PACS numbers: 42.50.Md; 42.50.Gy; 42.65.Tg
DOI: 10.1070/QE2004v034n07ABEH002643

Features of evolution of a 0π pulse in a medium with an inhomogeneously broadened resonance quantum transition

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***Abstract*. The results of numerical simulations are outlined for the evolution of a 0π pulse in a medium with an inhomogeneously broadened quantum transition neglecting irreversible relaxation. A rule is formulated for the prediction of the 0π-pulse structure at long distances, which relies on the notion of the 0π-pulse area as the area under the curve of the modulus of its envelope. Analytical relations between the duration, the peak value, and the propagation velocity are given for the 0π pulse of the breather type.**

Keywords: *self-induced transparency, inhomogeneous broadening, coherent effects.*

1. Introduction

Even early in the study of the effect of self-induced transparency (SIT), it was discovered that SIT can be effected not only by optical solitons – 2π pulses, but by some kinds of 0π pulses as well. Lamb [1] derived the 0π-pulse envelopes as particular solutions of the sine-Gordon equation in the study of SIT in the absence of inhomogeneous broadening. Numerical analysis [2] showed that the fundamental features of the 0π pulses remain unchanged even if inhomogeneous broadening is taken into account. An experimental study of the 0π-pulse propagation through a ruby crystal was undertaken in Ref. [3], in which this process was also numerically simulated.

In the numerical analysis performed in Ref. [2], boundary conditions of a rather general form were used. In particular, the input pulse was assumed to have a nonzero area. This hampered the systematisation of the results obtained. Furthermore, the computational capabilities were insufficient for investigating some details of 0π-pulse evolution at the time when study [2] was performed. This is the reason why the authors of Ref. [3] failed to perform the numerical simulation under conditions close to experimental (for more details, see Ref. [4]).

Our work is concerned with the numerical simulation of the evolution of a 0π pulse, this term being applied to any pulse with a zero area (in the sense of the SIT theory [5]).

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Received 27 February 2004
Kvantovaya Elektronika **34** (7) 652 – 656 (2004)
Translated by E.N. Ragozin

The boundary conditions of the calculation correspond to a 0π pulse obtained by the superposition of antiphase laser pulses shifted relative to each other in time. It was precisely this natural way of producing a 0π pulse that was realised in experiment [3]. We take into account the inhomogeneous broadening and restrict ourselves to the case of a perfect resonance, i.e., the case when the central frequency of the inhomogeneously broadened quantum transition coincides with the carrier frequency of the input pulse. Being concerned only with coherent effects, we ignore irreversible relaxation.

Studying the properties of 0π pulses is beneficial for the following reasons. The SIT theory, which is primarily aimed at the investigation of the properties of 2π pulses, is a part of the nonlinear wave theory and complements it with significant analytic results obtained, in particular, by the method of inverse scattering problem [6, 7]. The results of numerical analysis outlined below give promise that a similar role may be played by the study of 0π-pulse evolution, because it is clearly seen to obey some simple laws. Studying the properties of the 0π pulse is also useful because the SIT theory, in which it appears, is an important step towards the understanding of resonance processes in the femtosecond range of laser pulse durations [8]. Finally, it is difficult to interpret correctly the experimental data [3] without knowing the properties of the 0π pulse, even though the irreversible relaxation, which is neglected in our work, plays a significant role in these experiments.

2. Formulation of the problem

The resonance medium is represented as an ensemble of two-level quantum objects, whose nondegenerate levels are numbered by figures 1 and 2 in the order of increasing energy. The Gaussian profile of the spread of quantum transition frequencies about the central frequency ω is assumed to have a width $2/T$ at the e^{-1} level. Let $\boldsymbol{ijk}$ be the right orthonormal basis of the laboratory system of coordinates xyz and $p = (1/\sqrt{2})|p_{x12} - \mathrm{i}p_{y12}|$, where p_{x12} and p_{y12} are the x and y components of the vector of the electric dipole moment of the $1-2$ transition. The electric field intensity of circularly polarised laser radiation propagating along the z axis can be written in the form

$$\boldsymbol{E} = \mu(\boldsymbol{i} + \mathrm{i}\boldsymbol{j})a(z, t)\exp\{\mathrm{i}\omega[(z\eta/c) - t]\} + \mathrm{c.\,c.},$$

where $a(z, t)$ is the pulse envelope; η is the linear refractive index of the medium with quantum objects implanted in it;

and $\mu = 3\hbar/[2^{3/2}T(\eta^2+2)p]$. Let us introduce the dimensionless independent variables

$$s = \alpha T z, \quad w = (t - z\eta/c)/T,$$

where

$$\alpha = \frac{2\pi\omega p^2(\eta^2+2)^2 N}{9c\hbar\eta};$$

N is the concentration of the resonance objects. In the slowly varying envelope approximation [9, 10] we obtain a system of equations describing self-consistently the field – medium interaction:

$$\frac{\partial a}{\partial s} = \frac{\mathrm{i}}{\sqrt{\pi}}\int_{-\infty}^{\infty}\sigma_{21}\exp(-\varepsilon^2)\mathrm{d}\varepsilon,$$

$$\frac{\partial\sigma_{21}}{\partial w} + \mathrm{i}\varepsilon\sigma_{21} = \mathrm{i}a(\sigma_{11}-\sigma_{22}), \qquad (1)$$

$$\frac{\partial(\sigma_{11}-\sigma_{22})}{\partial w} = \mathrm{Im}(a\sigma_{21}^*),$$

where σ_{ik} $(i, k = 1, 2)$ are the amplitudes of the density matrix elements.

The system (1) was supplemented with the initial conditions $\sigma_{11}(s, w=0) = 1$, $\sigma_{12}(s, w=0) = \sigma_{22}(s, w=0) = 0$, $s \geqslant 0$, which describe the initially unexcited medium. The boundary condition specifying the radiation pulse at the input surface of the $(s = 0)$ resonance medium was of the form

$$a(s=0, w) = f(w) - f(w + \Delta w), \qquad (2)$$

$$f(w) = r\left[\exp\left(q\frac{w-w_0}{\tau}\right) + \exp\left(-3q\frac{w-w_0}{\tau}\right)\right]^{-1}, \qquad (3)$$

where r is the amplitude numerical factor; $q = 1.59$; and τ is the FWHM of the pulse $f(w)$. The envelope $f(w)$ describes a single bell-shaped pulse whose leading edge is much steeper than the trailing edge. Such an envelope corresponds in form to the envelopes of the laser pulses employed in SIT experiments in ruby crystals [3] and rubidium vapour [11]. The envelope (2) describes the 0π pulse obtained by the superposition of two antiphase laser pulses shifted relative to each other in time by Δw. Such a shaping of the input 0π pulse models the experimental technique of its preparation realised also in Ref. [3].

The boundary-value problem (1) – (3) was solved numerically. The code employed for this purpose was described in Refs [4, 12, 13], in which the methods of checking the validity of calculations were also discussed.

3. Method of presentation of the results of calculations

The results of calculations are represented as the plots of the function $A_s(w)$, where $A_s(w) = |a(s = \mathrm{const}, w)|$; and A_{sm} is the maximum value of the function $A_s(w)$. In the case of exact resonance under study, the quantity $a(s, w)$ is real, and therefore $a(s, w) = \pm A_s(w)$. The polarity of the envelope of $a(s, w)$ is specified by the signs '+' and '−' in the plot of the function $A_s(w)$. When the polarities alternate, these signs are indicated only by some of the subpulses.

As the integral characteristic of the envelope, we consider the function

$$\Theta(s) = \int_{-\infty}^{\infty} A_s(w)\mathrm{d}w,$$

which represents the area under the plot of the modulus of the function $a(s, w)$ for a fixed s. In the SIT theory [5], the pulse area $\bar{\Theta}(s)$ is determined by the integral of the function $a(s, w)$ itself. In the case of an exact resonance and the input 0π pulse, and the pulse (2) is precisely the one, according to the theorem of areas [5], the equality $\bar{\Theta}(s) = 0$ is fulfilled for any s.

To characterise the bell-shaped envelopes $F_s(w)$ obtained in calculations, we used two dimensionless quantities: the maximum value F_{sm} and τ_{p} the pulse half-width at the $\mathrm{sech}(1)F_{sm}$ level. If the above envelope is not deformed with increasing s, we introduce into consideration the velocity υ of motion of this envelope in the frame of reference s, w: $\upsilon = (\mathrm{d}w_{\mathrm{m}}/\mathrm{d}s)^{-1}$, where $w_{\mathrm{m}} = w_{\mathrm{m}}(s)$ is the dependence of the value of w, at which $F_s(w) = F_{sm}$, on s. When $F_s(w)$ is the 2π-pulse envelope, in accordance with the SIT theory [5]

$$F_s(w) = F_{sm}\,\mathrm{sech}\left(\frac{w - s/\upsilon}{\tau_{\mathrm{p}}}\right),$$
$$\upsilon = \sqrt{\pi}\left[\tau_{\mathrm{p}}^2\int_{-\infty}^{\infty}\frac{\exp(-\varepsilon^2)}{1+\tau_{\mathrm{p}}^2\varepsilon^2}\mathrm{d}\varepsilon\right]^{-1}, \qquad (4)$$

$$F_{sm} = 2/\tau_{\mathrm{p}}, \qquad (5)$$

F_{sm} being actually independent of s. In our work, a pulse discovered in the calculation is called the '2π pulse' if, on substitution of its parameters F_{sm}, τ_{p}, and υ, into expressions (4) and (5), these equalities are fulfilled to within 2 %.

As the energy characteristic, we consider the parameter $e_s = E(s)/E(0)$, where $E(s)$ is the energy transferred by radiation across a unit area of the cross section at a distance s to the entrance into the resonance medium.

4. Results of calculations

The calculations carried out for different combinations of the parameters r, τ, Δw subject to the condition $|\Delta w - \tau|/\tau < 0.3$ agree well with the following assumption: the qualitative pattern of radiation evolution depends on which of the intervals (Θ_{i-1}, Θ_i) $(i = 1, 2, 3, ...)$ the area $\Theta(0)$ of the input laser pulse belongs to. The numbers Θ_i are specified by the expressions

$$\Theta_i = \left(4\left[\frac{i}{2}\right] + 3\left\{\frac{i}{2}\right\}\right)\pi, \quad i = 0, 1, 2, 3, ..., \qquad (6)$$

where $[a]$ and $\{a\}$ are the integer and fractional parts of the number a. This statement is illustrated below by the calculations assuming that $\tau = \Delta w = 22$ in expressions (2) and (3). With this value of τ, the envelope (3) corresponds (for more details, see Ref. [13]) to the envelopes of the laser pulses employed in SIT experiments in rubidium vapour

[11]. Note that the relation $\Theta(0) = 19.89r$ is valid for the given parameters τ and Δw. By taking sequentially $i = 1, 2, 3, \ldots$, we consider the following cases.

Case 1: $i = 1$, $0 < \Theta(0) < 1.5\pi$. The results of calculations for $\Theta(0) = 1.43\pi$ are depicted in Fig. 1. The signs '+' and '−' indicate the polarity of the peaks, the polarities alternating in this case. The calculation showed that the pulse decreases for $s > 2$ with an insignificant increase in duration so that

$$\Theta(s) = \Theta(2)\exp[-\sqrt{\pi}(s-2)]. \quad (7)$$

Expression (7) has the form of the theorem of areas for a weak signal [14, 15], but differs from it in that the area $\bar{\Theta}(s)$ is replaced with $\Theta(s)$. For $s > 2$, the pulse is immobile in the frame of reference s, w, and in the laboratory frame of reference its velocity is therefore equal to c/η. Such a pulse is hereafter referred to as a decaying 0π pulse.

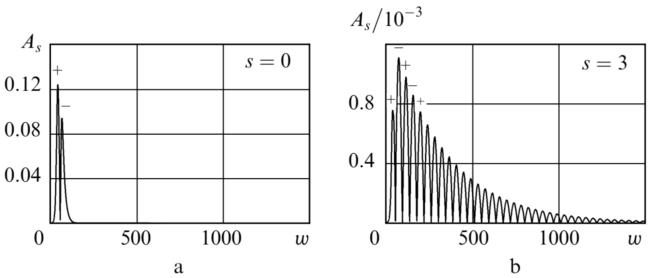

Figure 1. Formation of a decaying 0π pulse for $\Theta(0) = 1.43\pi$: the input 0π pulse (a) and the decaying 0π pulse for $s = 3$ (b).

Case 2: $i = 2$, $1.5\pi < \Theta(0) < 4\pi$. The results of calculations for $\Theta(0) = 1.8\pi$ are shown in Fig. 2. As in the previous case, the polarities of the peaks alternate. It follows from Fig. 3 that for $s > 2$ the pulse energy e_s is constant, while the peak value A_{sm} oscillates. In the nonlinear wave theory, a pulse of this type with a nonstationary form is called a breather [16, 17] (hereafter, a 0π breather). The thin line in Figs 2b and 2c depicts the envelope $F_s(w)$ of the function $A_s(w)$. Our calculation showed that this 'secondary envelope' is determined by expressions (4) with condition (5) replaced by the condition

$$F_{sm} = 4/\tau_p, \quad (8)$$

where, $F_{sm} = 0.0738$ is the peak value of the envelope $F_s(w)$. Note that the discrepancy between the values of υ

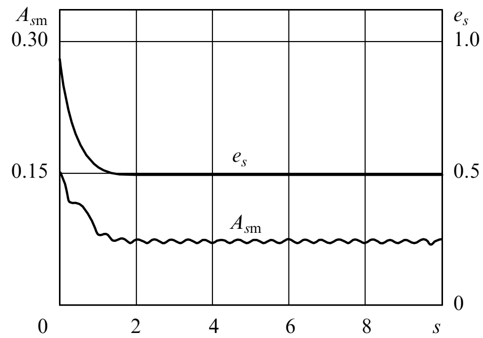

Figure 3. Peak value A_{sm} of the envelope $A_s(w)$ and energy e_s as functions of s in the 0π-breather formation for $\Theta(0) = 1.8\pi$.

[calculated by expressions (4) and (8) for the specified F_{sm} and obtained directly from the dependences $A_s(w)$] does not exceed 1%.

Since the envelope $F_s(w)$ is not deformed with increasing s, its velocity υ and duration τ_p are naturally taken as the velocity and duration [at the sech(1)F_{sm} level] of the 0π breather itself. Therefore, it follows from expressions (4), (5), and (8) that the velocities of the 0π breather and the 2π pulse are equal for equal τ_p, but the peak intensity of the electric field of the 0π breather is twice as strong as that of the 2π pulse. The possibility of existence of a pulse with such properties in the absence of the inhomogeneous broadening was predicted [1] in the analysis of particular solutions of the sine-Gordon equation. However, the relationship between υ and τ_p obtained in Ref. [1] is substantially different from that presented in our work. The numerical analysis with the inclusion of the inhomogeneous broadening performed in Ref. [2] also revealed the possibility of appearance of the 0π breather, but its structural details were not studied in Ref. [2].

The results of calculations for $\Theta(0) = 1.56\pi$ and $\Theta(0) = 3.7\pi$ are presented in Fig. 4. One can see that as $\Theta(0)$ increases, the 0π-breather duration and the number of subpulses in it decreases, whereas the peak value increases.

Case 3: $i = 3$, $4\pi < \Theta(0) < 5.5\pi$. The results of calculation for $\Theta(0) = 5\pi$ are presented in Fig. 5. The separate pulses shown in Fig. 5 are the 2π pulses of the opposite polarity. Having different durations, these pulses propagate at different velocities. A similar two-pulse field configuration was considered in Refs [1, 2] and was called a 'separating' 0π pulse.

Case 4: $i = 4$, $5.5\pi < \Theta(0) < 8\pi$. The results of calculation for $\Theta(0) = 7\pi$ are presented in Fig. 6a. In this case, a

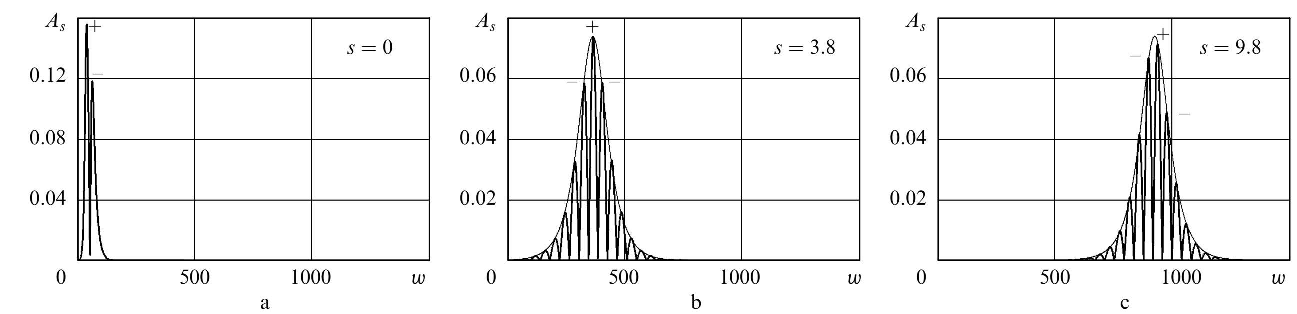

Figure 2. Formation of a 0π breather for $\Theta(0) = 1.8\pi$: the input 0π pulse (a), 0π breather for $s = 3.8$ (b) and $s = 9.8$ (c). The thin curves indicate the 'secondary envelope' of the 0π breather $F_s(w)$.

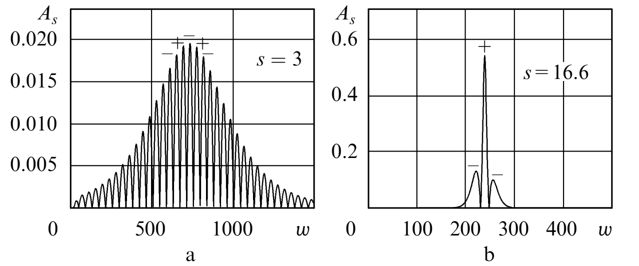

Figure 4. Structure of the 0π breather for $\Theta(0) = 1.56\pi$, $s = 3$ (a) and $\Theta(0) = 3.7\pi$, $s = 16.6$ (b).

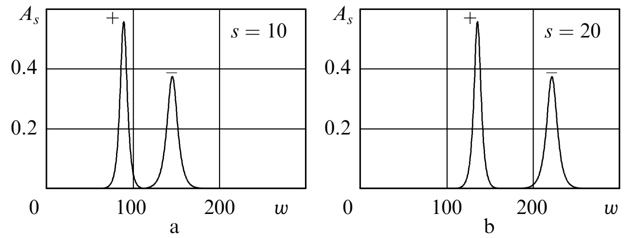

Figure 5. Evolution of a 'separating' 0π pulse for $s = 10$ (a) and $s = 20$ (b); $\Theta(0) = 5\pi$.

'separating' 0π pulse (pulses *1* and *2*) and a 0π breather [pulse (*3*)] are produced.

Case 5: $i = 5$, $8\pi < \Theta(0) < 9.5\pi$. The results of calculation for $\Theta(0) = 9\pi$ are presented in Fig. 6b. In this case, two 'separating' 0π pulses emerge, the first of them being formed by the first pair of pulses and the second one by the second pair.

Case 6: $i = 6$, $9.5\pi < \Theta(0) < 12\pi$. The results of calculation for $\Theta(0) = 10.5\pi$ are presented in Fig. 6c. Pulses *1* and *3* form the first 'separating' 0π pulse, while pulses *2* and *4* form the second 'separating' 0π pulse. As s increases, pulse *3* outruns pulse *2*, and the 2π pulses line up in the order of decreasing peak values. Pulse *5* is a 0π breather.

By extrapolating the results of calculations to larger values of i, we can draw the following conclusion. Let $\Theta(0)$ be the area under the plot of the modulus of the input 0π-pulse envelope. For $\Theta(0) < 1.5\pi$, the only decaying 0π pulse is formed. For $\Theta(0) > 1.5\pi$, several 'separating' 0π pulses and one 0π breather can be formed. The number of 'separating' 0π pulses is $[\Theta(0)/4\pi]$, and the 0π breather is formed only when $\{\Theta(0)/4\pi\} > 3/8$, i.e., when the residue of the division of $\Theta(0)$ by 4π exceeds 1.5π.

Note that our calculations enabled us to determine the critical values i of input 0π-pulse area with a precision of about 0.1π. Calculations carried out for larger values of Δw ($\Delta w = 4\tau$) showed that the 0π-breather production threshold $\Theta(0)$ is in the vicinity of 1.2π. That is why it is not inconceivable that Θ_i depends slightly on Δw.

According to the nonlinear wave theory [16, 17], breathers, and solitons, interact elastically in collisions. To verify this fact for a 0π breather, we performed calculations with $a(s = 0, w)$ in the form of input pulses (2) shifted relative to each other by 660 time units of w, with $\tau = \Delta w = 22$ and the areas $\Theta(0)$ equal to 1.8π (the first pulse) and 3π (the second pulse). The results of this calculation for large s, when the 0π breathers have already been formed, are presented in Fig. 7. One can see from Figs 7a and 7c that the 0π-breather forms prior to and after the collision are the same. The calculation showed that the 0π-breather velocities also remain

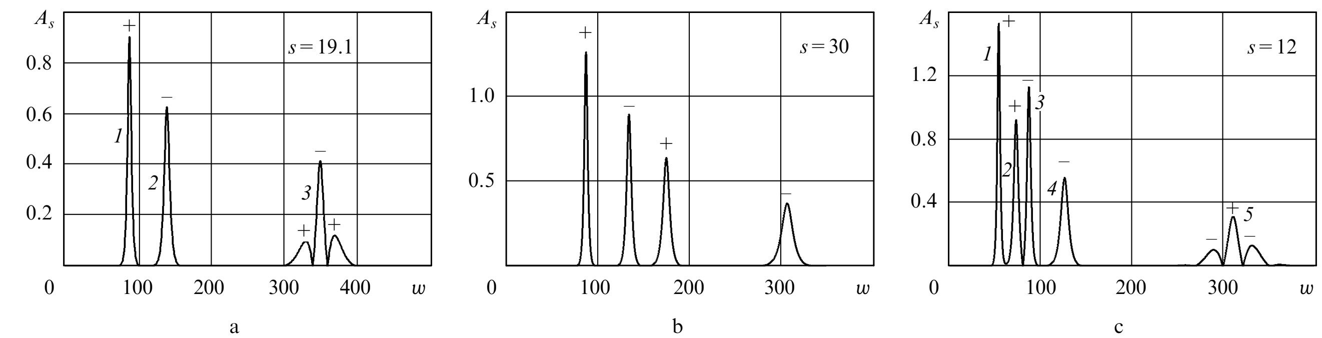

Figure 6. Complex 0π pulse consisting of a disintegrating 0π pulse (*1*, *2*) and a 0π breather (3) for $\Theta(0) = 7\pi$, $s = 19.1$ (a), of two disintegrating 0π pulses for $\Theta(0) = 9\pi$, $s = 30$ (b), and of two disintegrating 0π pulses (*1*, *3* and *2*, *4*) and a 0π breather (*5*) for $\Theta(0) = 10.5\pi$, $s = 12$ (c).

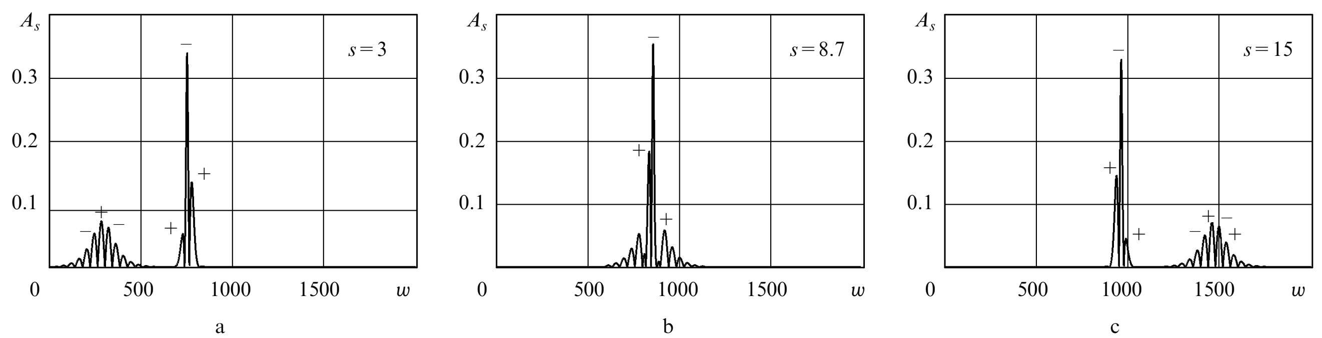

Figure 7. Interaction of 0π breathers: isolated 0π breathers prior to (a) and after (c) the collision of the pulses and structure of the envelope in the course of their collision (b).

unchanged. At the instant of overlap (Fig. 7b), the peak value of the envelope is smaller than the sum of the peak values of individual 0π breathers. Nevertheless, this does not suggest that the interaction is nonlinear, because the envelope $a(s, w)$ can assume both positive and negative values. The nonlinearity of the process now manifests itself in the change of velocities of both breathers inside the overlap region. Our calculation revealed that the greater 0π breather escapes from the collision region 13 time units of w earlier than is prescribed by its velocity prior to (and after) the collision. By contrast, the minor 0π breather stays in the collision region longer by 57 time units of w. Such a behaviour of the 0π breathers is similar to the behaviour of two solitons, whose interaction is accompanied by the phase shift of their spatiotemporal trajectories [16].

5. Conclusions

The calculations presented in our work are consistent with the well-known fact [1, 2] that SIT is realised not only by optical solitons – 2π pulses, but by some types of 0π pulses as well. For one of these types bearing the characteristic features of a breather, we derived analytic expressions relating its peak value, duration, and the propagation velocity in the case of the inhomogeneously broadened quantum transition. Another type of the 0π pulse, which does not lose its energy upon propagation, is a pair of 2π pulses of different duration and opposite polarity. Furthermore, the calculation revealed the possibility of existence of a decaying 0π pulse with the area under the modulus of its envelope decaying exponentially with distance.

The decay of the input 0π pulse was shown to obey a simple law. In its formulation, the input pulse is characterised by the area $\Theta(0)$ under the plot of the modulus of the envelope, while the area in the SIT theory is defined as the area under the plot of the envelope itself. The above-mentioned law is a rule for the determination of the 0π-pulse structure at long distances using the given value of $\Theta(0)$.

In our work, which is concerned with the study of coherent effects, the processes of irreversible relaxation were disregarded. A numerical analysis with the inclusion of these processes suggests that they exert a nontrivial effect on the evolution of the 0π pulse. These calculations, with application to the simulation of the experiment of Ref. [3] with the 0π-breather observation in a ruby crystal, will be detailed elsewhere. Departures from the perfect-resonance condition give rise to interesting features emerging in the course of the 0π-pulse formation. A study of this problem will also be the subject of further investigation.

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