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An analog + digital phase-frequency detector for phase locking of a diode laser to an optical frequency comb

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***Abstract*. Optical phase locking of a diode laser to a mode of a femtosecond optical comb has been achieved with a system that combines the advantages of analog and digital phase detectors. Low noise and fast response are combined with a broad phase range and lock reliability. Details of the circuit are illustrated, and properties of different phase detectors in view of optical frequency measurements are discussed.**

Keywords: *optical phase locking, diode lasers, femtosecond optical combs.*

1. Introduction

Absolute frequency measurements in the optical domain have recently acquired a great interest. Laser cooling and high-resolution laser spectroscopy on neutral atoms and on trapped ions opened new perspectives in frequency metrology [1, 2] as well as in the physics of fundamental constants [3].

To perform the absolute measurement of an optical frequency, it is necessary to connect the frequency of interest to that of a secondary standard. This generally requires to bridge very large frequency differences. Until few years ago, phase-coherent links between distant optical frequencies were obtained by means of complicated frequency chains. The recent advent of frequency combs based on mode-locked lasers [4, 5] has greatly simplified the structure of optical frequency chains. Nowadays it is possible to connect any optical frequency to a RF standard with the use of a single mode-locked laser [6].

2. Phase locking of diode lasers

Any optical frequency chain requires a number of phase-locked lasers. Because of their wide wavelength reliability, diode lasers are particularly suited for this application. One of the main difficulties in optical phase locking is the high loop bandwidth, required due to the presence of fast phase fluctuations. Free-running diode lasers have effectively a large linewidth (20 – 100 MHz, typically), caused by a very small cavity length; for the same reason, diode lasers present a high level of phase noise at high Fourier frequencies. Optical feedback can be used to reduce the linewidth. A very common setup for linewidth reduction is the extended cavity diode laser (ECDL); in most cases a diffraction grating is used to provide optical feedback. The presence of the grating also allows a large slow frequency tuning, while fast frequency control is possible through the diode laser's injection current. Nevertheless, the linewidth of an ECDL is still relatively large (typically of the order of several hundreds of kHz).

The limits imposed by the laser linewidth on stable phase-lock can be easily understood in the simple assumption of white frequency noise. In this case the two-sided spectral density of frequency fluctuations $S_{\rm w}$ (in Hz^2/Hz) is related to the laser linewidth by $\delta\nu = \pi S_{\rm w}$. The contribution of fast phase fluctuations to the root-mean-square phase noise can be obtained by integrating the two-sided spectral density of phase fluctuations

$$\Phi(\nu) = \frac{S_{\rm w}}{\nu^2} \quad (1)$$

for Fourier frequencies higher than a value ν_0, thus obtaining

$$\langle\varphi^2\rangle_{\nu>\nu_0} = \frac{2\delta\nu}{\pi\nu_0}. \quad (2)$$

It must be considered that phase detectors have a finite phase range of operation. In order to have a stable phase locked loop, phase fluctuations must be kept well within the range of the phase detector. So the poor spectral characteristics of the laser source impose to have either a very fast loop or a very broad-range phase detector. The fastest loops can be obtained with the use of an analog phase detector, while a digital phase detector can offer a very broad phase range. In this paper we used a mixed system, combining the advantages of both kinds of phase detectors.

3. Requirements for stable phase-lock

An optical phase-locked loop (OPLL) is used to stabilise the frequency difference between a slave laser and a master laser. A reference radiofrequency from a local oscillator and the beat note between the two lasers are sent into a phase detector, whose output gives the error signal to control the frequency of the slave laser. In closed-loop condition the output of the phase detector is maintained constant, so that

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the frequency difference between the two lasers equals the reference radiofrequency.

We indicate with $f(t)$ the instantaneous frequency difference between the beat note and the local oscillator, and with $S_f(\nu)$ its power spectral density. Under closed-loop conditions $S_f(\nu)$ is reduced by the factor

$$A(\nu) = \left| \frac{1}{1 + G(\nu)} \right|^2 \tag{3}$$

where $G(\nu)$ is the open-loop transfer function. A very simple model for $G(\nu)$ can be obtained by assuming that the error signal is directly applied to the slave laser and that the frequency response of the slave laser is flat. As the transfer function of the phase detector is proportional to $(2\pi i\nu)^{-1}$, we then have

$$G(\nu) = \frac{\nu_1}{i\nu}, \tag{4}$$

where ν_1 is the unity-gain frequency. Assuming a white frequency noise for the free-running lasers, we can calculate the closed-loop root-mean-square phase noise as:

$$\langle \varphi^2 \rangle = 2\int_0^\infty \frac{S_f(\nu)}{\nu^2} d\nu = 2\int_0^\infty \frac{S_w}{\nu^2 A(\nu)} d\nu = \frac{\pi S_w}{\nu_1}. \tag{5}$$

In a more realistic approach, both the diode laser response and the signal delay through the circuit should be included into $G(\nu)$, together with the transfer function of the necessary loop filter between the phase detector and the slave laser. Furthermore, low-frequency excess noise should be included into $S_f(\nu)$. However the root-mean-square residual phase noise basically increases with the free-running laser linewidth and decreases with the loop bandwidth.

Phase detectors have a finite range of operation $\Delta\varphi$, beyond which the phase reading is no more univocal. If the phase difference between the two inputs exceeds this phase range, the loop will lock to a different point. If $\langle \varphi^2 \rangle^{1/2}$ is comparable with $\Delta\varphi$, the occurrence of these 'cycle slips' may become quite frequent, resulting in random fluctuations of the frequency difference. For an analog mixer $\Delta\varphi = \pi$. In our simple model, the average time between cycle slips can be calculated as [7]:

$$t_s = \frac{\pi \exp(2\langle \varphi^2 \rangle^{-1})}{4\nu_1}. \tag{6}$$

Thus for typical values of ν_1 (a few MHz), cycle slipping can cause important frequency instabilities if $\langle \varphi^2 \rangle$ is greater than 0.1 rad^2. With noisy sources like diode lasers one thus must either build a fast OPLL to reduce $\langle \varphi^2 \rangle$, or use a phase detector with a broader phase range.

4. Analog and digital phase detectors

The output of an analog phase detector is proportional to the sine of the phase difference, so that the monotonic range is only π rad. Besides the problem of cycle slipping, analog phase detectors limit the capture range of the phase-locked loop, defined as the maximum admissible frequency difference to recover lock. When the input frequencies are not the same, the output oscillates at a rate equal to the frequency difference. As the loop filters out frequencies higher than ν_1, the capture range of an OPLL employing an analog phase detector is of the order of ν_1, which can hardly exceed a few MHz.

There are several ways to build a digital phase detector with a broader monotonic range. The most convenient one [8] is to send the beat note signal to an upward counter and the local oscillator signal to a downward counter. The outputs of the counters are finally sent to a digital adder, whose output, after D/A conversion and low-pass filtering, gives a linear response to the phase difference. The linear range only depends on the number of bits in the counters and the adder. Saturation of the response is obtained by blocking the appropriate counter when the adder reaches the minimum or maximum value: in this way when the input frequencies are different the detector gives a constant value indicating the sign of the frequency difference. Thus the system acts as a phase-frequency detector (PFD), whose scheme is presented in Fig. 1.

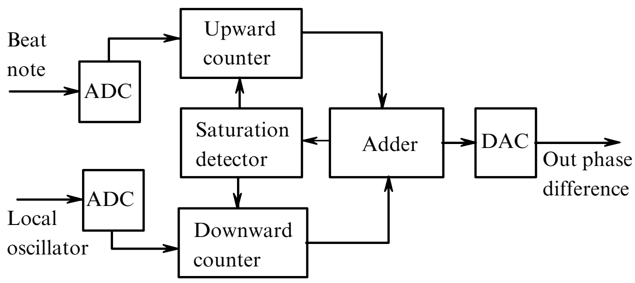

Figure 1. Block scheme of the digital PFD: (ADC) analog-to-digital converter; (DAC) digital-to-analog converter.

Compared to analog phase detectors, digital PFDs offer several advantages in the construction of an OPLL. The broader operating phase range allows slip-free phase locking even in presence of a large residual phase noise. This can be a frequent condition when operating with broad sources as diode lasers, or when the signal-to-noise ratio in the beat note is low [8], as in the case of optical frequency combs. Phase locking of diode lasers to optical combs can require a very narrow detection bandwidth [9], thus reducing the capture range. This limits the lock reliability, and imposes a careful reduction of external perturbations. The capture range of an OPLL that makes use of a PFD is simply limited by the speed of the A/D converter and of the logics, and can be of the order of 100 MHz. This larger capture range makes the system, in comparison with analog ones, more robust against external perturbations. In addition, a digital PFD allows for calibration of the phase sensitivity, and thus of the loop gain, by simply changing the bit number; it also allows for sideband selection, while in an analog OPLL the slave laser is indifferently locked to either side of the master laser.

Besides these advantages, digital PFDs present some important limitations concerning speed and signal-to-noise ratio. Conversion and logic operations introduce a time delay in the signal propagation. Time delay is one of the main fundamental limits to the loop bandwidth of an OPLL, as it produces a phase shift (linearly increasing with the Fourier frequency) that reduces the phase margin for stable loop operation. An analog phase-locked loop is intrinsically faster than a digital one, thus allowing a smaller residual phase error [see Eqn (5)]. Moreover, digital PFDs are generally noisier than analog phase detectors. This is stipulated by the fact that analog detectors are sensitive to the average phase difference over one period, while digital detectors only sense the edges of the input signals. Signal-to-noise ratio is another important limitation to the obtainable residual phase noise [8].

In conclusion, digital PFDs allow for phase-locks that are more stable and robust even in presence of broad and noisy sources, but analog detectors can give the smallest residual phase noise. We have implemented a novel mixed system that combines the advantages of both kinds of detector, and we have applied it to lock an ECDL to a mode of a femtosecond laser.

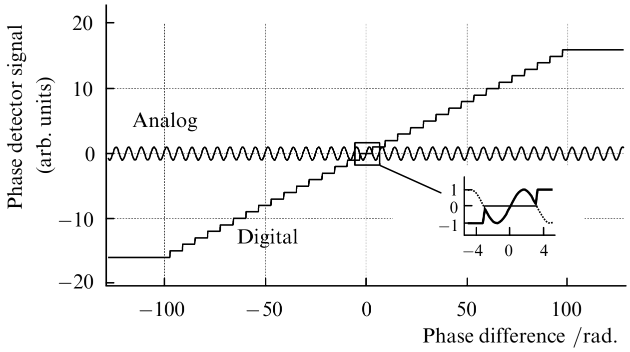

Figure 2. Generation of the phase error in the analog + digital detector. In the small box the solid curve represents the signal of the mixed detector. For clarity the digital monotonic range has been reduced to 32 cycles (5-bit logic).

5. Circuit description

5.1 Digital PFD

The main idea in our system is that the output from the analog phase detector is useful only when the phase error is below one cycle, while the output from the digital PFD can be used to extend the phase detection range and to allow for frequency detection when the loop is out of lock. In order to use both phase detectors together, the analog phase detector must be disabled when the phase error is more than one cycle, while the digital PFD must be disabled when the phase error is less than one cycle.

These features are obtained by adding few extra logics to the scheme of Fig. 1. The local oscillator input to the analog phase detector is switched by a logic signal revealing whether the phase error is below one cycle. In this way the input of the analog detector is disabled and its output is zero when the phase error is larger than one cycle. On the other hand, by using a synchronous adder clocked by the local oscillator signal, the PFD response becomes a staircase function of the phase difference and exhibits a 'dead zone' in the range where the analog detector is enabled. Finally, the local oscillator signal is sent to the analog detector after an appropriate delay, so that the analog locking point is in the exact center of the dead zone. The response of such a mixed detector to phase difference in the case of 5-bit logics is illustrated in Fig. 2.

In our system the functions of counters and adder are all performed by a single Complex Programmable Logic Device (CLPD), which also controls the signal enabling the analog detector. The CLPD (EPM7064 from the Altera MAX7000S family), can be easily re-programmed by connecting the circuit to a computer. Thus, it is possible to change the bit number of the counters and of the adder; it is also possible to add frequency scalers to the local oscillator or to the beat note signal. In our experiment we used a 7-bit adder. Thus the monotonic phase range of our PFD is 128 cycles (about 804 radians).

5.2 Mixed PFD and feedback circuit

The scheme of our circuit is shown in Fig. 3. The signals of local oscillator and beat note are squared by two fast comparators (LT1016) and sent as inputs to the CLPD together with two control signals, one for enabling the digital detector and the other for enabling the analog one.

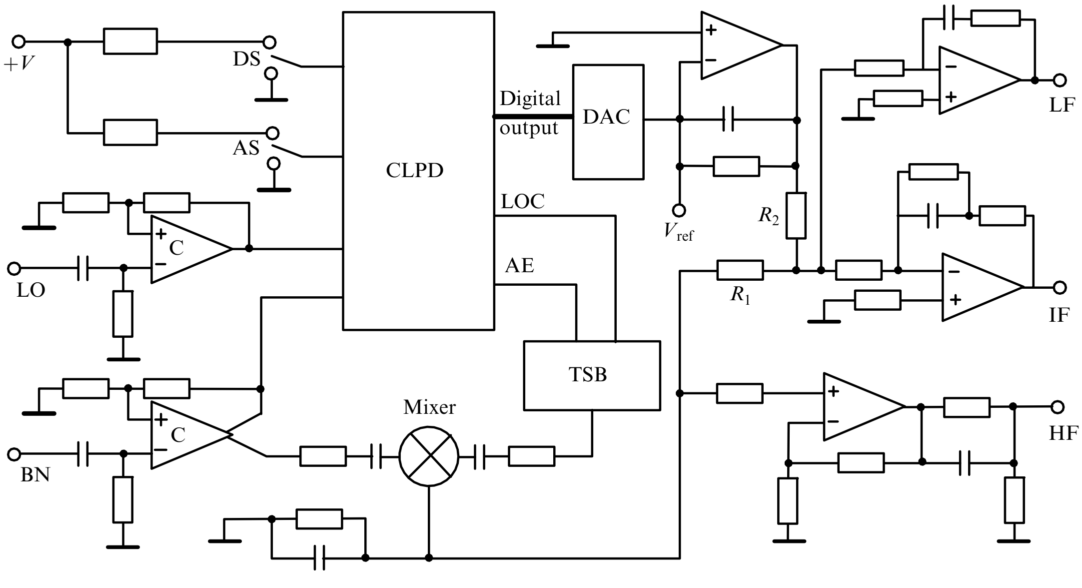

Figure 3. Electrical scheme of the phase-lock circuit: (DAC) digital/analog converter; (TSB) three-state buffer; (C) comparator; (DS) or (AS) switch for digital or analog operation; (LO) local oscillator; (BN) beat note; (LOC) copy of the local oscillator signal enabled by AS; (AE) control signal enabling LOC input to the mixer; (V_{ref}) voltage reference for the bias in the digital output; (LF) low-frequency output for PZT driver; (IF) intermediate-frequency output for diode laser current supply; (HF) high-frequency direct output to diode laser injection current.

Thus, the system allows four different operating modes: digital PFD alone, analog phase detector alone, mixed PFD and zero output.

The squared beat note is also sent to the analog phase detector (Minicircuits RPD-1). The CLPD has three outputs: the counter signal is sent to a D/A converter; a copy of the squared local oscillator and the signal enabling analog detection are sent to a three-state buffer (74ACT245), used as both a time delay and a switch. The output of the buffer goes to the analog phase detector as the local oscillator input.

The signals from the analog phase detector and the D/A converter are low-pass filtered and summed. The phase sensitivity should be the same under the two operating conditions (analog and digital phase detection) for closed loop stability. The ratio R_1/R_2 takes into account the different sensitivities of the two detectors.

The feedback loop is composed of three paths, to control three actuators in different Fourier frequency regions. The fastest path acts directly on the diode laser injection current. An RC phase-lead filter is used to balance the roll-off in the diode laser response and therefore to extend the loop bandwidth. For this path we use the analog phase detector output alone, as under tight lock conditions the digital PFD is not active however. The intermediate-frequency correction is sent to the current supply of the diode laser, providing high gain in the $10^1 - 10^4$-Hz region. The unity-gain point for this path is about 100 kHz. In order to keep the injection current around a constant value (the current-to-frequency characteristic of a diode laser, and therefore the loop gain, changes with the injection current), long-term corrections are performed by changing the cavity length of the ECDL through a PZT. An integrator increases the low-frequency gain.

6. Phase lock experiment

6.1 Lasers

We used the mixed PFD to lock an 850-nm ECDL to a mode of a femtosecond optical comb with a 100-MHz repetition rate. The ECDL is made of a GaAlAs diode laser (SDL-5400-C), a collimating lens, and a 1200-lines mm^{-1} diffraction grating in Littrow configuration at a distance of 25 mm from the laser. The diode laser power at the operating current (60 mA) is about 37 mW. The available power from the ECDL is about 6 mW. The system is closed into an aluminium box for thermal and acoustic isolation. The ECDL is temperature stable within 3 mK by means of Peltier elements. To prevent optical feedback we placed two Faraday isolators in cascade after the ECDL.

The optical comb is generated by a Kerr lens mode-locked (KLM) femtosecond Ti : Sapphire laser. The system, completely build by our group, is a standard four-mirror Z-folded resonator which consists of a Brewster-cut active medium, concave spherical mirrors, and two arms bounded by plane mirrors. One of the arms contains a prism pair for group velocity dispersion compensation, and one of the plane mirrors acts as an output coupler. The pump laser is a 10-W Millennia system, which provides the 532-nm pump radiation. With a pump power of about 6.5 W and an output power of 0.5 W, the system has a conversion efficiency of about 8 %. The pulse duration is 35 fs, the spectrum is about 45-nm wide ($\sim$ 19 THz) and the central wavelength can be set within the range $855 - 880$ nm by moving one of the prisms. The laser operates with a 97.7-MHz repetition rate which can be changed up to 5 MHz by translating one of the mirrors.

6.2 Experimental setup

The experimental setup is shown in Fig. 4. We combined the two laser beams in a polarising beamsplitter, followed by a polariser that could be rotated in order to obtain the best signal-to-noise ratio in the beat note. The beat note was detected with an avalanche photodiode (APD), filtered, amplified and sent to the PFD for phase-lock.

The output signal from the APD also presents the beat notes between the different comb modes at all the harmonics of the repetition rate f_r. The power on these notes can be many orders of magnitude larger than the useful signal. Furthermore, the unused comb modes increase the photodetector shot noise. Filtering this background electronically (in particular at the lowest harmonics of f_r) without reducing the capture range of the OPLL is not an easy

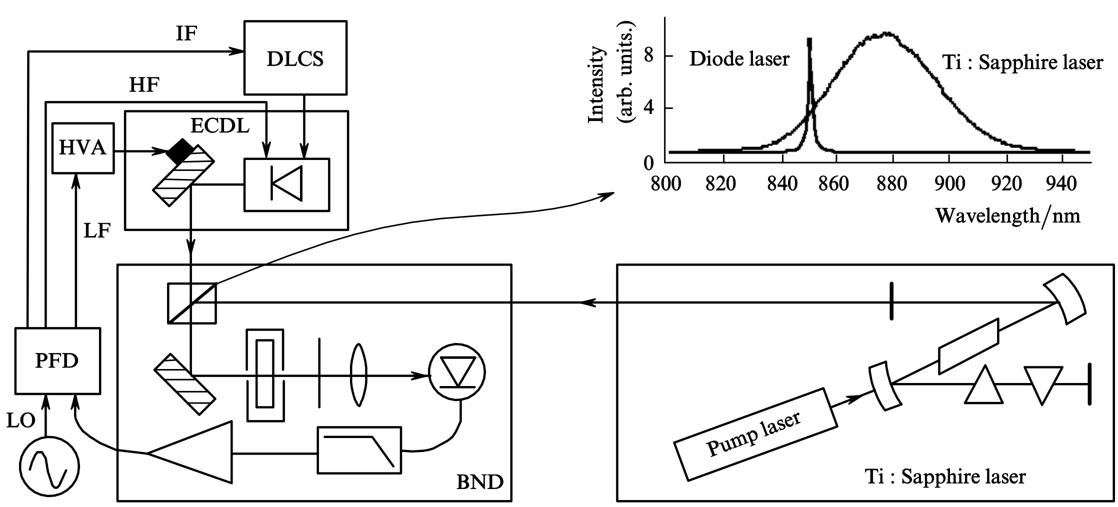

Figure 4. Experimental setup for the OPLL: (BND) beat-note detection; (PFD) phase-frequency detector and feedback loop; (HVA) high-voltage amplifier for PZT driving; (DLCS) diode laser current supply.

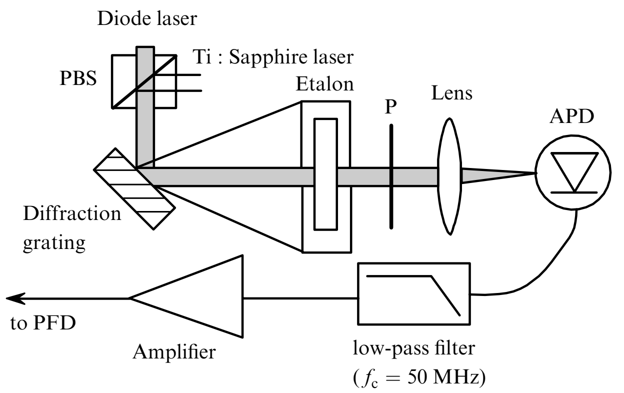

Figure 5. Optical setup for detection of the beat note: (PBS) polarising beamsplitter; (P) polariser; (APD) avalanche photodiode.

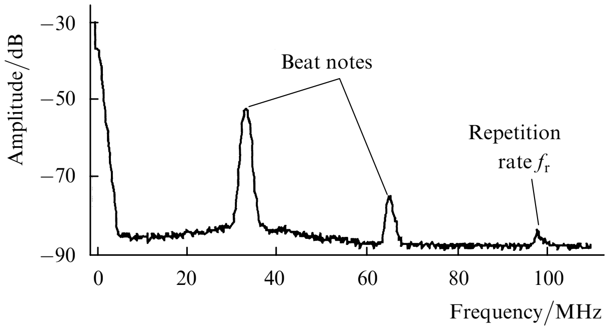

Figure 6. Typical spectrum analyser signal after low-pass filter and amplifier.

task. It is then necessary to prevent the unused comb modes from reaching the APD. This is usually performed by dispersing the radiation on a grating and selecting a small number of modes with a slit. In order to achieve a very efficient selection without large loss of power, we used a two-stage optical filter, as shown in Fig. 5.

The combined laser beams are first sent on a 1200-lines mm^{-1} diffraction grating, then through a thick etalon. The latter is an 8-mm reflective coated quartz disk. The free spectral range is 18.4 GHz, while the width of the resonant modes is 440 MHz (finesse > 40). In this way we filtered one out of about forty comb modes. By simply tilting the etalon we could obtain a peak transmission of about 70% for the diode laser. The measured temperature sensitivity of the resonant frequencies is 2.0 ± 0.1 GHz K^{-1}. The temperature of the etalon was stabilised at the 10 mK level, so to keep the diode laser in resonance within 5 % of the etalon linewidth. As the spacing of the comb modes is equal to f_r, the maximum capture range of our phase-lock system is obviously limited to 100 MHz (see Sec. 6.3), that is 22 % of the etalon resonance width. Within this frequency range the transmission for the diode laser changes less than 1 %. A slight adjustment of the etalon tilt angle is required after some hours, due to the slow frequency drift of the Ti : Sapphire comb. With this method we reduced the power in the first harmonic f_r to the level of the useful beat note.

The transmitted beams were focused on an avalanche photodiode. The resultant beat note was low-pass filtered and amplified. The corner frequency of 50 MHz was chosen for the filter to reach the maximum capture range of the OPLL. This limit is imposed by the comb repetition rate: if the useful beat-note has a frequency f_b, the beat with the next closest comb mode will have a frequency $f_r - f_b$. A typical record of the filtered signal on a spectrum analyser is shown in Fig. 6.

6.3 Results

We used a 30-MHz oscillator as a reference frequency for the phase-locked loop. With the etalon tilt angle set for maximum diode laser transmission, one beat note is always present in our detection band (0 – 50 MHz). Phase lock could be automatically obtained by closing the loop, whatever the initial frequency of the beat note. Actually, as our PFD is sideband selective, the diode laser can be on the wrong side of the closest comb mode; in this case the loop carries the diode laser away from the closest mode and locks it to the adjacent mode. That is, for each comb mode the capture range is 30 MHz on one side and 70 MHz on the other.

Under open-loop conditions the observed beat-note is broadened to a few MHz by acoustic jitter and is subject to a frequency drift of the order of 10 MHz min^{-1}, essentially due to mechanical relaxation processes in the ECDL. Closing the loop with the PFD set for simple digital operation efficiently reduces the beat-note frequency noise in the spectral region below 10 kHz. A typical record of the beat-note spectrum under this condition is shown in Fig. 7. The −3-dB full width is about 500 kHz and can be attributed mainly to fast frequency fluctuations of the ECDL.

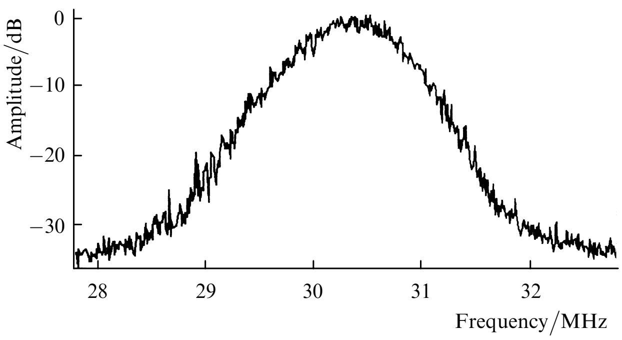

Figure 7. Spectrum of the closed-loop beat note when the analog phase detector is disabled; the record is the average of four scans of the spectrum analyser; the resolution bandwidth is 10 kHz.

When the PFD is set for the analog + digital operation, the loop bandwidth increases up to 1.2 MHz, as shown in Fig. 8. The lock is stable even in the presence of strong mechanical perturbations on the optical board. The locking stability was observed for some hours, which was mainly limited by the finite range of the HV amplifier for the PZT. With pure analog operation of the phase detector a beat-note spectrum similar to Fig. 8 is obtained, but with much lower locking stability and capture range.

We calculated the residual root-mean-square phase error $\langle\varphi^2\rangle$ by the relation [8]

$$\langle\varphi^2\rangle = -\ln\eta, \tag{7}$$

where η is the fraction of power in the carrier of the beat note. We measured η by integrating directly the beat-note spectrum of Fig. 8. For the wings of the spectrum we

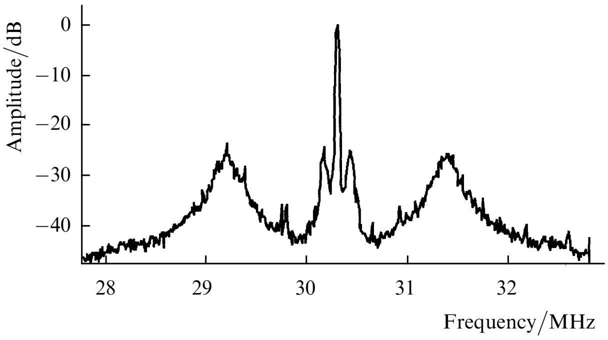

Figure 8. Spectrum of the closed-loop beat note when the PFD is set for analog + digital operation; the record is the average of four scans of the spectrum analyser; the resolution bandwidth is 10 kHz.

assumed a Lorentzian of width equal to that of the curve in Fig. 7. The result was $\eta = 0.75$ and thus $\langle \varphi^2 \rangle = 0.29$ rad.

7. Conclusions

The properties of the analog + digital PFD make it particularly well suited for OPLLs where a high spectral purity is required together with reliable and stable lock, but in the presence of broad and low-power beat notes. Typical applications are the optical frequency measurements by means of femtosecond combs.

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