PACS numbers: 42.68.Ay; 42.68.Bz; 42.65.Hw; 42.68.Wt
DOI: 10.1070/QE2004v034n09ABEH002661

Phase and amplitude – phase control of a laser beam propagating in the atmosphere

B.P. Lukin, F.Yu. Kanev, V.A. Sennikov,
N.A. Makenova, V.A. Tartakovskii, P.A. Konyaev

***Abstract*. Phase and amplitude – phase corrections of laser beam distortions during their propagation in a turbulent atmosphere under conditions of strong intensity fluctuations are compared. The effect of wavefront dislocations and the possibility of controlling the amplitude and phase of an optical wave are studied. Two approaches are analysed: phase correction using amplitude control and two-mirror phase correction. The efficiency of both methods is demonstrated.**

Keywords: *amplitude, phase, turbulence, wavefront dislocation, adaptive correction, wavefront phase-conjugation, two-mirror correction.*

1. Introduction

A strict fulfilment of the reciprocity principle during the formation of a backgward laser beam requires an exact reproduction of the phase and amplitude components of the wave received from the reference source (beacon). In actual practice, however, one has to remain content with information about phase only. In this case, a phase-conjugate wavefront (WF) is formed at the adaptive mirror illuminated by a Gaussian beam.

We performed numerical simulation of the propagation of a backward laser beam under conditions of strong intensity fluctuations, when WF discontinuities are observed. The difference between the wave reversal and phase conjugation (PC) at an ideal adaptive mirror faithfully reproducing all the WF discontinuities is shown. It is proposed that discontinuity lines be cut out during formation of the backward beam. This can be done by extracting the WF areas, corresponding to deep attenuation regions in the light wave received from the reference source, from the adaptive system channel. It is concluded that the removal of such areas does not deteriorate the properties of the beam formed due to PC and allows a solution of the problem of reproducing the WF dislocations at the adaptive mirror.

B.P. Lukin, F.Yu. Kanev, V.A. Sennikov, N.A. Makenova, V.A. Tartakovskii, P.A. Konyaev Institute of Optics and Atmosphere, Siberian Branch, Russian Academy of Sciences, prosp. Akademicheskii 1, 634055 Tomsk, Russia; Tel.: (382) 2 491 606, (382) 2 492 086; e-mail: lukin@iao.ru

Received 25 March 2004
Kvantovaya Elektronika **34** (9) 825 – 832 (2004)
Translated by Ram Wadhwa

2. Numerical solution of the problem

To apply the reciprocity principle to the adaptive formation of laser beams in atmosphere, the adaptive system must reproduce the complete amplitude – phase profile of the wave received from the reference source [1, 2]. In actual practice, however, only the phase component of the wave is considered, and the complicated amplitude profile is replaced by a simple Gaussian profile. The aim of our study is to analyse the effect of the amplitude approximation error, caused by such a replacement, on the efficiency of correction. We used for this purpose the numerical simulation of laser beam propagation in a turbulent atmosphere [2, 3]. Figure 1 shows schematically the numerical experiment. A collimated Gaussian beam serving as the reference source propagates in a turbulent medium (with the Kolmogorov spectral density). A backward beam was formed in the adaptive mirror plane due to PC or wave reversal and returned back. We call the beam propagating in the forward direction the forward beam, while the beam passing twice through the atmosphere is called the backward beam. The phase-conjugation error ε_0, which is equal to the normalised root-mean-square deviation of the amplitude $\tilde{A}_0$ of the conjugated field from the amplitude A_0 of the reference field in the source plane, was calculated as well as the energy E in a circle with the effective beam radius:

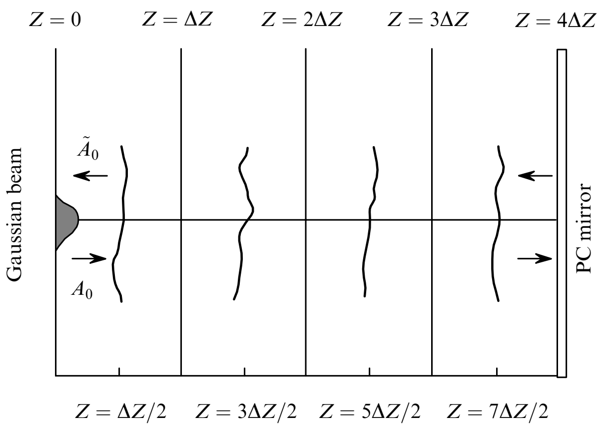


Figure 1. Scheme of the numerical experiment on the phase conjugation of a Gaussian beam propagating through a turbulent medium. The number of phase screens is $n = 4$, the grid has a size 512×512, the ratio of outer and inner turbulence scales is $L_0/l_0 = 10^3$.

$$\varepsilon_0^2 = \sum_S (\tilde{A}_0 - A_0)^2 \Big/ \sum_S A_0^2,$$
$$E = \sum_{S_{\rm eff}} \tilde{A}_0^2, \qquad (1)$$

where S is the area of the circle and $S_{\rm eff}$ is the area of the circle with the effective radius. Calculations were made by using the splitting technique [1–3] and fast Fourier transform method. The paraxial approximation was used to solve the dimensionless parabolic quasi-optical equation in the form

$$\frac{\partial W}{\partial z} = \frac{\rm i}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \delta n\right) W, \qquad (2)$$

where $W(x, y; z)$ is the complex wave amplitude, and δn is the normalised deviation of the refractive index n of the medium. The bicyclical splitting technique was used, the number of steps along the path was equal to 4, and the matrix of the grid was of the order of 512.

The spectral density of the refractive index had the form

$$F(\varkappa_x, \varkappa_y) = C_n^2(\varkappa_0^2 + \varkappa_x^2 + \varkappa_y^2)^{-11/6} \exp\left(-\frac{\varkappa_x^2 + \varkappa_y^2}{\varkappa_m^2}\right), \qquad (3)$$

where C_n^2 is the structural constant of the refractive index corresponding to the Kolmogorov turbulence model and having the dimensions m$^{-2/3}$; $\varkappa_0 = 2\pi/L_0$; $\varkappa_m = 2\pi/l_0$; L_0 and l_0 are the outer and inner scales of atmospheric turbulence normalised to the beam radius a_0. The wave parameter $Z = L\lambda/(2\pi a_0^2)$, where λ is the wavelength and L is the path length. The parameters of the numerical experiment were: $L_0 = 100$, $l_0 = 0.1$, and $Z = 0.1$. For a beam of radius $a_0 = 10$ cm and $\lambda = 0.63$ μm, this corresponds to a surface layer of turbulent atmosphere along a horizontal path of length $L = 1$ km. The effect of turbulence on a beam is characterised by a beam scintillation index β^2 calculated in the phase-conjugation plane. The dependence of β^2 on the parameter C_n^2 is shown in Fig. 2.

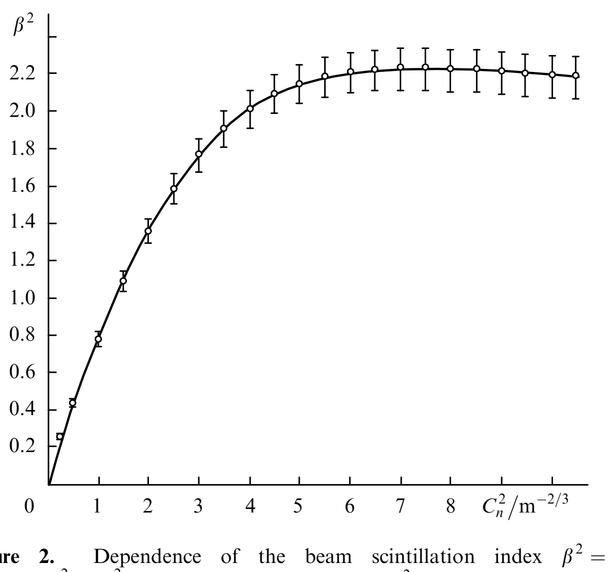


Figure 2. Dependence of the beam scintillation index $\beta^2 = (\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2$ on the structural constant C_n^2 obtained during the numerical experiment. Weak fluctuations correspond to $\beta^2 < 0.3$, while strong fluctuations correspond to $\beta^2 > 1.0$.

3. Comparison of the results of simulation of PC and wave reversal

We performed a numerical simulation of PC at an adaptive mirror and propagation of the backward beam in a turbulent atmosphere. The backward beam was formed by the Gaussian beam at the adaptive mirror that reproduces conjugately the form of the WF of the reference Gaussian beam propagating through a turbulent atmosphere.

Three different methods of formation of the backward beam at an ideal adaptive mirror were compared. These methods are based on the use of the wave reversal, PC with tracking of the centre of gravity of the arriving beam, and a Gaussian beam propagating through an atmospheric turbulence layer without phase correction. Figure 3 shows the phase-conjugation error ε_0 and parameter E calculated for the beams formed during propagation through an atmospheric layer as a function of the structural constant C_n^2.

Figure 4 shows the intensity and phase distributions for a Gaussian beam, propagating through a turbulent medium, in the adaptive mirror plane. The beam is split into streams separated by deep attenuation regions containing optical vortices. Figures 5 and 6 show the intensity and phase distributions for the field of the backward beam in the source plane (compared to the initial Gaussian beam with a plane WF). For the backward wave obtained due to a wave

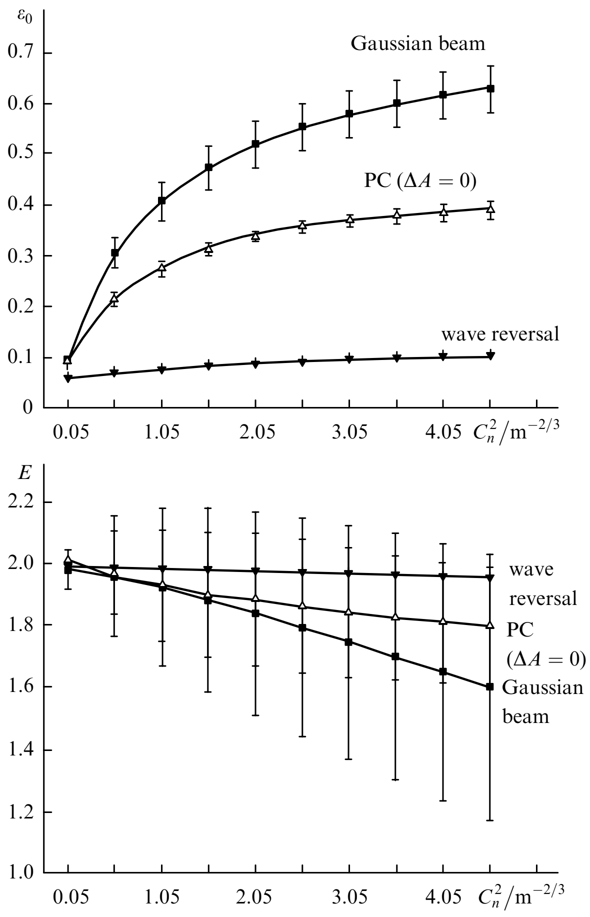


Figure 3. Dependence of the phase-conjugation error ε_0 and the parameter E on C_n^2 for various methods of backward beam formation (with the help of wave reversal, PC, and a Gaussian beam without phase correction) for $N = 100$ realisations.

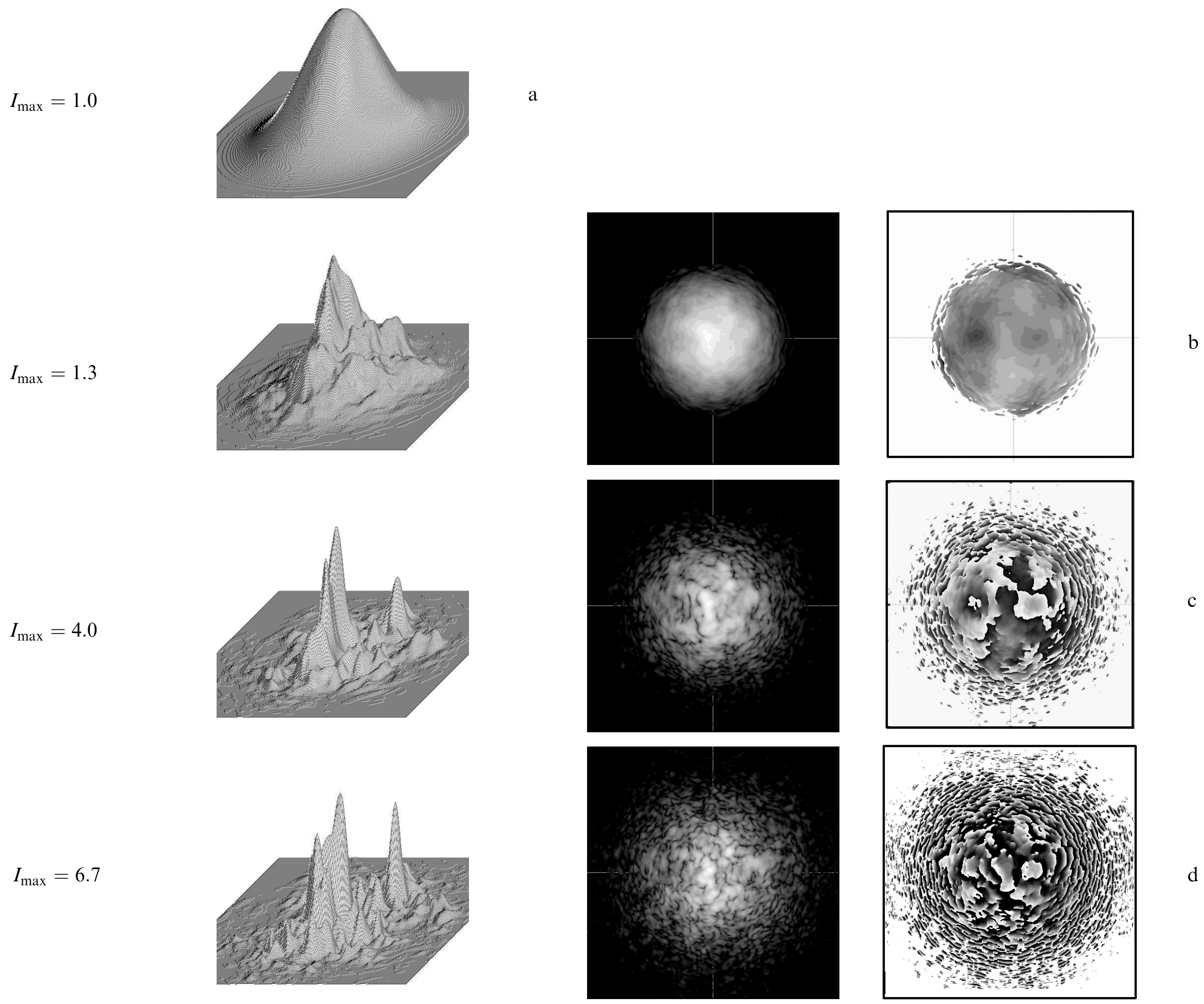


Figure 4. Three-dimensional (left) and two-dimensional (middle) intensity distribution patterns, as well as phase distributions (right) obtained in the adaptive mirror plane for the initial Gaussian beam (a) and for a beam propagating through the turbulent medium for values of β^2 equal to 0.1 ($C_n^2 = 0.1$ m$^{-2/3}$) (b), 1.0 ($C_n^2 = 1.5$ m$^{-2/3}$) (c), and 2.0 ($C_n^2 = 4$ m$^{-2/3}$) (d).

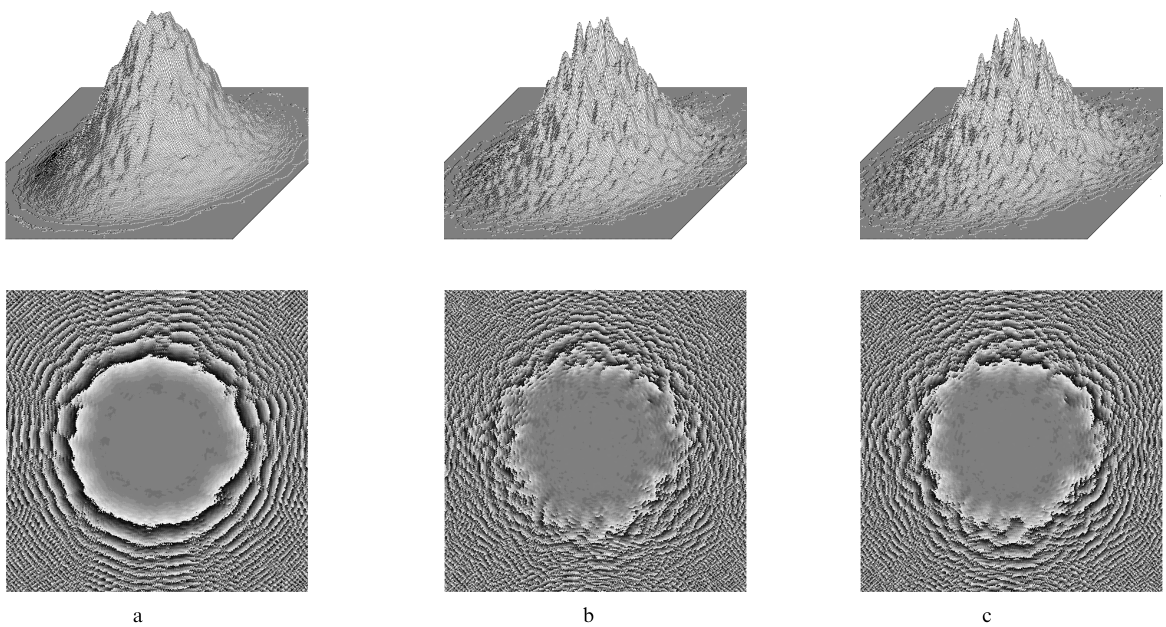


Figure 5. Intensity (above) and phase (below) distribution patterns for a beam after wave reversal for β^2 equal to 0.1 ($C_n^2 = 0.1$ m$^{-2/3}$) (a), 1.0 ($C_n^2 = 1.5$ m$^{-2/3}$) (b), and 2.0 ($C_n^2 = 4$ m$^{-2/3}$) (c).

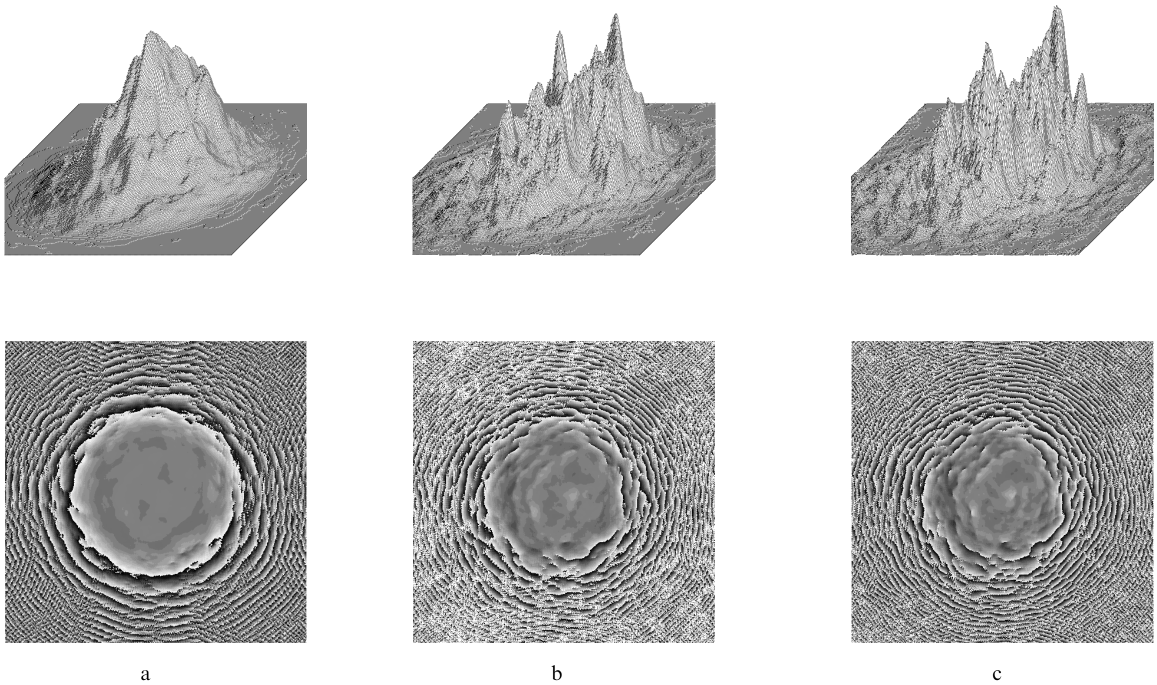


Figure 6. Intensity (above) and phase (below) distribution patterns for a beam after PC for the same values of the parameters β^2 and $C_n 2$ as in Fig. 5.

reversal, the intensity distribution is described by a Gaussian profile covered by a fine ripple, whereas a pile-up of peaks is observed in the case of PC. For a wave reversal, the phase surface of the backward beam is nearly plane, except at the periphery, while the beam is severely distorted during PC and dislocations appear even in the central part of the beam. Thus, it can be concluded that a replacement of the complicated amplitude distribution of the arriving wave by a Gaussian amplitude distribution during PC leads to a larger error than in the case of an analogous substitution during the wave reversal. This may lead to constraints on the fulfillment of the reciprocity principle that forms the basis of the adaptive optics.

In this case, the ability to reproduce faithfully a WF with dislocations during PC may not be strictly necessary in the case of backward beam formation using only the information about the phase of a beam propagating through a turbulent atmosphere. This is due to the fact that the amplitude information is not used [2, 3]. The replacement of the complicated amplitude profile of the beam by a smooth Gaussian profile during a detailed reproduction of the WF peculiarities will lead to the transfer of a large amount of energy into the regions where it should not exist, in particular, to the regions with optical vortices. Note here that a simple neglect of dislocations (the use of a smoothed phase obtained with the help of the root-mean-square WF reconstructors) during PC of a beam with Gaussian amplitude also does not solve the problem of excess energy.

Figure 7 shows the dependences of the phase-conjugation error ε_0 and parameter E on the structural constant C_n^2, calculated in the case of PC for thresholds $\Delta A = 0$, 0.1 and 0.2. The threshold ΔA is measured in dimensionless units and characterises the level at which the field amplitude of the beam during amplitude – phase control is assumed to be equal to zero. For $\Delta A = 0$, the amplitude is reproduced in details. For $\Delta A \neq 0$, the measured amplitude is assumed to

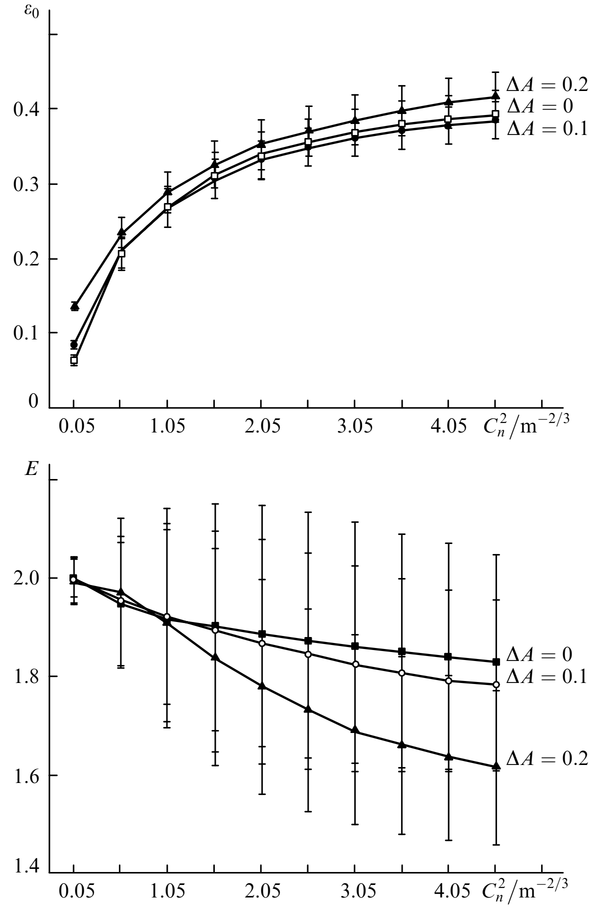


Figure 7. Dependence of the phase-conjugation error ε_0 and the parameter E on C_n^2 during PC for various values of ΔA and for $N = 100$ realisations.

be equal to zero at points where it is below the threshold value. Note that the beam intensity and phase distributions obtained using the field cutoff at the threshold $\Delta A = 0.1$ almost coincide with those obtained without cutoff. For a higher threshold ($\Delta A = 0.2$), however, these distributions differ quite significantly. For comparison, Fig. 8 shows the dependence of the phase-conjugation error ε_0 on the parameter C_n^2 calculated in the case of wave reversal for $\Delta A = 0.05$, 0.1, 0.2, 0.3 and 0.4. Technically, this could be accomplished in a system with two mirrors. A flexible adaptive mirror reproduces a smoothed WF without dislocations, while a segmented adaptive mirror forms holes in a Gaussian beam at places where strong attenuation regions would have been formed.

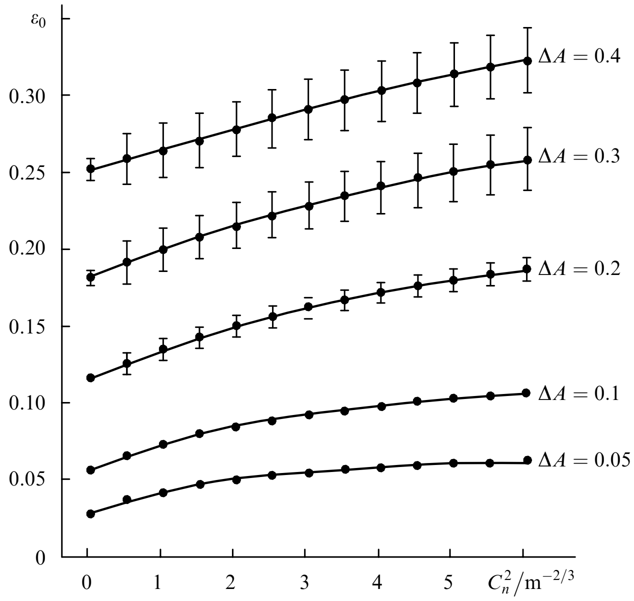


Figure 8. Dependence of the phase-conjugation error ε_0 on C_n^2 during wave reversal for various values of ΔA and for $N = 100$ realisations.

Thus, it is shown that the replacement of the complicated amplitude distribution of the arriving wave by Gaussian distribution during PC at an adaptive mirror introduces a larger error than that in the case of wave reversal. This may lead to constraints on the fulfillment of the reciprocity principle, which forms the basis of adaptive optics [2, 3]. Compared to this, the inability to reproduce the 2π-discontinuity lines, associated with the presence of WF dislocations, on the flexible adaptive mirror is a less significant problem. Nevertheless, it can be solved successfully by extracting strong attenuation regions containing dislocations from the adaptive system channel through an appropriate choice of the threshold ΔA.

4. Amplitude – phase control of a beam in a two-mirror adaptive system

Despite a number of papers devoted to this subject [1, 4, 5], one of the main theoretical problems appearing during the wave reversal in a two-mirror system remains unsolved so far [4, 6 – 10]. There is no exact fast algorithm that would allow one to obtain, by controlling phase in one plane, the required amplitude distribution in a plane at a certain distance from this plane. The algorithms proposed for solving this problem are either iterative [6, 7] or do not ensure a high accuracy [4].

We consider here another method making it possible to determine the beam phase in the plane of the first mirror that would lead to the required amplitude profile in the plane of the second mirror. The algorithm developed by us was used to study the compensation of turbulent distortions of coherent radiation, and the wave reversal efficiency was compared with that of pure phase control [7].

In this section also, calculations were also performed numerically. The intensity of atmospheric turbulence is characterised by the Fried radius

$$r_0 = \left(0.423k^2 \int_0^L C_n^2(l)\mathrm{d}l\right)^{-3/5}, \quad (4)$$

where L is the thickness of the turbulent layer, and k is the wave number of the radiation. The efficiency of compensation for the produced and residual distortions was characterised by the focusing criterion [1]

$$J(t) = \frac{1}{P_0}\int\int \rho(x,y)I(x,y,z_0,t)\mathrm{d}x\mathrm{d}y, \quad (5)$$

representing the relative fraction of the radiation power falling within the aperture of radius a_0 in the z_0 plane. Here, P_0 is the total power of the beam, $I(x,y,z_0,t)$ is the radiation intensity, and $\rho(x,y) = \exp[-(x^2+y^2)/a_0^2]$ is the aperture function.

The wave reversal is possible in an adaptive system [4, 7] consisting of two mirrors separated by a distance over which the beam propagates without distortions (Figs 9, 10). First mirror (*1*) defines the beam phase. Phase variations during the beam propagation in free space lead to amplitude variations. Thus, the required distribution of the light field is attained in the plane of mirror (*2*) (at the input to the medium). Mirror (*2*) compensates the variations introduced into the phase and performs phase conjugation. Thus, a beam with the required amplitude distribution and phase profile is formed at the input to the medium. The main problem in realising this operation lies in specifying the phase profile providing the required amplitude distribution.

The problem is solved quite easily [6, 11] if only one distorting screen is placed in the middle of the path. The following property of optical radiation was used for the solution: the beams having identical initial phase and amplitude profiles and propagating along identical paths acquire identical amplitude and phase distributions. The possibility of realising the amplitude – phase control by using this property is demonstrated in Figs 9 and 10. The reference beam propagates from the detection plane (Fig. 9). As the beam falls on the phase screen, the difference in the amplitude of this beam from the initial Gaussian amplitude, and of its phase from the phase of a wave with a plane WF (plane phase), is determined by diffraction only. The amplitude remains Gaussian immediately behind the screen, and passage through the screen leads only to phase variation. The incidence of the reference beam on mirror (*2*) leads to amplitude distortions. If mirror (*2*) is used for phase conjugation of the reference beam with an unchanged amplitude, a wave exactly similar to the one existing behind the distorting screen, i.e., a beam having Gaussian amplitude and a certain phase differing from the plane phase, will

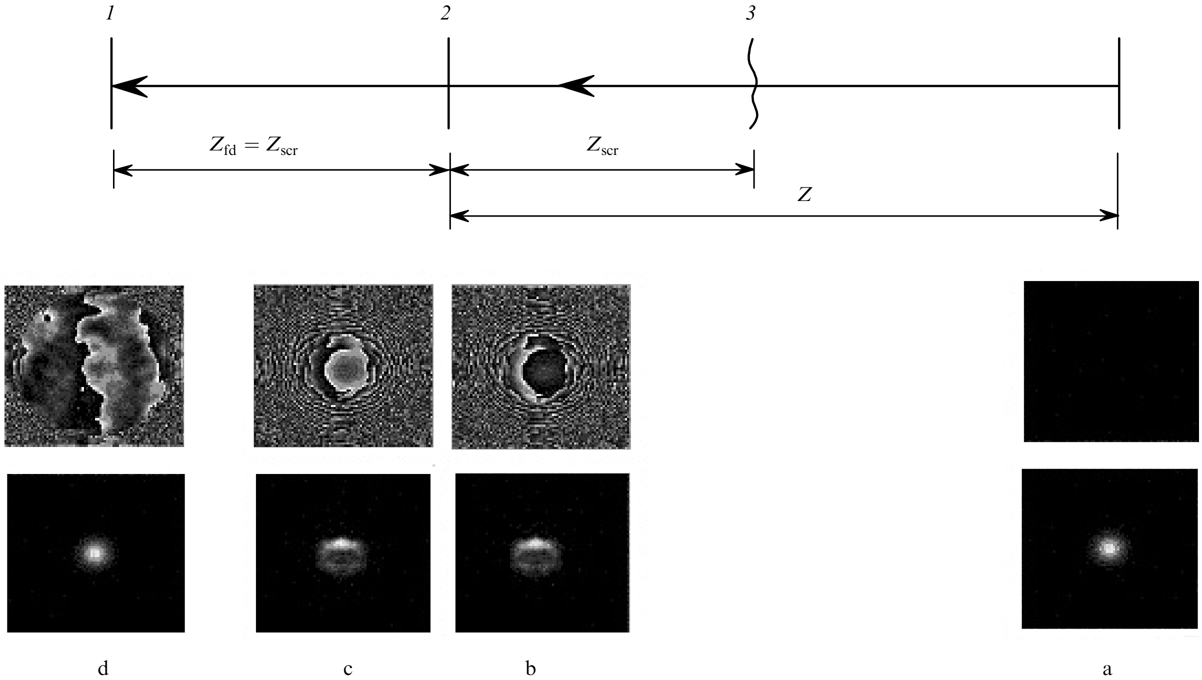


Figure 9. Propagation of reference beam in a two-mirror adaptive system (the distortions are simulated by a single screen): the phase (above) and amplitude (below) of the reference beam in the detection plane (a), and in the plane of mirror (*2*) before (b) and after (c) conjugation, as well as in the plane of mirror (*1*) (d). The phase screen is shown as (*3*).

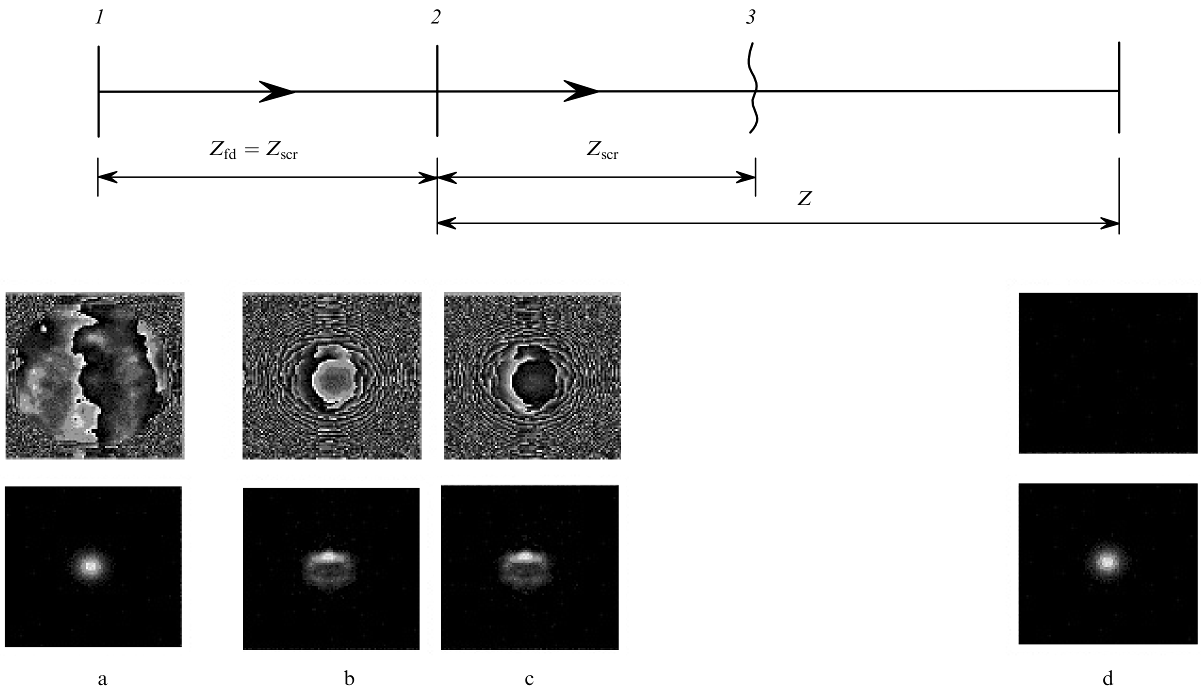


Figure 10. Propagation of reference beam in a two-mirror adaptive system (the distortions are simulated by a single screen): the phase (above) and amplitude (below) of the direct beam in the plane of mirror (*1*) after correction (a), and in the plane of mirror (*2*) before (b) and after (c) conjugation, as well as in the detection plane (d). The phase screen is shown as (*3*).

appear in the plane of mirror (*1*). In the plane of mirror (*1*), the reference beam is replaced by a direct beam whose phase is conjugate to the reference phase and the amplitude is conjugate to the Gaussian amplitude (Fig. 10). All the requirements of the optical reversibility principle are now satisfied. In the plane of mirror (*2*), the amplitude distribution is the same as in the reference beam, while the phase is conjugate to the phase of the reference beam.

The next problem considered in this work is the correction of distributed turbulence of a lens [11–13]. The solution is based on the assumption that distortions of beams passing through a phase screen placed in the middle of the path are approximately equal to the distortions of beams passing through a set of identical phase screens. This statement is confirmed by the data presented in Tables 1 and 2 for these two cases. The tables contain values of the criterion J, beam energy radius $\sigma(t)$ and displacements X_c and Y_c of the energy centre along the axes perpendicular to the direction of propagation:

$$\sigma(t) = \left[\frac{1}{P_0 a_0^2} \int\int (\boldsymbol{r}_\perp - \boldsymbol{r}_c)^2 I(x, y, z, t) \mathrm{d}x \mathrm{d}y \right]^{1/2}, \quad (6)$$

$$X_c = \frac{1}{P_0 a_0^2} \int\int x I(x, y, z, t) \mathrm{d}x \mathrm{d}y, \quad (7)$$

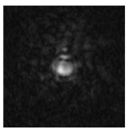
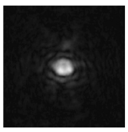
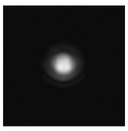
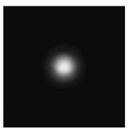
Table 1. Beam parameters obtained during simulation of turbulence by a single screen placed in the middle of the beam path ($Z = 0.5$).

r_0/m	J	σ	X_c/a_0	Y_c/a_0
0.1	0.28	1.40	−0.22	0.10
0.08	0.25	1.48	−0.27	0.12
0.06	0.21	1.60	−0.34	0.16
0.04	0.15	1.80	−0.47	0.22
0.02	0.08	2.59	−0.83	0.38

Table 2. Beam parameters obtained during simulation by a distributed lens formed by identical screens ($Z = 0.5$).

r_0/m	J	σ	X_c/a_0	Y_c/a_0
0.1	0.30	1.35	−0.25	0.12
0.08	0.27	1.40	−0.30	0.14
0.06	0.23	1.49	−0.37	0.18
0.04	0.17	1.64	−0.51	0.25
0.02	0.08	2.10	−0.84	0.43

Table 3. Correction of a distributed turbulent lens simulated by a set of identical screens in an adaptive system with two mirrors.

r_0/m	J	J_{cor}	ε_0 ($Z = 0.5$)	Light field distribution
0.005	0.002	0.341	0.66	
0.01	0.039	0.462	0.46	
0.05	0.192	0.531	0.13	
0.1	0.292	0.532	0.07	

Note: J_{cor} are the values of the criteria obtained during beam control.

Table 4. Correction of a distributed turbulent lens simulated by a set of identical screens, based on the PC algorithm.

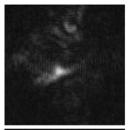
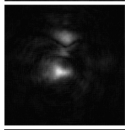
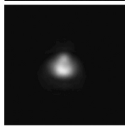
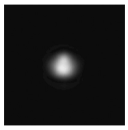
r_0/m	J	J_{cor}	ε_0 ($Z = 0.5$)	Light field distribution
0.005	0.002	0.105	1.06	
0.01	0.039	0.163	0.92	
0.05	0.192	0.452	0.32	
0.1	0.292	0.501	0.21	

Table 5. Parameters of a beam obtained during compensation by a distributed turbulent lens simulated by a set of various screens.

r_0/m	J	J_{cor}	ε_0 ($Z = 0.5$)
0.005	0.049	0.41	0.45
0.01	0.143	0.47	0.31
0.02	0.262	0.50	0.17
0.03	0.310	0.50	0.12

where $\boldsymbol{r}_\perp$ is the radius vector of a point in the plane; $\boldsymbol{r}_c$ is the radius vector of the energy centre of gravity of the beam; and Y_c is defined by a formula similar to (7). The results are obtained for various values of turbulent distortions. It can be seen from Tables 1 and 2 that all the quantities introduced for characterising the radiation are indeed quite close for these two cases.

Because the correction for one screen is absolute, a fairly good correction is also obtained as a result of compensation for a distributed lens simulated by a set of identical random screens (Table 3). In this case, absolute compensation is not achieved, but the values of the criteria obtained as a result of correction for small values of the Fried radius are much higher than those obtained in the case of PC (cf. Tables 3 and 4). Subsequent errors are encountered during compensation of the distributed lens formed by different screens generated by using a set of random numbers. However, in this case also, the results [13] obtained by using a two-mirror system (Table 5) are found to be quite good.

5. Conclusions

The results obtained in this work lead to the conclusion that the amplitude – phase method of beam control proposed here does not provide absolute compensation of a distributed turbulent lens, but the obtained values of the focusing criterion are higher than in the case of pure phase control. Thus, the calculations made in this work have shown a higher efficiency of the amplitude–phase correction compared to the pure phase correction. The amplitude control is realised by using only linear transformations of the field of the distorted wave.

References

1. Vorontsov M.A., Shmal'gauzen V.I. *Printsipy adaptivnoi optiki* (Principles of Adaptive Optics) (Moscow: Nauka, 1985).
2. Lukin V.P. *Atmosphernaya adaptivnaya optika* (Atmospheric Adaptive Optics) (Novosibirsk: Nauka, 1986).
3. Lukin V.P., Fortes B.V. *Adaptivnoe formirovanie puchkov i izobrazhenii* (Adaptive Formation of Beams and Images) (Novosibirsk: Publishing House of the Siberian Branch of Russian Academy of Sciences, 1999).
4. Vysotina N.V., Rozanov N.N., Semenov V.E., Smirnov V.A. *Izv. Vyssh. Uchebn. Zaved. Ser. Fiz.*, **28** (11), 42 (1985).
5. Lukin V.P., Charnotskii M.I. *Kvantovaya Elektron.*, **9**, 952 (1982) [*Sov. J. Quantum Electron.*, **12**, 602 (1982)].
6. Kanev F.Yu., Lukin V.P., Lavrinova L.N. *Proc. SPIE Int. Soc. Opt. Eng.*, **4341**, 135 (2000).
7. Kanev F.Yu, Lukin V.P. *Opt. Atmos.*, **4** (12), 1237 (1991).
8. Roggermann M.C., Lee D.J. *Appl. Opt.*, **37**, 4577 (1998).
9. Barcher J.D. *J. Opt. Soc. Am. A*, **18** (12), 423 (2001).
10. Vysloukh V.A., Kandidov V.P., Chesnokov S.S., Shlenov S.A. *Izv. Vyssh. Uchebn. Zaved. Ser. Fiz.*, **28** (11), 30 (1985).
11. Lukin V., Fortes B. *Appl. Opt.*, **41** (27), 1 (2002).
12. Lukin V.P. *Usp. Fiz. Nauk*, **173** (8), 11 (2003).
13. Kanev F.Yu., Lukin V.P., Makenova N.A. *Opt. Atmos. Okean.*, **15** (12), 1073 (2002).