

# Dynamics of wave packets in fibres with amplification and inhomogeneous distribution of dispersion parameters

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**Abstract.** The dynamics of radiation pulses is studied in a fibreoptic system with amplification and an arbitrary dependence of the real and imaginary components of dispersion parameters on the longitudinal coordinate. The dependence of compression regimes of a wave packet on the chirp and imaginary components of dispersion parameters related to the frequency dependence of the gain increment is analysed.

**Keywords:** optical fibre, group velocity dispersion, ultrashort pulse.

## 1. Introduction

Fibreoptic communication links (FCLs) with a variable group velocity dispersion are currently considered as most probable candidates for the development of long ultrafast communication links with the data transfer rate above 10 Gbit s<sup>-1</sup> [1–5]. The use of fibres of different types with chromatic dispersion of opposite signs in such systems allows one to limit the dispersion broadening of a pulse [6–11]. A special role in such systems is played by fibre amplifiers. When they are used, losses are no longer a key factor restricting the operation of FCLs. Until the present time erbium-doped fibre amplifiers were mainly employed in FCLs [12, 13]. These amplifiers are called lumped amplifiers because the length at which amplification occurs is much shorter than the distance between them. However, at present Raman amplifiers [14], which have a number of important advantages over erbium-doped amplifiers, are gaining increasing acceptance. Thus, the gain band of Raman amplifiers can be broadened and the gain can be smoothed over the entire band by increasing the number of pump waves and adjusting their input powers. They also provide a comparatively low level of losses and, which is especially important, the optical fibre itself is used as an active medium [15]. Recently highly efficient Raman fibre lasers, which emit virtually at any wavelength in the range from 1.2 to 1.5 μm [16, 17], and Raman amplifiers based on heavily germanium-doped fibres with low optical losses were elaborated [18]. Fibreoptic systems with the specified dispersion distribution over the fibre length can be developed [19, 20]. A number of experimental and theo-

retical papers appeared devoted to the study of inhomogeneous FCLs with a variable dispersion and distributed (as a rule, Raman) amplification [21–27].

However, the study of the dynamics of light pulses propagating in amplifying media has been performed in many papers, as a rule, by neglecting the frequency dispersion of the gain. At the same time, it was shown [28, 29] that the imaginary components of dispersion parameters can cause both the compression of a laser pulse (decrease in its duration) and its broadening. In this paper, we study the dynamics of an optical pulse propagating in a fibre with an arbitrary dependence of the real and imaginary components of dispersion parameters of the first and second orders on the longitudinal coordinate. We also analyse the possibility of obtaining efficient compression regimes in continuously inhomogeneous and cascade fibre structures and study the conditions required for achieving superluminal velocities of the envelope of a wave packet propagating in an inhomogeneous fibre.

## 2. Basic equations of the wave packet dynamics

Consider the dynamics of optical radiation in an optical fibre inhomogeneous over its length, which contains in the general case both active (with gain) and passive regions with the normal and anomalous material dispersion. Taking the complex propagation constant  $\beta(z) = \beta'(z) - i\beta''(z)$  into account, we can represent the field of a wave packet propagating in the fibre as

$$\mathbf{E}(t, r, z) = \frac{1}{2} \mathbf{e} R(r) \left\{ \mathcal{A}(t, z) \right. \\ \left. \times \exp \left[ i \left( \omega_0 t - \int_0^z \beta'(\xi) d\xi \right) \right] + \text{c. c.} \right\}, \quad (1)$$

where  $\mathbf{e}$  is the unit vector of the light-field polarisation; the function  $R(r)$  describes the radial distribution of the field in the fibre; and  $\omega_0$  is the carrier frequency of a wave packet coupled to the fibre. Taking into account the first- and second-order dispersion effects, we obtain in the linear approximation the equation

$$\frac{\partial \mathcal{A}}{\partial z} + k(z) \frac{\partial \mathcal{A}}{\partial t} - i \frac{d(z)}{2} \frac{\partial^2 \mathcal{A}}{\partial t^2} = -\beta''(z) \mathcal{A}, \quad (2)$$

for the complex envelope of the wave packet  $\mathcal{A}(t, z)$ , where  $k(z) = (\partial \beta(z) / \partial \omega)_0$  and  $d(z) = (\partial^2 \beta(z) / \partial \omega^2)_0$  are the complex dispersion parameters of the first and second orders;

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Received 31 March 2004; revision received 30 June 2004

Kvantovaya Elektronika 34 (9) 852–856 (2004)

Translated by M.N. Sapozhnikov

the values of these derivatives are taken at the carrier frequency  $\omega_0$  of the wave packet.

In the case of the real propagation constant, the first dispersion parameter determines the group velocity, and the second parameter determines the group velocity dispersion. Taking the complex propagation constant into account, the slowly varying temporal envelope of the wave packet can be conveniently written in the form

$$\mathcal{A}(t, z) = \mathcal{A}(t, z) \exp \left( - \int_0^z \beta''(\xi) d\xi \right), \quad (3)$$

where  $\beta''(z) > 0$  for the absorbing segment of the fibre and  $\beta''(z) < 0$  for the amplifying segment. By substituting (3) into (2), we obtain the equation for the amplitude of the wave packet envelope

$$\frac{\partial \mathcal{A}}{\partial z} - ik''(z) \frac{\partial \mathcal{A}}{\partial \tau} - i \frac{d(z)}{2} \frac{\partial^2 \mathcal{A}}{\partial \tau^2} = 0, \quad (4)$$

where  $\tau = t - \int_0^z k'(\xi) d\xi$  is the time in the running coordinate system and  $k'^{'''} = (\partial \beta^{'''} / \partial \omega)_0$  [similarly,  $d'^{'''} = (\partial^2 \beta'^{'''} / \partial \omega^2)_0$ ].

We assume that a frequency-modulated Gaussian pulse with the temporal envelope amplitude

$$A(\tau, 0) = A_0 \exp[-(1 + i\alpha_0 \tau_0^2) \tau^2 / 2\tau_0^2] \quad (5)$$

is incident on the fibre input ( $z = 0$ ), where  $\alpha_0$  is the frequency modulation rate and  $\tau_0$  is the input pulse duration. The solution of Eqn (4) for the initial conditions of fibre excitation (5) can be written in the form

$$A(\tau, z) = \rho(\tau, z) \exp[i\phi(\tau, z)], \quad (6)$$

where the expressions

$$2\phi(\tau, z) = \frac{S\tau_s^2 + 2\tau_s K''(1 + S^2) + K''^2 S(1 + S)}{\tau_p^2} - \arctan(S + \alpha_0 \tau_0^2), \quad (7)$$

$$\rho(\tau, z) = A_0 \left( \frac{\tau_0}{\tau_p} \right)^{1/2} \exp \left[ \frac{(1 + S^2) K''^2 - \tau_s^2}{2\tau_p^2} \right]$$

for the phase and amplitude of the complex quantity are introduced.

Here,

$$\tau_p = \tau_0^2 \left[ \frac{(1 - \chi_1)^2 + \chi_2^2}{\tau_0^2 + D''(1 + \alpha_0^2 \tau_0^4)} \right]^{1/2} \quad (8)$$

is the pulse duration;  $\tau_s = \tau - SK''$ ;  $\chi_1 = \alpha_0 D' - D'' \tau_0^{-2}$ ; and  $\chi_2 = \alpha_0 D'' + D' \tau_0^{-2}$ ;

$$D' = \int_0^z d'(\xi) d\xi; \quad D'' = \int_0^z d''(\xi) d\xi; \quad K'' = \int_0^z k''(\xi) d\xi;$$

$$S = \frac{(\alpha_0^2 \tau_0^2 + \tau_0^{-2}) D' - \alpha_0 \tau_0^2}{1 + (\alpha_0^2 \tau_0^2 + \tau_0^{-2}) D''}.$$

It follows from the above expressions that the pulse broadens or is compressed during its propagation depending on the relation between the parameters  $\tau_0$ ,  $\alpha_0$ ,  $D'$ ,  $D''$ , and also acquires the additional phase modulation. The condition under which local compression occurs at the

point  $z$  [ $(\partial \tau_p / \partial z) < 0$ ] in the case of complex dispersion parameters, taking (8) into account, is determined by the inequality

$$2\alpha_0 \tau_0^2 d'(z) + (\alpha_0^2 \tau_0^4 - 1) d''(z) > 0. \quad (9)$$

It follows from (9) that for  $\alpha_0 = 0$ , the pulse compression can occur when  $d'' < 0$ , while for  $d'' = 0$ , a classical situation takes place, i.e., the condition  $\alpha_0 d'(z) > 0$  should be satisfied.

### 3. Spectral representation of the pulse dynamics

We will analyse the radiation dynamics in the fibre under study by using the spectral representation of the complex amplitude of the temporal envelope of the wave packet

$$\tilde{A}(\omega, z) = \int_{-\infty}^{\infty} A(\tau, z) \exp(i\omega\tau) d\tau. \quad (10)$$

For a Gaussian frequency-modulated pulse, taking (6) into account, the spectral component of the wave packet takes the form

$$\tilde{A}(\omega, z) = A_0 \left( \frac{\tau_0}{2\pi\Delta\omega_s} \right)^{1/2} \times \exp \left[ -\frac{(\omega_s - \omega)^2}{2\Delta\omega_s^2} + \frac{\Omega_s^2}{2\Delta\omega_s^2} + i\varphi(\omega) \right], \quad (11)$$

where  $\omega_s = \omega_0 \pm \Omega_s(z)$  is the effective carrier frequency;  $\Omega_s(z) = K''(z)\Delta\omega_s^2$  is the carrier frequency shift; and

$$\Delta\omega_s(z) = \left[ \frac{1 + \alpha_0^2 \tau_0^4}{\tau_0^2 + D''(z)(1 + \alpha_0^2 \tau_0^4)} \right]^{1/2} \quad (12)$$

is the spectral width of the wave packet. The minus sign in the expression for the effective carrier frequency corresponds to the amplifying medium, and the plus sign corresponds to the absorbing medium.

It follows from the above expressions that the effective carrier frequency of the pulse shifts during its propagation in the fibre and the spectral width of the pulse changes. For  $D'' < 0$ , the spectral broadening of the pulse occurs on the path  $z < z_m = \tau_0^2 [D''(1 + \alpha_0^2 \tau_0^4)]^{-1}$ . Because the carrier frequency  $\omega_0$  of the pulse coupled to the fibre cannot coincide with the central frequency of the gain (absorption) line in the region  $D'' < 0$ , the effective carrier frequency  $\omega_s$  in the absorbing medium is displaced outside the resonance line, while in the amplifying medium, the opposite situation takes place – the effective carrier frequency  $\omega_s$  is pulled into the gain band.

The shift of the carrier frequency substantially limits the possibilities of the compression mechanism because, for  $K'' \neq 0$ , the carrier frequency can be displaced from the region with the negative value of the imaginary component of the group velocity dispersion parameter. This problem can be solved by either using media with a complicated profile of the gain increment, having a local extremum in the frequency region with  $D'' < 0$ , or using cascades for which the average value  $\langle K''(z) \rangle$  over the pulse propagation length is close to zero. Also, it is possible to use a fiberoptic link containing a sequence of amplifying segments, whose

resonance frequencies increase on passing from one segment to another, thereby providing the fulfilment of the condition  $D'' < 0$  despite the increase in the carrier frequency.

The pulse compression can be achieved and the transform-limited pulse can be produced at the given distance  $z$  from the point of radiation coupling into the fibre when the condition  $\alpha(z) = -(\partial^2 \phi / \partial t^2) = 0$  is satisfied. In the case of an inhomogeneous distribution of dispersion parameters considered here, this condition takes the form

$$(1 + \alpha_0^2 \tau_0^4) D'(z) - \alpha_0 \tau_0^4 \simeq 0. \quad (13)$$

When this condition is fulfilled, the pulse duration becomes minimal and it is related to the spectral width (12) of the pulse by the expression  $\tau_p = 1/\Delta\omega_s$ . If  $D'' < 0$ , the pulse duration can substantially decrease over the length  $z$  down to  $\tau_p \ll \tau_0(1 + \alpha_0^2 \tau_0^4)^{-1/2}$ . When

$$D' = \frac{\alpha_0 \tau_0^4}{1 + \alpha_0^2 \tau_0^4}, \quad D'' = -\frac{\tau_0^2}{1 + \alpha_0^2 \tau_0^4}, \quad (14)$$

the pulse can be strongly compressed, and  $\tau_p \rightarrow 0$ . However, in this case the negative effects can appear due to the development of a spontaneous amplitude modulation [12, 30] and the action of nonlinear aberrations. For this reason, it is desirable to perform the pulse compression not in the active segment of the fibre but in the next passive segment, where the chirp compensation occurs.

Of some interest is also the possibility of pulse self-monochromatization, i.e., a significant decrease in the spectral width of the pulse and an increase in the spectral density for  $D'' > 0$ . According to (12), the pulse duration substantially increases in this case.

#### 4. Compression of radiation in cascade schemes

Consider the properties of the dynamics of optical pulses in cascade systems consisting of a sequence of active and passive elements for which the imaginary and real components of dispersion parameters are constants.

As a simplest example illustrating the properties of the dynamics of radiation propagating in a fibre with amplification, we consider first an elementary cascade compression scheme, when radiation propagated through the first amplifying fibre with the complex parameters  $k_1$ , and  $d_1$  and the length  $L_1$  is coupled into the second fibre with the real dispersion parameters  $k_2$  and  $d_2$ . Let us assume that the pulse coupled to the first amplifying fibre has the duration  $\tau_0$  and the initial frequency modulation rate  $\alpha_0$ . At the output of the amplifying fibre and, hence, at the input of the second fibre, the frequency modulation rate of the pulse is

$$\begin{aligned} \alpha_1 &= -\left.\frac{\partial^2 \phi}{\partial t^2}\right|_{z=L_1} \\ &= \frac{\alpha_0 \tau_0^2 - (\alpha_0^2 \tau_0^2 + \tau_0^{-2}) d'_1 L_1}{\tau_0^2 \{[1 - (\alpha_0 d'_1 - d''_1 \tau_0^{-2}) L_1]^2 + (\alpha_0 d''_1 + d'_1 \tau_0^{-2})^2 L_1^2\}}. \end{aligned} \quad (15)$$

The pulse duration after propagation over the distance  $z$  in the second passive fibre is [30]

$$\tau_2(z) = \tau_1 \left[ (1 - \alpha_1 d'_2 z)^2 + \left( \frac{d'_2 z}{\tau_1^2} \right)^2 \right]^{1/2}, \quad (16)$$

where  $\tau_1 = \tau_p(L_1)$  is the pulse duration at the output from the first amplifying fibre. It follows from (16) that the pulse compression in the second fibre can occur when the inequality  $\alpha_1 d'_2 > 0$  is satisfied.

Consider the case demonstrating most clearly the possibility of pulse compression in the absence of its frequency modulation at the fibre input, i.e., when  $\alpha_0 = 0$ . In this case, the duration and frequency modulation rate of the pulse at the output of the first fibre are

$$\tau_1 = \left[ \frac{(\tau_0^2 + d''_1 L_1)^2 + (d'_1 L_1)^2}{\tau_0^2 + d''_1 L_1} \right]^{1/2}, \quad (17a)$$

$$\alpha = -\frac{d'_1 L_1}{(\tau_0^2 + d''_1 L_1) + (d'_1 L_1)^2}. \quad (17b)$$

From expressions (16) and (17b), the compression condition in the cascade scheme follows in the form of the inequality  $d'_1 d_2 < 0$ . In this case, the maximum compression of the pulse occurs at the second fibre length,

$$z_{\text{com}} = \frac{\alpha_1 \tau_1^4}{d_2 (1 + \alpha_1^2 \tau_1^4)}, \quad (18)$$

and the pulse duration is

$$\tau_{\min} = \frac{\tau_1}{(1 + \alpha_1^2 \tau_1^4)^{1/2}}. \quad (19)$$

In the case of high compression degrees, the inequality  $|\alpha_1| \tau_1^2 \gg 1$  should be fulfilled, and the above expressions can be written in the form

$$z_{\text{com}} \simeq \frac{1}{|d_2 \alpha_1|} \simeq \tau_0^2 \left| \frac{d'_1}{d_2 d''_1} \right|, \quad (20)$$

$$\tau_{\min} \simeq \frac{1}{|\alpha_1 \tau_1|} = (\tau_0^2 + d''_1 L_1)^{1/2} \left[ 1 + \left( \frac{\tau_0^2 + d''_1 L_1}{d'_1 L_1} \right)^2 \right]^{1/2}. \quad (21)$$

To produce the efficient pulse compression in the cascade mechanism, the length  $L_1$  of the amplifying fibre should be selected so as to provide the fulfilment of the inequality  $|\tau_0^2 + d''_1 L_1| \ll |d'_1 L_1|$ . In this case, expression (21) can be written in the simple form  $\tau_{\min} \simeq (\tau_0^2 + d''_1 L_1)^{1/2}$ .

By using expressions (8), (12), and (13), we can also easily calculate a three-element cascade consisting of a fibre for radiation coupling into an amplifier, the amplifier itself, and a compensating fibre located after the amplifier. In this case, the length of the compensator (the third element of the cascade), at which output the transform-limited, compressed pulse should be obtained, is, according to (13),

$$L_3 = \frac{\alpha_0 \tau_0^4 (1 + \alpha_0^2 \tau_0^4)^{-1} - d'_1 L_1 - d'_2 L_2}{d'_3} > 0, \quad (22)$$

where  $L_i$  is the length of the  $i$ th element of the cascade and  $d'_i$  is the real component of the material dispersion of the  $i$ th element. In this case, the minimal duration of the transform-limited pulse at the compressor output is determined by the expression

$$\tau_{\min}(L_3) = \frac{1}{\Delta\omega_s(L_3)} \simeq \left[ \frac{1 + \alpha_0^2 \tau_0^4}{\tau_0^2 + d''_2 L_3 (1 + \alpha_0^2 \tau_0^4)} \right]^{-1/2}. \quad (23)$$

Therefore, the minimal duration of the pulse (as in the case of the two-element cascade) is completely determined by the amplifier parameters ( $d_2''$ ,  $L_2$ ,  $k_2''$ ) and the initial frequency modulation rate  $\alpha_0$  (chirp) of the pulse coupled to the cascade.

If the cascade consists of  $N$  dispersion elements, among which  $k$  elements are amplifiers and  $(N - k)$  elements are passive fibres (the first- and second-order parameters for which of them are constants over the length of a given segment), the condition for the chirp compensation [condition for the production of a transform-limited (compressed) pulse] at the output of the  $N_{th}$  element takes the form

$$\sum_{i=1}^N d'_i L_i = \frac{\alpha_0 \tau_0^4}{1 + \alpha_0^2 \tau_0^4}. \quad (24)$$

Here,  $L_i$  and  $d'_i$  are the length and the real component of the material dispersion of an arbitrary  $i$ th element of the cascade (passive or active). The duration of the transform-limited pulse at the output of the  $k$ th element of the cascade is

$$\tau_p(z = L) = \left[ \frac{\tau_0^2 + (1 + \alpha_0^2 \tau_0^4) \sum_{j=1}^N (d_j'' L_j)}{1 + \alpha_0^2 \tau_0^4} \right]^{1/2}, \quad (25)$$

where  $d_j''$  is the imaginary component of the material dispersion of an arbitrary  $j$ th amplifier.

The compression scheme considered above neglects, however, the shift of the carrier frequency of the wave packet. The elimination of a considerable shift of the carrier frequency is in fact a key problem to be solved for the implementation of the pulse compression scheme involving amplifying fibres. For the case  $\alpha_0 = 0$ , the condition  $|\Omega_s| \ll \Delta\omega$ , which determines the restriction on the shift of the carrier frequency in the cascade compression scheme, can be written in the form

$$\tau_0 \left( \frac{\Delta\omega_1}{|k_1''| L_1} \right)^{1/2} \gg \eta_{\max}, \quad (26)$$

where  $\eta_{\max} = \tau_0 / \tau_{\min}$  is the maximal degree of pulse compression. Condition (22) can be fulfilled by varying the parameter  $k_1''$ , which can be taken indefinitely small.

## 5. Velocity of the wave packet envelope

The question about the time dependence of the position of the maximum of the pulse envelope is important for the description of the pulse dynamics in a fibre. The velocity of the maximum of the wave packet envelope also depends on the coordinate  $z$  and, according to (17b), can be described by the expression

$$u_m = z \left[ \int_0^z \frac{\partial \beta'(\xi)}{\partial \omega} d\xi + S(z) \int_0^z \frac{\partial \beta''(\xi)}{\partial \omega} d\xi \right]^{-1}. \quad (27)$$

The time dependence of the maximum of the wave packet envelope is described by the relation

$$\int_0^l \frac{dz}{u_m(z)} = t_m, \quad (28)$$

where  $t_m$  is the time interval measured from the instant of arrival of the pulse maximum into the fibre;  $l$  is the distance propagated by the pulse maximum from the entrance point during the time interval  $t_m$ . Thus, if  $\beta'$  is independent of the coordinate  $z$  and  $\beta'' = 0$ , then the position of the envelope maximum is described by the standard expression  $l = (\partial \beta' / \partial \omega)_0^{-1} t_m$ . If the parameters  $\beta'$  and  $\beta''$  are constant along the fibre length, we obtain the expression obtained earlier in Refs [29, 31]

$$l = \frac{(\partial \beta' / \partial \omega)_0^{-1} t_m}{1 + S(\partial \beta'' / \partial \beta')_0}. \quad (29)$$

Therefore, the average velocity  $\langle u_m \rangle = l/t_m$  of the maximum of the wave packet envelope can be described by the expression

$$u_m = \frac{u_1}{1 + S u_1 (\partial \beta'' / \partial \omega)_0}, \quad (30)$$

where  $u_1 = c/N$  is the quantity, which is usually treated as the group velocity of the pulse, and  $N$  is the real part of the effective refractive index of the mode forming the pulse. It follows from (30) that, when  $S(\partial \beta'' / \partial \omega)_0 < 0$  and also  $K'' < 0$ , a superluminal pulse can be produced in the fibre [29, 31].

The possibility of phase conjugation is also interesting. In this situation,  $S u_1 (\partial \beta'' / \partial \omega)_0 < -1$  and, therefore,  $u_m < 0$ . Phase conjugation can appear due to strong amplification and dispersion. As a result, the maximum of the wave packet is formed at the very onset of the pulse and shifts in the direction opposite to the pulse propagation. Such a situation was experimentally observed in paper [32].

Note that one of the substantial difficulties in the realisation of the superluminal propagation of waves is the above-considered shift of the carrier frequency to the region where  $(\partial \beta'' / \partial \omega) \simeq 0$  and therefore the superluminal regime cannot be realised (for Gaussian pulses). To 'overcome' this problem, sufficiently long frequency-modulated pulses can be used, for which the inequality  $|\alpha_0 \tau_0^2| \gg 1$  is valid and we can assume that  $S \simeq -\alpha_0 \tau_0^2$ . In this case, relation (29) takes the form

$$u_m = \frac{u_1}{1 - \alpha_0 \tau_0^2 u_1 (\partial \beta'' / \partial \omega)_0}. \quad (31)$$

It follows from (31) that, when the condition  $\alpha_0 \tau_0^2 \times u_1 (\partial \beta'' / \partial \omega)_0 \simeq 1$  is fulfilled which provides indefinitely large propagation velocities of the maximum of the wave packet envelope, the shift of the carrier frequency is determined by the expression  $|\Omega_s| \simeq |\alpha_0| z / u_1$ . If  $u_1 \simeq 10^8 \text{ m s}^{-1}$  and  $\alpha_0 \simeq 10^{10} \text{ s}^{-2}$ , then  $|\Omega_s| < 10^5 \text{ s}^{-1}$  for  $z < 10 \text{ m}$ . Taking into account that the width of the gain line is  $\Delta\omega_1 = 10^{11} - 10^{14} \text{ s}^{-1}$ , the shift of the carrier frequency by  $10^5 \text{ s}^{-1}$  (and even a significantly greater shift) can be considered negligible small.

In addition, the complexity of the achievement of superluminal velocities can be related to the modulation instability of the wave packet [12, 30] and the spontaneous development of noise [33]. This problem can be solved by using media and corresponding spectral ranges with large values of the real component of the group velocity dispersion, i.e., highly dispersive media.

Because, according to the Kramers–Kronig relation, media with the large imaginary component of the permittivity are virtually always strongly dispersive, the conditions for the pulse propagation with a large group velocity dispersion in amplifying media can be easily realised.

## 6. Conclusions

Our analysis has shown that the use of optical fibres with a specified inhomogeneous (along the fibre length) distribution of dispersion parameters for the efficient compression of optical pulses offers a number of advantages over the schemes based on the pulse compression directly in the amplifying fibre. Thus, to produce a greater pulse compression in a single amplifying fibre, it is necessary to have a greater value of  $|d''/d'|$ , which is restricted by two substantial factors (except the above-considered effect of the carrier frequency displacement). The first factor involves purely technological difficulties related to the formation of the required shape of the gain line. The second factor is the rapid development of the modulation instability at large values of  $|d''/d'|$ , which was studied in detail for such fibres in Ref. [34]. In addition, a strong pulse compression directly in the amplifying medium requires the consideration of nonlinear effects, which lead, as a rule, to additional undesirable aberrations. These restrictions can be eliminated by using an inhomogeneous fibre. Thus, they are absent in the above-considered cascade schemes consisting of alternating homogeneous active and passive fibre segments. In this case, first, the value of the parameter  $|d''/d'|$  is no longer decisive for obtaining strong compression. Moreover, if the carrier frequency is fixed, a quite strong compression can be achieved even when  $|d''/d'| \ll 1$ . Second, the stronger is the pulse broadening in the amplifying fibre, the higher is the efficiency of the two-cascade compression scheme, i.e., the closer is the active fibre length to  $z_m$ , the shorter pulse can be obtained at the cascade output. In this case, the restrictions caused by the possible negative effect of nonlinear parameters are eliminated.

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