ω_0, propagating in the stripped plasma. The effective frequency of Coulomb electron–ion collisions for any space–time shape of the pulse and the Maxwell distribution of electrons can be obtained from expression (13) determining ν (cf. [2–4])

$$\nu(u_E^2) = \nu_C(T_e)\frac{3}{2}\int_0^1 \sqrt{x}\exp\left(-\frac{m_e u_E^2}{2T_e}x\right)dx. \quad (24)$$

Note that for $m_e u_E^2 \ll 2T_e$, expression (24) transfers to the known limiting expression $\nu = \nu_C(T_e)$ for weak electromagnetic field (see, for example, Ref. [11]). By substituting (24) into (19) and taking (20) into account, we obtain

$$\Delta = \int_V d^3\boldsymbol{r}\,\frac{n_e T_e \nu_C(T_e)}{\varepsilon_{out}}\int_{-\infty}^{+\infty} dt\, G\left(\frac{m_e|\boldsymbol{V}_E(\boldsymbol{r},t)|^2}{4T_e},\ \eta\right),$$

$$G(\xi,\eta) = P(\eta)\frac{6}{\sqrt{\xi}}\int_0^{\xi}\sqrt{x}\,e^{-x} I_1(P(\eta)x)dx, \quad (25)$$

where $I_1(b)$ is the modified first-order Bessel function. We will assume below for definiteness that the EMP is linearly polarised: $\eta = 0$, $P = 1$.

For low EMP intensities, when the inequality $\delta = I_{max}/(2cn_c T_e) \ll 1$ is fulfilled (where I_{max} is the EMP peak intensity and $n_c = m_e\omega_0^2/(4\pi e^2)$ is the critical electron density), expression (25) transfers to (22) at $n_a = 0$. The shift Δ of the square of frequency for $\delta \ll 1$ proves to be linearly dependent on I_{max} because $[m_e|\boldsymbol{V}_E|^2/(2T_e)]^2 \propto I_{max}^2$ and $\varepsilon_{out} \propto I_{max}$.

We estimate the shift Δ for arbitrary δ in the three-dimensional geometry by assuming that the EMP, neglecting absorption ($e_0 \ll \varepsilon_{in}$), has a Gaussian spatial shape. Namely, the intensity $I = c|\boldsymbol{E}_0|^2/(8\pi) = I_{max} f(z, \boldsymbol{r}_\perp, t)$ of the pulse propagating along the z axis in the plasma is described in time t and direction $\boldsymbol{r}_\perp$ perpendicular to the z axis by the Gaussian function

$$f(z,\boldsymbol{r}_\perp,t) = \frac{1}{1+(z/z_R)^2} \times \exp\left\{-2\frac{(t-z/c)^2}{\tau^2} - 2\frac{\boldsymbol{r}_\perp^2}{r_0^2[1+(z/z_R)^2]}\right\}, \quad (26)$$

where $z_R = \omega_0 r_0^2/(2c)$ is the Rayleigh length of the pulse. Expression (26) describes in the parabolic approximation the propagation of the EMP taking into account its diffraction in the direction perpendicular to the z axis, the pulse radius being minimum and equal to r_0 at the point of tightest focusing with the coordinate $z = 0$. By substituting (26) into (25), taking into account the expression $\varepsilon_{in} = (\pi/2)^{3/2} I_{max}\tau r_0^2$, we obtain

$$\Delta = \frac{n_e}{n_c}\frac{z_R}{c}\nu_C(T_e)F,$$

$$F = \frac{\sqrt{2}}{\pi^{3/2}}\frac{1}{\delta}\int_V\frac{d^3\boldsymbol{r}}{r_0^2 z_R}\int_{-\infty}^{+\infty}\frac{dt}{\tau}G(\delta f(\boldsymbol{r},t),\eta=0). \quad (27)$$

By calculating the integral over the volume V in (27), we assume that this volume is a layer of thickness L ($-L/2 \leqslant z \leqslant L/2$). Then, the quantity F proves to be a function of δ and $\zeta = L/(2z_R)$:

$$F(\delta,\zeta) = \int_0^{\zeta}\Phi(\delta,s)ds,$$

$$\Phi(\delta,s) = \frac{4}{3}\left(\frac{2}{\pi}\right)^{1/2}\frac{s^2}{\delta}\frac{3+s^2}{1+s^2}\int_0^{\infty}G\left(\delta\frac{\exp(-2y^2)}{1+s^2},\eta=0\right)dy, \quad (28)$$

where $s = z/z_R$ is the normalised spatial coordinate. The distribution of the function $\Phi(\delta,s)$ inside a plasma layer for $\delta = 100$ is shown in Fig. 1. The function $\Phi(\delta,s)$ has a maximum whose position z_{max} depends on δ as $z_{max} \approx 0.5\sqrt{\delta}\,z_R$. One can see that the plasma layer can be comparatively thin [$\zeta \lesssim z_{max}/(2z_R) \approx \sqrt{\delta}$] or thick [$\zeta \gg z_{max}/(2z_R)$]. The dependence of the shift Δ of the square of frequency (27) on δ proves to be different in these two cases. Figure 2 shows the function $F(\delta,\zeta)$ [see (28)] for a thick plasma layer (for all presented values of δ) [curve (*1*)] and for $\zeta = 5$ [curve (*2*)].

Note that, according to expressions (28), the vicinity of the maximum of the function $\Phi(\delta, s)$ [when $s \sim \sqrt{\delta}$ (see Fig. 1)] makes the main contribution to the shift Δ (27) (main contribution to the integral over s). For $s \sim \sqrt{\delta}$, the oscillatory energy $m_e u_E^2/2$ of an electron in the electromagnetic field proves to be of the order of T_e. Therefore, the region of comparatively small energies, where $m_e u_E^2/2 \sim T_e$, makes the main contribution to the shift Δ of squares of frequencies in a thick layer. This is explained by the fact that the frequency of Coulomb electron–ion collisions is proportional to u_E^{-3} for large values of u_E.

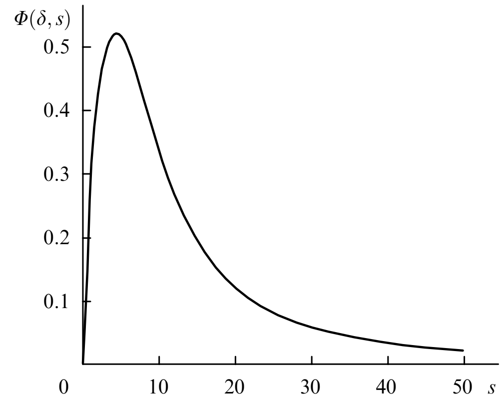

Figure 1. Dependence of the function $\Phi(\delta, s)$ (28) on $s = z/z_R$ for $\delta = 100$.

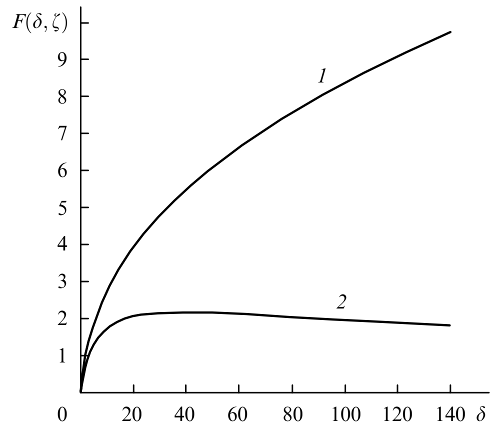

Figure 2. Dependences of the function $F(\delta, \zeta)$ (28) entering the expression for the shift of the square of frequency (27) on δ for a thick (for all presented values of δ) plasma layer (1) and for $\zeta = 5$ (2).

4.1 Estimate of collision absorption of electromagnetic energy and electron heating

Expressions (27) and (28) were derived assuming a weak relative collision absorption of the electromagnetic pulse ($A = e_0/\varepsilon_{in} \ll 1$) and a weak relative local change in the electron temperature due to ohmic heating [$\Delta T_e(\boldsymbol{r}, t = +\infty)/T_e \ll 1$]. A change in the electron temperature $\Delta T_e(\boldsymbol{r}, t = +\infty)$ can be estimated from the local equation (cf. Ref. [12])

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e T_e\right) = n_e \nu m_e u_E^2 . \quad (29)$$

Assuming that $\Delta T_e(\boldsymbol{r}, t = +\infty) \ll T_e$ and $n_e = \text{const}$, we obtain from (29)

$$\Delta T_e(\boldsymbol{r}, t = +\infty) = \frac{2}{3} m_e \int_{-\infty}^{+\infty} \nu u_E^2 \mathrm{d}t \approx \frac{2}{3} m_e \int_{-\infty}^{+\infty} \overline{\nu u_E^2} \mathrm{d}t. \quad (30)$$

The latter relation in (30) is obtained by neglecting terms of the order of $(\omega_0 \tau)^{-1}$, which are small compared to unity. Note that Eqn (29) integrated over the time from $-\infty$ to $+\infty$ and over the plasma volume V gives, together with (18) and (7), the energy conservation law

$$\frac{3}{2} \int_V [n_e T_e(\boldsymbol{r}, t = +\infty) - n_e T_e(\boldsymbol{r}, t = -\infty)] \mathrm{d}^3 \boldsymbol{r} = e_0 . \quad (31)$$

From expression (24) for ν, we find (cf. Ref. [12])

$$\overline{\nu \frac{m_e u_E^2}{2T_e}} = \frac{3}{4} \nu_C(T_e) H\left(\frac{m_e |\boldsymbol{V}_E(\boldsymbol{r}, t)|^2}{4T_e}, \eta\right), \quad (32)$$

where

$$H(\xi, \eta) = \frac{2}{\sqrt{\xi}} \int_0^{\xi} \sqrt{x}\, \mathrm{e}^{-x} [I_0(P(\eta)x) - P(\eta) I_1(P(\eta)x)] \mathrm{d}x;$$

$I_0(b)$ is the modified zero-order Bessel function. By substituting (32) into (30), we obtain

$$\frac{\Delta T_e(\boldsymbol{r}, t = +\infty)}{T_e} = \nu_C(T_e) \tau q(\boldsymbol{r}),$$
$$q(\boldsymbol{r}) = \int_{-\infty}^{+\infty} H\left(\frac{m_e |\boldsymbol{V}_E(\boldsymbol{r}, t)|^2}{4T_e}, \eta\right) \frac{\mathrm{d}t}{\tau}. \quad (33)$$

For a linearly polarised electromagnetic Gaussian pulse ($\eta = 0$) with the space–time shape (26), q is the function of δ, $R = r/r_0$, and $s = z/z_R$:

$$q = q(\delta, s, R)$$
$$= 2 \int_0^{\infty} H\left(\frac{\delta}{1+s^2} \exp\left(-2y^2 - \frac{2R^2}{1+s^2}\right), \eta = 0\right) \mathrm{d}y. \quad (34)$$

The spatial distribution of q for $\delta = 100$ is shown in Fig. 3. The maximum value $q_{max} \approx 1.7$ is achieved at the point $r = 0$, $s_{max} \approx 2.2$. The value of q_{max} virtually does not depend on δ, while s_{max} slowly increases with increasing δ.

By substituting $\Delta T_e(\boldsymbol{r}, t = +\infty)$ from (33) into (31), we find the relative EMP energy absorbed in collisions (absorption coefficient)

$$A = \frac{3}{2} \frac{n_e T_e}{\varepsilon_{in}} \nu_C(T_e) \tau \int_V q(\boldsymbol{r}) \mathrm{d}^3 \boldsymbol{r}.$$

By using (34), we find for a linearly polarised Gaussian pulse

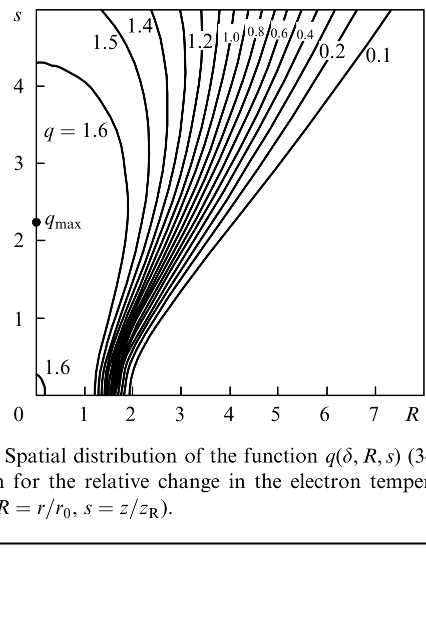

Figure 3. Spatial distribution of the function $q(\delta, R, s)$ (34) entering the expression for the relative change in the electron temperature (33) for $\delta = 100$ ($R = r/r_0$, $s = z/z_R$).

$$A = \frac{n_e}{n_c}\frac{z_R}{c}\,\nu_C(T_e)a(\delta,\zeta), \quad a(\delta,\zeta) = \int_0^{\zeta}\Psi(\delta,s)\mathrm{d}s, \qquad (35)$$

where

$$\Psi(\delta,s) = 96\left(\frac{2}{\pi}\right)^{1/2}\int_0^{\infty}k(y)K\left(\delta\,\frac{\mathrm{e}^{-2y^2}}{1+s^2}\right)\mathrm{d}y;$$

$$k(y) = y\mathrm{e}^{-3y^2}\int_0^{y}x^2\mathrm{e}^{x^2}\mathrm{d}x;$$

$$K(b) = \mathrm{e}^{-b}[I_0(b) - I_1(b)].$$

The distribution of the function $\Psi(\delta,s)$ inside a plasma layer for $\delta = 100$ is shown in Fig. 4. The function $\Psi(\delta,s)$ saturates in the vicinity of the point $s_s \sim \sqrt{\delta}$. The dependence of a on δ is shown in Fig. 5 for $\zeta = 5$, smaller than $s_s = 10$. In the interval $10 \leqslant \delta \leqslant 150$, the value of a changes approximately as $1/\sqrt{\delta}$, in accordance with the fact that the frequency of electron–ion collisions is proportional to u_E^{-3} for large u_E.

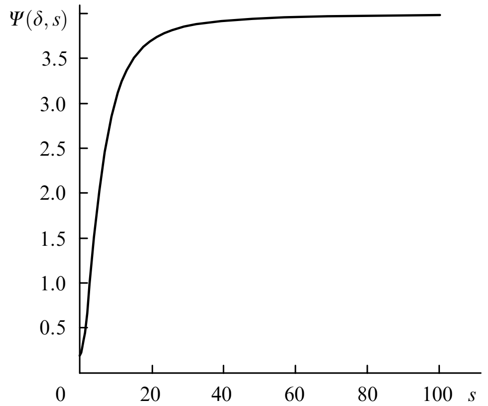

Figure 4. Dependence of the function $\Phi(\delta,s)$ on $s = z/z_R$ for $\delta = 100$.

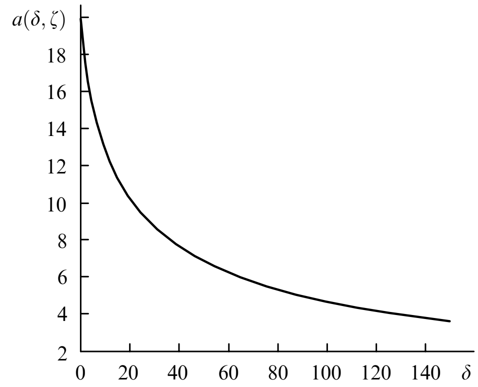

Figure 5. Dependence of the function $a(\delta,\zeta)$ determining the coefficient of collision absorption of the EMP energy (35) on δ for $\zeta = 5$.

Let us make some estimates for the EMP and plasma parameters that can be easily achieved at present. For the plasma temperature $T_e = 10$ eV, the electron density $n_e = 2.7 \times 10^{18}$ cm^{-3}, the layer thickness $L = 10z_R$ for the EMP with the wavelength $\lambda = 0.8$ μm, $r_0 = 20$ μm, and $I_{max} = 1.7 \times 10^{16}$ W cm^{-2}, we have the parameter $\delta \approx 100$, $z_R \approx 1.6$ mm, $L \approx 1.6$ cm, and the relative shift $\Delta \approx 0.02$. In this case, $A \approx 0.04 \ll 1$, and for the pulse duration $\tau = 30$ fs, the maximum ratio $\Delta T_e/T_e \approx 0.06 \ll 1$. The pulse power $P_p \approx 0.11$ TW is comparatively low and is much lower than the critical power $P_c \approx 11$ TW required for relativistic self-focusing [the value of P_c in GW was estimated from the known relation $P_c \approx 17(n_c/n_e)$ (see, for example, review [13]). In addition, the EMP intensity proves to be low for excitation of SRS. Indeed, by estimating the space–time gain g, determining the exponential increase in SRS according to [13], we obtain

$$g \approx \frac{n_e}{n_c}\,a_0\omega_0\left(\frac{z_R\tau}{2c}\right)^{1/2} \approx 0.09,$$

where the normalised vector potential of the EMP is $a_0 \approx 8.6 \times 10^{-10}\lambda I_{max}^{1/2} \approx 0.09$ (λ is measured in μm, I_{max} in W cm^{-2}). Therefore, the nonlinear amplification of SRS is absent at the parameters chosen.

5. Conclusions

The frequency of an EMP propagated through matter is shifted because the frequency of collisions of electrons with ions or neutral particles in the EMP field is nonstationary. The direction of the frequency shift characterises the type of interaction of electrons with heavy particles. In the case of the Coulomb electron–ion interaction, the blue shift is observed, whereas upon collisions of electrons with neutral particles, the EMP shifts to the red.

The relative frequency shift (27) in a stripped plasma depends substantially on the gas temperature itself [dependence $\nu_C(T_e)$ (23)]. In addition, the frequency shift depends on the relation between the maximum oscillatory energy of an electron in the electromagnetic field and its thermal energy δ, as well as the plasma layer thickness. From the point of view of optical measurements, the value $\Delta \approx 0.02$ (see the above estimates) is comparatively large and can be readily detected in experiments. The shift Δ can be used for estimating the temperature of the electron component of the plasma.

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