

Spectral – threshold characteristics of a distributed-feedback laser with the sinusoidal modulation of coupling coefficient

A.A. Afanas'ev, S.Yu. Mikhnevich

Abstract. The spectral-threshold characteristics of a distributed-feedback (DFB) laser with the sinusoidal modulation of the complex coupling coefficient are studied within the framework of the linear theory of coupled modes. The analysis is performed in a broad range of amplitudes of the coupling coefficient for phase, amplitude, and amplitude–phase gratings providing distributed feedback in the active medium of the laser. It is shown that the eigenmode spectrum of the DFB laser on a phase grating with a large-scale modulation of the coupling coefficient is similar to the eigenmode spectrum of the DFB laser on an amplitude grating with a constant coupling coefficient. The DFB laser under study is promising for obtaining stable single-frequency lasing and can find applications in devices in integrated optics.

Keywords: distributed-feedback lasers, complex coupling coefficient, deep sinusoidal amplitude–phase modulation.

1. Introduction

Progress in optical laser technologies imposes new stringent requirements on the parameters of laser sources of coherent radiation. In particular, the stability of single-frequency lasing, compactness and simplicity of coupling of laser sources with different elements of optical systems are very important for integrated optics and fibreoptic communication networks. The distributed-feedback (DFB) lasers meet these requirements in many respects. They use instead of a conventional mirror resonator a fibre Bragg grating, whose period can be changed to tune the lasing frequency within the gain band of the active medium. Semiconductor DFB lasers have found wide applications in integrated optics as sources of narrow-band coherent radiation in the range from 1.3 to 1.5 μm [1–7].

It is known [3–8] that stable single-frequency lasing cannot be obtained with the help of DFB based on a periodic modulation of the refractive index of the active medium of the laser. A laser with the distributed resonator based on the phase grating has a high spectral selectivity due to a narrow spectral width of the Bragg resonance and

possesses the lowest self-excitation threshold simultaneously for two modes with frequencies located symmetrically with respect to the Bragg frequency ω_{Br} . A part of a smooth fibre in the middle of the Bragg structure (defect) changes the intermode distance between these modes [9]. In Refs [3, 6, 9–11], a method was proposed and realised for obtaining stable single-frequency generation in a DFB laser on the so-called phase-shifted structure consisting of two sinusoidal gratings of the refractive index with identical periods and amplitudes and the phase shift equal to π in the middle of the distributed resonator. The DFB laser based on this structure has the lowest self-excitation threshold for one Bragg mode at the frequency $\omega = \omega_{\text{Br}}$. A substantial disadvantage of this laser is a very complicated multi-stage technological process for fabricating a combination of refractive-index gratings with the phase shift π in the middle of the distributed resonator.

The authors of Refs [1, 12] considered the alternative and technologically simpler concept of a single-mode DFB laser on the phase grating with a low-frequency sinusoidal modulation of the coupling coefficient of the counter-propagating laser waves. By selecting appropriately the modulation of the coupling coefficient in such a DFB laser, the longitudinal inhomogeneity of inversion burning in the active medium [1] resulting in the broadening of the laser line can be eliminated [13]. The periodic structure of the refractive index [and (or) the gain] of the active medium of the laser providing the sinusoidal modulation of the coupling coefficient can be simply realised by the one-stage holographic method. By using the scheme of three-beam holographic recording, two harmonic gratings with equal amplitudes and close periods can be recorded comparatively easily, whose combination provides the low-frequency sinusoidal modulation of the coupling coefficient [12]. The technological advantages in the manufacturing of such a relief of the refractive index open up wide opportunities for practical applications of single-mode DFB lasers with the periodically modulated coupling coefficient.

The approximate solution of the corresponding system of equations in the case of weak coupling was obtained in Ref. [12], where the spectral-threshold parameters of the DFB laser on the phase grating with the sinusoidal modulation of the coupling coefficient were analysed. The spectral-threshold parameters were compared for the DFB laser on the harmonic grating with the sinusoidal modulation of the coupling coefficient and the DFB laser on the phase-shifted structure with the phase shift π in the middle of the distributed resonator. The approximate relations obtained in Ref. [12] describe quite accurately

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the spectral-threshold parameters of the DFB laser for small amplitudes of the modulated coupling coefficient. However, due to the recent progress in new technologies for fabricating media with a deep modulation of the dielectric constant [14], it is necessary to generalise the theory proposed in Ref. [12] to the case of DFB lasers with large amplitudes of the modulated coupling coefficient.

In this paper, we analysed numerically the system of equations describing the linear generation regime of the DFB laser on the harmonic grating with the sinusoidal modulation of the coupling coefficient of counterpropagating waves. We studied the spectral–threshold parameters of this laser in a broad range of variation in the amplitude of the coupling coefficient. The analysis was performed for the phase, amplitude, and amplitude–phase gratings providing the DFB for waves in the active medium of the laser.

2. Basic equations

According to Ref. [12], we represent the modulation of the refractive index n and the gain k in the active medium as a combination of two harmonic gratings with equal amplitude \bar{n} and \bar{k} with slightly different periods A_1 and A_2 ($A_1 < A_2$):

$$n(z) = n_0 + \bar{n} \left[\cos\left(\frac{2\pi}{A_1}z\right) + \cos\left(\frac{2\pi}{A_2}z + \varphi\right) \right], \quad (1)$$

$$k(z) = k_0 + \bar{k} \left[\cos\left(\frac{2\pi}{A_1}z\right) + \cos\left(\frac{2\pi}{A_2}z + \varphi\right) \right],$$

where n_0 and k_0 are the unperturbed values of the refractive index and the gain, and φ is an arbitrary phase shift between the gratings. By introducing the notation

$$g = \pi \left(\frac{1}{A_1} + \frac{1}{A_2} \right), \quad G = \pi \left(\frac{1}{A_1} - \frac{1}{A_2} \right)$$

and moving along the direction of counterpropagating laser waves from the point $z = 0$ to the point $z = (\varphi - \pi)/(2G)$, we can rewrite expressions (1) in the convenient form

$$n(z) = n_0 + 2\bar{n} \sin(Gz) \cos(gz + \phi), \quad (2)$$

$$k(z) = k_0 + 2\bar{k} \sin(Gz) \cos(gz + \phi),$$

where

$$\phi = \frac{\varphi}{2} - \frac{A_2 + A_1}{A_2 - A_1} \frac{\varphi + \pi}{2}.$$

We assume in (2) that $|k| \ll n_0 \kappa_0$ and $|\bar{n}| \ll n_0$, where $\kappa_0 = \omega/c$, and $\sin(Gz)$ is a slowly varying function of the coordinate z ($G \ll g$). By using the method of coupled waves [15], we will seek the solution of the wave equation

$$\frac{d^2 E}{dz^2} + \frac{\omega^2}{c^2} \left[n(z) - i \frac{c}{\omega} k(z) \right]^2 E = 0 \quad (3)$$

in the form of two counterpropagating waves

$$E(z) = E_+(z) \exp\left(i \frac{g}{2} z\right) + E_-(z) \exp\left(-i \frac{g}{2} z\right), \quad (4)$$

where $E_{\pm}(z)$ are slowly varying amplitudes and ω is the lasing frequency [$\omega \approx \omega_{\text{Br}} = gc/(2n_0)$]. By substituting (4) into (3) and taking (2) into account, we obtain the system of truncated equations [12]

$$\pm \frac{dE_{\pm}}{dz} - \left(\frac{k_0}{2} + i\delta \right) E_{\pm} = i\kappa(z) e^{\pm i\phi} E_{\mp}(z), \quad (5)$$

describing the linear generation regime in the DFB laser with the sinusoidal modulation of the coupling coefficient

$$\kappa(z) = \frac{\kappa_0}{2} \left(\bar{n} - \frac{i\bar{k}}{2\kappa_0} \right) \sin(Gz) \equiv \kappa_0 \sin(Gz) \quad (6)$$

and the Bragg detuning $\delta = (\kappa_0 n_0 - g/2)$.

The solution of the system of linear equations (5) determines the spectral–threshold parameters (the eigenmode frequencies and threshold gains) of the DFB laser. Note that, to study lasing itself, it is necessary to add constitutive equations to system (5), which take into account the saturation of the gain $k = k(|E|^2)$ in the active medium by laser radiation.

To find the solutions of the system of equations (5), they should be supplemented with the boundary conditions

$$E_+(-L/2) = E_-(L/2) = E_0, \quad (7)$$

where L is the active medium length. An arbitrary specification of amplitudes on the medium boundaries does not affect the dispersion equation, which determines the spectral–threshold parameters of the DFB laser [15]. However, in the case of biharmonic modulation under study [$\kappa = \kappa(z)$], the choice of the boundary values of amplitudes can no longer be arbitrary because of an arbitrary phase shift between the gratings ($\phi \neq 0$). One can easily see that, by making the substitution $E_{\pm}(z) = A_{\pm}(z) \exp(\pm i\phi/2)$ in equations (5), the phase factors can be excluded. In this case, boundary conditions (7) lead to certain phase relations for waves at the active medium boundaries. To exclude the difference between amplitudes at the boundaries, which is not essential, we will assume below that $\phi = 0$.

The analytic solution of the system of equations (5) with boundary conditions (7) cannot be obtained without additional simplifying assumptions. As mentioned in introduction, the solution of system (5) in the weak coupling approximation ($|\kappa(z)| \ll k_0$) was obtained in Ref. [12], where the spectral–threshold parameters of the DFB laser on the phase grating ($\bar{k} = 0$) were analysed. We performed the numerical simulation of the system of equations (5) without any additional simplifying assumptions. Three different situations upon the formation of DFB were considered with the use of phase ($\bar{k} = 0$), amplitude ($\bar{n} = 0$), and amplitude–phase ($\bar{k} \neq 0, \bar{n} \neq 0$) gratings.

3. Results of numerical analysis

To perform the numerical simulation of equations (5), it is convenient to pass to the real variables $\mathcal{E}_{\pm}(z)$ and ψ_{\pm} :

$$E_{\pm}(z) = \mathcal{E}_{\pm}(z) e^{i\psi_{\pm}(z)}. \quad (8)$$

By substituting (8) into (5), we obtain the following equations for amplitudes $\mathcal{E}_{\pm}(z)$ and phases $\psi_{\pm}(z)$ of the counterpropagating laser waves

$$\pm \frac{d\mathcal{E}_{\pm}}{dz} - \frac{k_0}{2} \mathcal{E}_{\pm} = \sin(Gz)(\kappa_0'' \cos \gamma \pm \kappa_0' \sin \gamma) \mathcal{E}_{\mp}, \quad (9)$$

$$\pm \frac{d\psi_{\pm}}{dz} = \frac{\mathcal{E}_{\mp}}{\mathcal{E}_{\pm}} \sin(Gz)(\kappa_0' \cos \gamma \mp \kappa_0'' \sin \gamma) + \delta,$$

where $\kappa_0' = \text{Re } \kappa_0$; $\kappa_0'' = \text{Im } \kappa_0$; $\gamma = \phi + \psi_- - \psi_+$. The system of equations (9) was solved by the Runge–Kutta method, which was modified for problems with the boundary conditions $\mathcal{E}_{\pm}(\mp L/2) = \mathcal{E}_0$, $\psi_{\pm}(\mp L/2) = 0$ specified at the opposite boundaries of the active medium. At the first stage of numerical simulation, we determined the eigenmode frequencies ω_j of the DFB laser, i.e., the frequencies of the narrow peaks of the output amplitudes of the waves depending on the Bragg detuning δL . At the second stage of calculations, we found the values of output amplitudes at eigenfrequencies ω_j depending on k_0 , from which threshold gains $k_{0\text{th}}(\omega_j)$ were determined.

Phase grating. The numerical analysis of equations (9) showed that the spectral–threshold characteristics of the DFB laser on the phase grating with the sinusoidal modulation of the coupling coefficient at small values of the parameter GL (for example, for $GL = \pi$) are similar to the corresponding characteristics of the DFB laser on the amplitude grating with a constant coupling coefficient [15]. The discrete spectrum of eigenmodes with frequencies $\omega_{\pm j}$ ($j = 0, 1, 2, \dots$ is the mode number) of the laser under study consists of the central (Bragg) mode with $\omega_0 = \omega_{\text{Br}}$ and higher-order modes with $\omega_{-j} < \omega_0 < \omega_j$ ($j \geq 1$) symmetrically located with respect to the central mode (Fig. 1a). One can see from Figs 1a, b that for $GL = \pi$, the Bragg mode has the lowest threshold amplification. As the mode number j increases, the value of $(k_0L)_{\text{th}}$ also increases, i.e., the discrimination of modes is observed, and $k_{0\text{th}}(\omega_j) = k_{0\text{th}}(\omega_{-j})$. In this case, the amplitude of the coupling coefficient increases with decreasing monotonically $(k_0L)_{\text{th}}$.

Because the values of $(k_0L)_{\text{th}}$ for the Bragg mode (ω_0) and the first-order modes ($\omega_{\pm 1}$) considerably differ in the region of large κ_0' (for example, for $\kappa_0' = 1 \text{ cm}^{-1}$, we have $k_{0\text{th}}(\omega_1)/k_{0\text{th}}(\omega_0) \approx 1.2$), the dynamic range of variation in the gain $\Delta k_{0\text{th}}$ for the realisation of single-frequency operation of the DFB laser is sufficiently broad. The dynamic range of $\Delta k_{0\text{th}}$ also increases with increasing κ_0' . As expected, in the region of small κ_0' (for $\kappa_0' \leq 1 \text{ cm}^{-1}$), the obtained values of $(k_0L)_{\text{th}}$ are close to those calculated by approximate expressions in Ref. [12]. As κ_0' increases, these differences become greater. Figure 1c shows the dependence of the intermode distance $\Omega_j = |\omega_{j+1} - \omega_j|L/c$ on the amplitude of the coupling coefficient κ_0' .

Due to the symmetry of the eigenmode spectrum with respect to the Bragg frequency $\omega_0 = \omega_{\text{Br}}$, we will restrict ourselves to the consideration of the frequency region $j > 0$ with the positive indices j . In the region $\kappa_0' \leq 1 \text{ cm}^{-1}$, the frequencies of the first-order modes are removed from the frequency ω_{Br} with increasing κ_0' : the distance Ω_1 monotonically increases. In this case, the dependence of the intermode distance Ω_2 for the first- and second-order modes on the parameter κ_0' is not monotonic. In the region $\kappa_0' > 0.2 \text{ cm}^{-1}$, the frequencies of these modes tend to draw together.

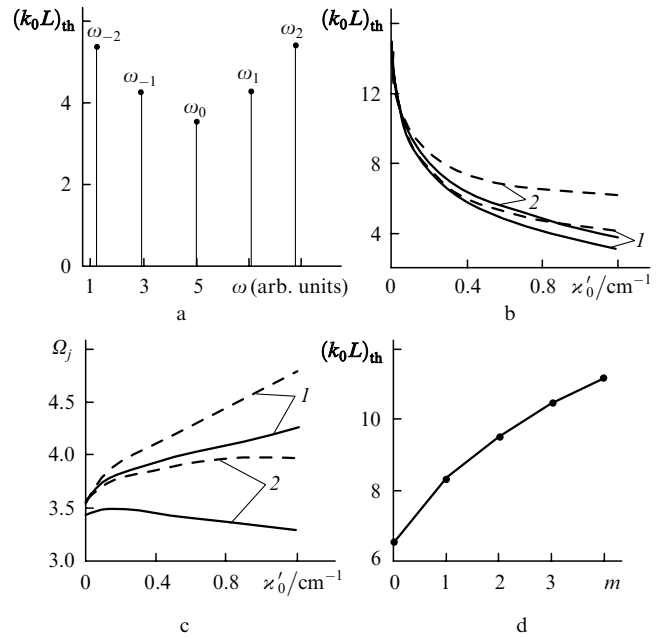


Figure 1. Spectral–threshold characteristics of the DFB laser on the phase grating ($\bar{k} = 0$): the eigenmode spectrum of DFB laser with frequencies ω_j for $\kappa_0' = 1 \text{ cm}^{-1}$ and $GL = \pi$ (a); dependences of the threshold gain $(k_0L)_{\text{th}}$ of eigenmodes at frequencies $\omega_{\pm j}$ on the amplitude κ_0' of the coupling coefficient for $GL = \pi$ ($m = 0$) for the Bragg mode ($j = 0$, $\omega_0 = \omega_{\text{Br}}$) (1) and the first-order mode ($j = 1$, $\omega_{\pm 1}$) (2) (b); dependences of the intermode distance $\Omega_j = |\omega_{j+1} - \omega_j|L/c$ on the amplitude κ_0' of the coupling coefficient for $j = 0$ (1) and 1 (2) (c); and the dependence of $(k_0L)_{\text{th}}$ for the Bragg mode at the frequency $\omega_0 = \omega_{\text{Br}}$ on the number m of full modulation periods of the coupling coefficient over the active medium for $\kappa_0' = 0.3 \text{ cm}^{-1}$ (d). Solid curves in Figs 1b, c are numerical solutions of equations (9), the dashed curves are calculated by approximate formulas (37) and (38) from [12].

Figure 1d shows the dependence of the threshold gain for the Bragg mode on the number $m = (GL/\pi - 1)/2$ of full periods of the low-frequency modulation of the coupling coefficient of waves over the length L of the active medium of the DFB laser. One can see that the threshold gain of the Bragg mode is minimal for $m = 0$. As m increases, the value of $(k_0L)_{\text{th}}$ for the Bragg mode substantially increases. Our numerical calculations showed that a further increase in m results in the reverse discrimination of modes, i.e., the self-excitation thresholds for higher-order modes become lower than the threshold for the zero-order mode. In this case, the greater m , the higher is the order of modes with the lowest excitation threshold. The dynamics of mode discrimination is illustrated in Fig. 2, where the spectral dependences of the output amplitudes of counterpropagating waves are shown for different m . The greatest output amplitudes at the corresponding frequencies are achieved for modes with the minimal threshold gain.

Amplitude grating. The typical eigenmode spectrum of the DFB laser on the amplitude grating ($\bar{n} = 0$) with the sinusoidal modulation of the coupling coefficient for $GL = \pi$ is shown in Fig. 3a. In this case, the Bragg mode at the frequency $\omega_0 = \omega_{\text{Br}}$ is forbidden. The two lowest first-order modes at frequencies $\omega_{\pm 1}$, which are located symmetrically with respect to the Bragg frequency ω_{Br} , have the same lowest threshold gain. The spectral–threshold characteristics of the DFB laser for the given modulation frequency of the coupling coefficient are similar

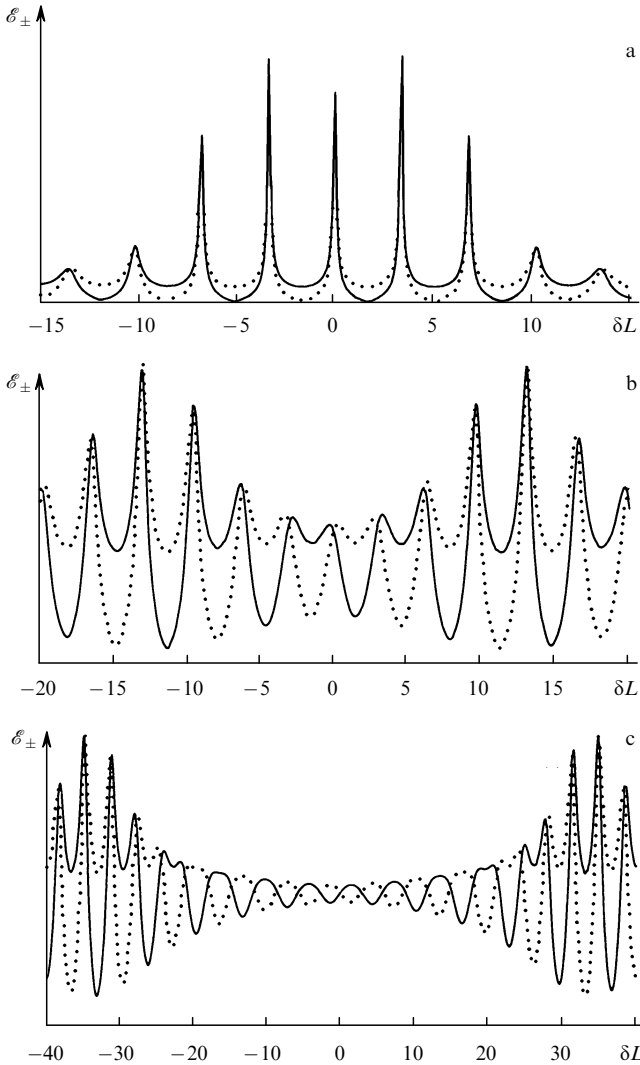


Figure 2. Output amplitudes $\varepsilon_{\pm}(L/2)$ (solid curves) and $\varepsilon_{\pm}(-L/2)$ (dotted curves) of generation waves in the DFB laser on the phase grating as functions of the Bragg detuning δL for $m = 1$ (a), 3 (b), and 10 (c) and $\varkappa'_0 = 10^{-2} \text{ cm}^{-1}$ and $k_0 L = 15.2$.

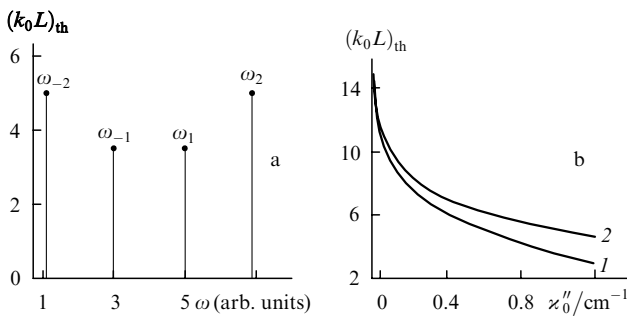


Figure 3. Spectral–threshold characteristics of the DFB laser on the amplitude grating ($\bar{n} = 0$) for $GL = \pi$ ($m = 0$): the eigenmode spectrum with frequencies $\omega_{\pm j}$ for $\varkappa'_0 = 1 \text{ cm}^{-1}$ (a) and dependences of the threshold gain $(k_0 L)_{\text{th}}$ on the parameter \varkappa''_0 for the first- ($j = 1, \omega_{\pm 1}$) (1) and second-order ($j = 2, \omega_{\pm 2}$) (2) modes (b).

to those of the DFB laser on the phase grating with a constant coupling coefficient [15]. Figure 3b shows the typical dependence of the threshold gain for the lowest eigenmodes with $k_{0\text{th}}(\omega_{+j}) = k_{0\text{th}}(\omega_{-j})$ on the amplitude of the coupling coefficient \varkappa''_0 . Note that the values of the

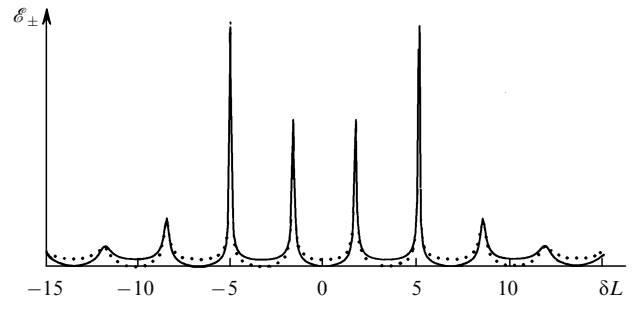


Figure 4. Output amplitudes $\varepsilon_{\pm}(L/2)$ (solid curves) and $\varepsilon_{\pm}(-L/2)$ (dotted curves) of generation waves in the DFB laser on the phase grating as functions of the Bragg detuning δL for $\varkappa''_0 = 10^{-2} \text{ cm}^{-1}$, $k_0 L = 15.24$, and $m = 1$.

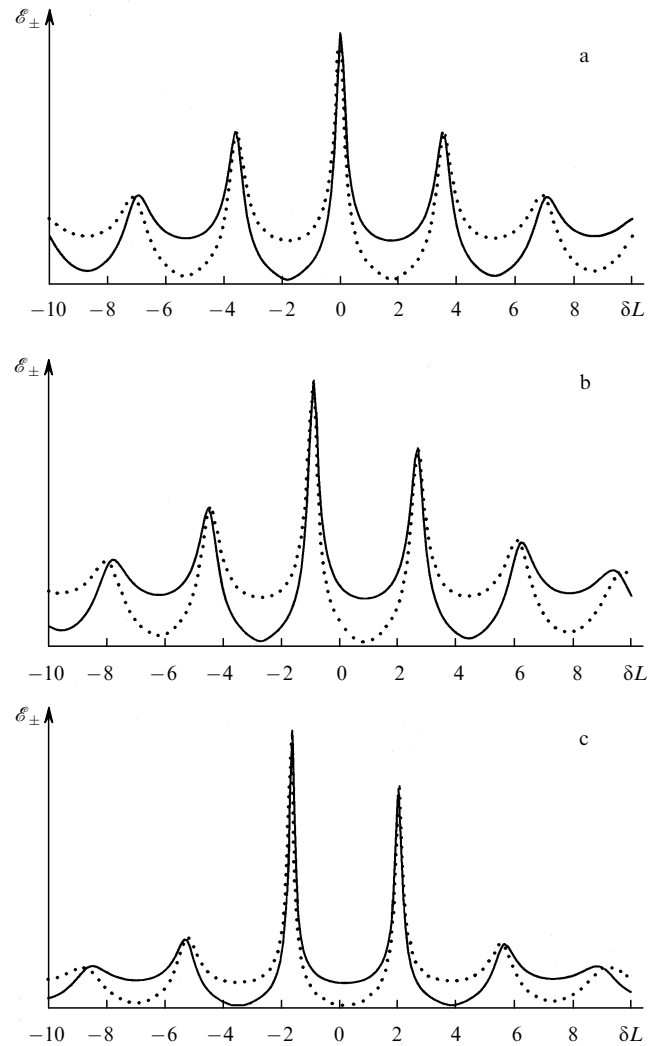


Figure 5. Dynamics of the spectral dependence of the output amplitudes $\varepsilon_{\pm}(L/2)$ (solid curves) and $\varepsilon_{\pm}(-L/2)$ dotted curves of generation waves in the DFB laser with increasing the amplitude (imaginary) component of the coupling coefficient for $\varkappa'_0 = 10^{-2} \text{ cm}^{-1}$, $GL = \pi$ ($m = 0$) and $k_0 L = 14.4$, $\varkappa''_0 = 0$ (a), $k_0 L = 13.6$, $\varkappa''_0 = 0.01 \text{ cm}^{-1}$ (b) and $k_0 L = 10.8$, $\varkappa''_0 = 0.05 \text{ cm}^{-1}$ (c).

threshold gain for the Bragg mode for DFB on the phase grating (Fig. 2b) and for the first-order modes for DFB on the amplitude grating (Fig. 3a) are virtually the same if $\varkappa'_0 = \varkappa''_0$. Our calculations showed that in this case the intermode distances very weakly depend on \varkappa''_0 . As in the

case considered above ($\kappa''_0 = 0$), the threshold gains for the first two modes in the DFB laser on the amplitude grating increase with increasing m , whereas the gains for higher-order modes decrease, i.e., the reverse discrimination of modes takes place. Figure 4 shows the spectral dependences of the output amplitudes of the laser waves for $m = 1$.

Amplitude–phase grating. The evolution of the spectral–threshold characteristics of the DFB laser with increasing the amplitude component κ''_0 of the coupling coefficient is shown in Fig. 5. One can see that the spectral picture of amplitudes of the output waves of the DFB laser on the phase grating (Fig. 5a) monotonically transforms with increasing κ''_0 to the picture corresponding to the DFB laser on the amplitude grating (Figs 5b, c). In this case, the Bragg mode ($\omega_0 = \omega_{Br}$) transforms to the minus first-order mode (ω_{-1}) for the DFB laser on the amplitude grating. Correspondingly, the plus first-order mode (ω_{+1}) shifts to the left from the axis δL and is located symmetrically to the ω_{-1} mode with respect to the point $\delta L = 0$. In this case, the threshold gains for these modes become identical.

4. Conclusions

Based on the numerical solution of the wave equations for counterpropagating laser waves, we have studied the spectral–threshold characteristics of the DFB laser with a deep sinusoidal modulation of the complex coupling coefficient $\kappa(z) = (\kappa'_0 + i\kappa''_0) \sin(Gz)$. We have analysed the dependence of these characteristics on the coupling coefficient $\kappa(z)$. It has been shown that the maximum difference between the lasing thresholds for the Bragg mode and first-order modes is achieved in the case of the phase grating ($\kappa''_0 = 0$). The spectrum of the DFB laser for $\kappa''_0 = 0$ is similar to that of the DFB laser on the amplitude grating with a constant coupling coefficient $\kappa'(z) = \text{const}$. The stability of single-frequency lasing, compactness, and the possibility of coupling this laser with different elements of optical schemes open up opportunities for its application in integrated optics.

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