

PACS numbers: 42.65.Pc; 78.66.-w
DOI: 10.1070/QE2004v034n04ABEH002680

Effect of α-factor on the dynamics of a bilayer semiconductor structure

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***Abstract*.** **A system consisting of two thin films interacting resonantly with a light field is considered. The model formulated for such a system is based on the approximation of two-level atoms and takes into account the peculiarities of the semiconductor medium, in particular, the effect of the ratio of dispersions of real and imaginary parts of the susceptibility (α-factor). It is shown that the inclusion of the α-factor leads to an additional nonlinearity and a considerable modification of the set of steady states of the system. The effect of α-factor on the dynamic regimes emerging in the system is discussed.**

Keywords: *thin films, bistability, symmetry breaking.*

1. Introduction

Semiconductor thin-film systems are used widely at present in systems for data transfer and processing because of their small size and large potentialities for light control. A significant role in the behaviour of such systems is played by the nonlinearity arising during interaction of material layers with the light field. Even in the case of a single layer with a feedback mechanism, nontrivial regimes such as bistability [1, 2], self-pulsation [2 – 4], and formation of transverse static and moving spatial structures [5 – 7] may emerge in the nonlinear regime.

An important feature of such systems is their phase sensitivity, i.e., the dependence of dynamic properties of the system on phase relations of the light field in the feedback loop. This dependence may become more pronounced when the feedback is realised in the system not through a passive element in the dynamic sense (e.g., a mirror), but through an element whose behaviour varies in time. This type of feedback is realised in multilayer optical structures. In this work, we consider the simplest version of a multilayer structure, namely, a system consisting of two thin nonlinear semiconductor films.

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Received 17 September 2003; revision received 12 January 2004
Kvantovaya Elektronika **34** (4) 355 – 360 (2004)
Translated by Ram Wadhwa

It was shown in our earlier publications that in addition to bistability, such systems may also display effects like symmetry breaking [8] (when identical fields incident on both sides of the system are reflected with different amplitudes), self-pulsations [9, 10], as well as the emergence of spatial structures with different degrees of symmetry (in systems with a large aperture) [10 – 12]. Note that the spontaneous formation of spatial structures and pulsations is due to the same feedback mechanism in which the phase relations in the feedback loop play a key role.

In Refs [8 – 12], a nonlinear medium was considered as an ensemble of two-level atoms whose interaction with the light field is described by the Maxwell – Bloch equations. The parameters of the medium for which the above-mentioned phenomena occur correspond best of all to the parameters of semiconductor layers [1, 3]. However, a semiconductor medium is characterised by mechanisms responsible for coupling between the amplitude and phase of the light field that differs from the analogous coupling in a medium of two-level atoms, playing a significant role in systems with a phase-sensitive feedback [13, 14]. However, this mechanism can be taken into account in the two-level model by introducing a coefficient characterising the relative variation in the refractive index and absorption coefficient upon a variation in the density of free carriers. This coefficient is called the α-factor [15].

In this paper, we consider the effect of the α-factor on the dynamic characteristics of a system with a small aperture (when only one transverse mode participates in the dynamics) as well as a large aperture. While describing the dynamics of semiconductor systems, the active medium polarisation dynamics can be excluded adiabatically from the analysis by assuming it to be inertialess [16]. In the thin film approximation (when the thickness of a layer is smaller than the wavelength [2]), this leads to a system of differential equations for the evolution of charge carrier densities with a delay equal to the transit time for the field between the films.

An analysis of the obtained system shows that an increase in the α-factor, which leads to an enhancement of the amplitude – phase coupling between the field and population, is equivalent to an increase in the nonlinearity of each of the layers. The threshold of symmetry breaking is lowered in this case. Moreover, an increase in the α-factor reduces the threshold of emergence of self-pulsations and the threshold of the formation of transverse spatial structures. This means that the amplitude – phase coupling between the field and the population density enhances the instability of the system under study.

2. Model equations

Consider a system of two nonlinear thin films separated by a distance d in a linear medium with a complex refractive index $n' - \mathrm{i}n''$. The system is illuminated from both sides by spatially homogeneous monochromatic light fields (Fig. 1).

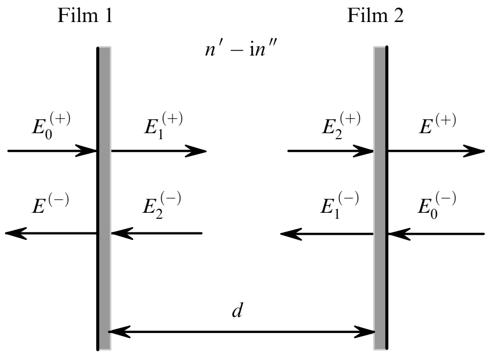

Figure 1. A system consisting of two films separated by a distance d: $E_0^{(+)}$, $E_0^{(-)}$ and $E_1^{(+)}$, $E_1^{(-)}$ are the amplitudes of fields incident on the first and second films and transmitted through them, respectively; $E_2^{(-)}$, $E_2^{(+)}$ are the amplitudes of fields reaching the opposite film, and $E^{(+)}$, $E^{(-)}$ are the amplitudes of fields at the system output.

To derive equations describing a system of two identical thin semiconductor films, we consider a system obtained for an active medium formed by two-layer atoms as the initial system. The Bloch equations for the interaction of a two-level medium (characterised by polarisations R_1, and R_2 of atoms in each film and the difference in level populations W_1, and W_2) with the light field can be written in the normalised form as [10]

$$\dot{r}_j = \gamma(-1 + \mathrm{i}\varDelta)r_j + \mathrm{i}\gamma e_j w_j, \tag{1}$$

$$\dot{w}_j = -(w_j + 1) + \mathrm{i}(e_j^* r_j - r_j^* e_j)/2, \tag{2}$$

where

$$\gamma = \frac{T_1}{T_2}; \quad \varDelta = T_2(\omega - \omega_0); \quad w = W;$$

$$r = R\sqrt{\gamma}; \quad e = E\frac{\mu(T_1T_2)^{1/2}}{\hbar}; \tag{3}$$

μ is the transition dipole moment; T_1, and T_2 are the longitudinal and transverse relaxation times; ω is the frequency of the incident radiation; and ω_0 is the resonance frequency of the two-level atoms. The current time t is normalised to the quantity T_1. The effective fields e_1, e_2 in the films are related with the fields incident on the films by the expressions [9, 17]

$$e_1 = e_0^{(+)} + e_2^{(-)} - \mathrm{i}\vartheta r_1, \tag{4}$$

$$e_2 = e_0^{(-)} + e_2^{(+)} - \mathrm{i}\vartheta r_2, \tag{5}$$

where

$$\vartheta = \frac{2\pi\tilde{n}NL\omega\mu^2 T_2}{\hbar c} \tag{6}$$

is a parameter characterising the nonlinearity of the system; N is the concentration of atoms in the medium; L is the thickness of the nonlinear film; c is the velocity of light; and $\tilde{n}$ is the refractive index of the two-level medium.

The result of light propagation between films in a linear medium is described by the equations

$$e_2^{(\pm)}(\boldsymbol{r}_\mathrm{t}, t) = \rho\exp(\mathrm{i}s)\exp\left(-\mathrm{i}\frac{d}{k}\varDelta_\mathrm{t}\right)e_1^{(\pm)}(\boldsymbol{r}_\mathrm{t}, t - \tau), \tag{7}$$

where $\varDelta_\mathrm{t} = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse part of the Laplace operator; $\rho = \exp(kn'd)$ are the losses; $s = knd$ is the phase shift; τ is the time of light propagation between the films; $\boldsymbol{r}_\mathrm{t} = (x, y)$ is the transverse component of the radius vector of the point (x, y, z); and k is the magnitude of the wave vector $\boldsymbol{k}$ of the light field. Equation (7) is a formal solution of the equation describing the propagation of light in a linear medium in the paraxial approximation [10, 11]. The amplitudes of the fields transmitted through the first and second films are $e_1^{(-)} = e_0^{(-)} - \mathrm{i}\vartheta r_2$ and $e_1^{(+)} = e_0^{(+)} - \mathrm{i}\vartheta r_1$, respectively.

For typical semiconductor materials, $T_1 \sim 10^{-9}$ s, and $T_2 \sim 10^{-12}$ s. Thus, $T_1 \gg T_2$, and hence $\gamma \gg 1$. Consequently, when processes occurring in a time shorter than the characteristic time T_2 are considered, polarisation of the medium plays only a passive role in the dynamics of the process and relaxes rapidly to its quasi-stationary state determined by the values of the population and the field. This means that the polarisation dynamics can be excluded adiabatically from the equations assuming that $\dot{r}_1 = 0$, and $\dot{r}_2 = 0$. In this case, r_j can be determined from the algebraic equations

$$r_j = \mathrm{i}\beta e_j w_j(1 + \mathrm{i}\varDelta), \tag{8}$$

$$r_j^* = -\mathrm{i}\beta e_j^* w_j(1 - \mathrm{i}\varDelta). \tag{9}$$

Here, $\beta = 1/(1 + \varDelta^2)$ is the Lorentzian contour describing the dependence of the susceptibility $\chi = \beta(1 + \mathrm{i}\varDelta)w$ of a two-level medium on frequency $\varDelta$. In semiconductor media, the shape of the absorption line differs from Lorentzian and is asymmetric relative to the sign of $\varDelta$. Moreover, the absorption coefficient and the refractive index are connected through the α-factor. Taking this into account, the susceptibility of a semiconductor medium near the centre of the absorption line can be written as $\chi = \beta(\varDelta)[1 - \mathrm{i}\alpha(\varDelta)]w$ with a corresponding line shape $\beta(\varDelta)$ [15, 18]. In this case, the α-factor ($\alpha = [\partial\mathrm{Re}(\chi)/\partial w][\partial\mathrm{Im}(\chi)/\partial w]^{-1}$) describes the relative variation in the phase shift of the field upon a variation in the density of free carriers caused by the fluctuations of the field amplitude, as well as the self-focusing effects in a nonlinear semiconductor system. For typical semiconductor systems, the numerical value of the α-factor near the centre of the gain line varies from 2 to 6 [15].

In this case, the system of equations (4), (5) is transformed as follows:

$$e_1(t) = e_0^{(+)}(t) + \rho\mathrm{e}^{\mathrm{i}s}[e_0^{(-)}(t - \tau) + \beta\vartheta e_2(t - \tau) \times w_2(t - \tau)(1 - \mathrm{i}\alpha)] + \beta\vartheta e_1(t)w_1(t)(1 - \mathrm{i}\alpha), \tag{10}$$

$$e_2(t) = e_0^{(-)}(t) + \rho\mathrm{e}^{\mathrm{i}s}[e_0^{(+)}(t - \tau) + \beta\vartheta e_1(t - \tau) \times w_1(t - \tau)(1 - \mathrm{i}\alpha)] + \beta\vartheta e_2(t)w_2(t)(1 - \mathrm{i}\alpha). \tag{11}$$

This system of difference equations, together with the field equations (7) as well as Eqns (2) written in the form

$$\dot{w}_1 = -(w_1 + 1) - \beta e_1^* e_1 w_1, \tag{12}$$

$$\dot{w}_2 = -(w_2 + 1) - \beta e_2^* e_2 w_2, \tag{13}$$

provides a model for describing systems of two semiconductor thin films.

3. Steady states and stability

Equations (8), (9) modified appropriately for the semiconductor medium, and the expression

$$w_j = -\frac{1}{1 + \beta |e_j|^2} \tag{14}$$

for the carrier density give expressions for the corresponding steady-state values in terms of the effective field e_1, and e_2 in each of the films. Equations (10) and (11) form a system of nonlinear algebraic equations in the complex field amplitudes e_1, and e_2, which may have several solutions for the same value of the incident field, i.e., may display multistability, observed for a quite large (10 and higher) value of the nonlinearity parameter, which can be achieved in semiconductor materials [1]. We are especially interested in the case when the fields incident on both sides are identical. To obtain a solution for equal fields in both films, the system of equations (10), (11) is reduced to a single equation displaying a standard optical bistability. However, system (10), (11) may also have solutions with unequal fields e_1, and e_2 and, hence, with unequal fields transmitted and reflected from the system. Such states are states with a broken symmetry and are formed due to bifurcation of the symmetry breaking. The symmetry is broken in the same range of system parameters as the bistability, and depends on the magnitude of the phase shift of the light field during its propagation between the films. Bistability is preferable for a phase shift equal to an integer of wavelengths. Otherwise symmetry breaking takes place.

Let us analyse the stability of the system of equations (7), (10)–(13) to the plane-wave perturbations characterised by a certain inclination to the system axis, determined by the transverse component $\boldsymbol{k}_t = (k_x, k_y)$ of the wave vector. For this purpose, we linearise the system in the vicinity of the homogeneous equilibrium state $(w_{01}, w_{02}, e_{01}, e_{02})$ by putting

$$w_j = w_{0j} + \delta w_j, \quad e_j = e_{0j} + \delta e_j. \tag{15}$$

Having written $\varsigma = (\delta w_1, \delta w_2, \delta e_1, \delta e_1^*, \delta e_2, \delta e_2^*)$ in the form $\varsigma = \varsigma_0 \exp(\lambda t + \boldsymbol{k}_t \boldsymbol{r}_t)$, we arrive at the matrix equation

$$\Lambda \varsigma = M \varsigma. \tag{16}$$

Here the matrix M has the form

$$\begin{pmatrix} \xi w_{01} & 0 & \xi e_{01} & \eta w_{02} e^{-\lambda\tau} & 0 & \eta e_{02} e^{-\lambda\tau} \\ 0 & \xi^* w_{01} & \xi^* e_{01}^* & 0 & \eta^* w_{02} e^{-\lambda\tau} & \eta^* e_{02}^* e^{-\lambda\tau} \\ -\beta e_{01}^* w_{01} & -\beta e_{01} w_{01} & -1 - \beta e_{01}^* e_{01} & 0 & 0 & 0 \\ \eta w_{01} e^{-\lambda\tau} & 0 & \eta e_{01} e^{-\lambda\tau} & \xi w_{02} & 0 & \xi e_{02} \\ 0 & \eta^* w_{01} e^{-\lambda\tau} & \eta^* e_{01}^* e^{-\lambda\tau} & 0 & \xi^* w_{02} & \xi^* e_{02}^* \\ 0 & 0 & 0 & -\beta e_{02}^* w_{02} & -\beta e_{02} w_{02} & -1 - \beta e_{02}^* e_{02} \end{pmatrix}, \tag{17}$$

where $\eta = \beta\rho e^{is}\vartheta(1 - i\alpha)$; $\xi = \beta\vartheta(1 - i\alpha)$. The matrix Λ (not shown here) is diagonal, with λ in the third and sixth rows and with unity in the remaining rows. The condition for the existence of a nontrivial solution of Eqn (16) leads to an equation for determining λ:

$$\det(M - \Lambda) = A(\lambda) + B(\lambda)e^{-2\lambda\tau} + C(\lambda)e^{-4\lambda\tau} = 0, \tag{18}$$

where $A(\lambda)$, $B(\lambda)$, $C(\lambda)$ are second-degree polynomials in λ; and the coefficients depend on $\boldsymbol{k}_t$ and coefficients of the system of equations (7), (10)–(13). The stability boundaries of the system are determined by the condition $\mathrm{Re}\lambda = 0$. Below we will study the effect of the α-factor on the stability boundaries of the system and its steady-state characteristics.

4. Effect of the *α*-factor on the steady-state characteristics and stability of the system

As before, we consider the case when the fields incident on both sides are identical: $e_0^{(+)} = e_0^{(-)} \equiv e_0$. In this case, one of the possible equilibrium states of system (7), (10)–(13) is a symmetric state with $e_1 = e_2 \equiv e$ determined from the equation

$$e = (1 + \rho e^{is})\left[e_0^{(+)} + \beta\vartheta(1 - i\alpha)\frac{e}{1 + \beta|e|^2}\right]. \tag{19}$$

This is a standard equation describing the bistability as a function of standard characteristics of the incident field. However, this equation is complicated due to the presence of an additional phase shift introduced by the α-factor. Figures 2a, b show the typical steady-state curve for $s = \pi$ in different coordinate systems. In this case, we obtain a symmetric branch of the solution as well as an asymmetric branch in which $e_1 \neq e_2$, and there is no bistability on the symmetric branch. Upon a phase change by π, the asymmetric branch vanishes and is replaced by a bistability (Figs 2c, d).

One can see from Eqn (19) that an increase in the α-factor is equivalent to an increase in the nonlinearity of the

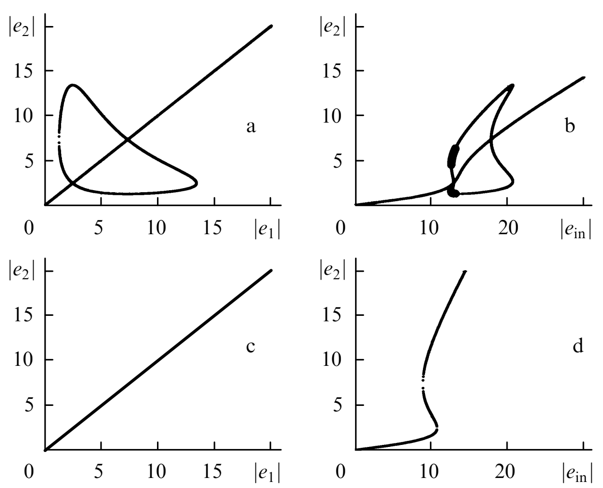

Figure 2. Steady states at the planes (e_1, e_2) (Figs a, c) and (e_{in}, e_2) (Figs b, d) for $\vartheta = 20.0$, $\alpha = 2.0$, $\rho = 0.5$ and $s = \pi$ (a, b) as well as $s = 0$ (c, d). The asymmetric branch region that is unstable to Andronov–Hopf perturbations is marked by bold segments in the curve in Fig. b; $e_{in} = (1 + \rho e^{is})e_0^{(+)}$.

system (determined by $\beta\vartheta$) by a factor $(1+\alpha^2)^{1/2}$. However, an additional phase shift is introduced in the system response in this case. For positive values of α, which are typical of a fairly large part of the absorption line [18], an increase in the α-factor causes a broadening of the range of incident field values in which bistability or symmetry breaking is observed. This is illustrated in Fig. 3 showing the dependence of the boundaries of symmetry breaking and bistability on the α-factor and the nonlinearity parameter ϑ for typical values of parameters for the semiconductor materials.

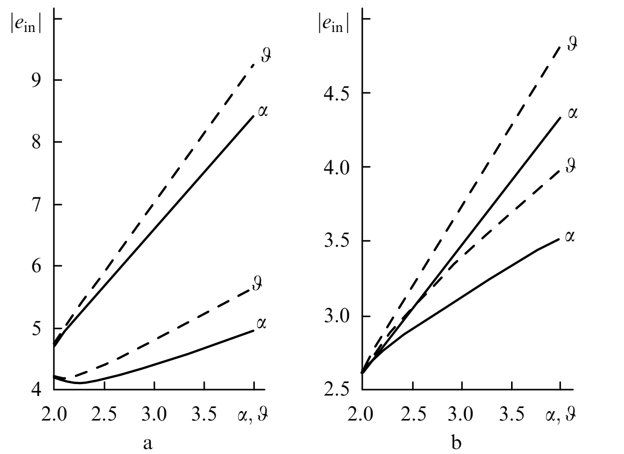

Figure 3. Dependences of the upper and lower boundaries of symmetry breaking (a) and bistability (b) on nonlinearity parameter ϑ and α-factor for $\rho = 0.5$ and $s = \pi$.

The α-factor may affect the Andronov–Hopf bifurcation boundaries in systems with a delayed feedback [13, 14]. For a system formed by two thin films, the Hopf instabilities are suppressed by static instabilities, as a rule. In a certain region, however, regimes periodic in time may emerge due to supercritical Andronov–Hopf bifurcation [10]. This occurs when the nonlinearity in the system is quite large and the bifurcation of the symmetry breaking itself is subcritical. In this case, a region of instability to time-periodic perturbations is formed on the asymmetric branch (see Fig. 2b). The corresponding range of field values increases with increasing the α-factor similarly to the instability region of symmetric and asymmetric steady-state regimes (Fig. 4a). In addition, the threshold of this instability in τ is lowered, as shown in Fig. 4b, where the dependence of the bifurcation of τ is presented for various values of α. The corresponding frequencies of pulsations emerging at the bifurcation point are shown in Fig. 4a. Numerical computations reveal that the Andronov–Hopf bifurcation is subcritical at the upper field boundary and supercritical at the lower field boundary. However, the limiting cycle is unstable for an incident field below the symmetry breaking bifurcation point, and the system relaxes to the symmetric equilibrium state.

For a system with a large aperture, spatial structures may emerge on the asymmetric branch for the same range of field values. The stability boundary as a function of $\boldsymbol{k}_{\text{t}}$ is determined from the condition $\lambda = 0$ leading to a much simpler equation than the equation describing the Andronov–Hopf field bifurcation boundaries. However, the maximum values of the field in the films for which the system is unstable are virtually identical. One can see this

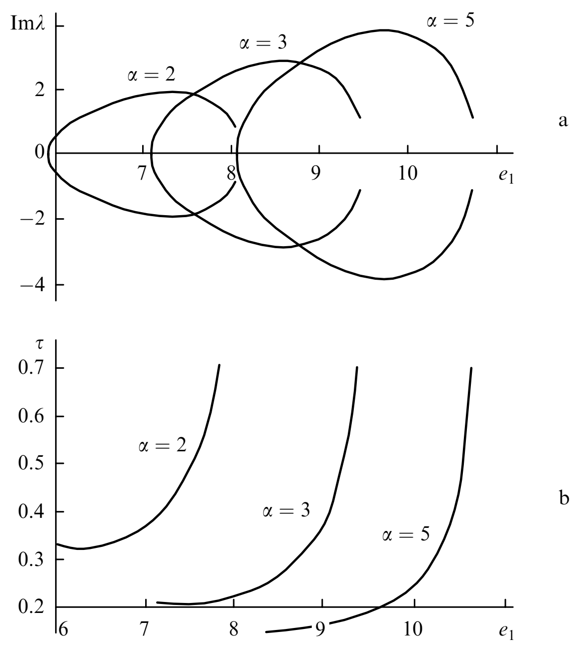

Figure 4. Dependence of the frequency of pulsations emerging at the Andropov–Hopf bifurcation threshold on field amplitude e_1 (a) and the boundaries of this bifurcation with respect to τ (b) for various values of the α-factor. Part of the steady-state characteristic for which the Andropov–Hopf bifurcation takes place is shown in Fig. 2b.

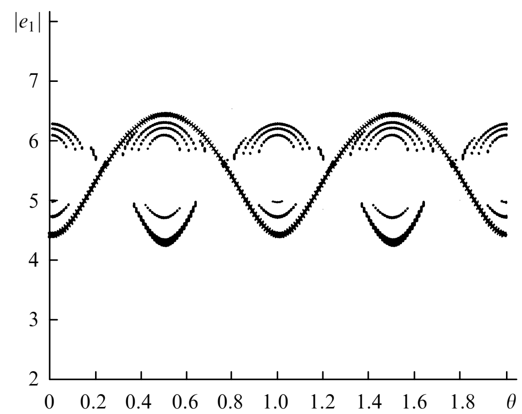

Figure 5. Boundaries of the static instability (marked by crosses) and Andropov–Hopf instability (dotted curve) for a system with a wide aperture as functions of the parameter $\theta = dk_{\text{t}}^2/k$.

from Fig. 5 showing the boundaries of stability with respect to both kinds of perturbations at the asymmetric region of the branch as functions of $\theta = dk_{\text{t}}^2/k$.

The phase disbalance caused by the presence of the α-factor does not affect the position of the instability peaks on the θ axis. This means that the size of spatial structures formed in the beam cross section due to instability to transverse perturbations remains unchanged.

5. Time dynamics

It was shown in Ref. [10] that pulsations whose period tends to 4τ with increasing τ appear in a system formed by two thin films. We shall consider here in detail the dynamics of the system for comparatively small delay times. In the region corresponding to bold segments in the curve in Fig. 2b (see also Figs 4 and 5), asymmetric pulsations may arise from the asymmetric equilibrium state. Figure 6 shows the time dependence of the system. Two cycles with large amplitudes emerge near the upper boundary (with respect to the field in the film) of the bifurcation point which is subcritical (Fig. 6a). The domains of attraction for these cycles in the $e_1(e_2)$ plane are separated by a bisectrix.

A weakening of the incident field corresponding to a departure of the working point from the threshold brings the cycles closer to each other (Fig. 6b). Apart from a large loop revolving around each of the asymmetric equilibrium states, a small loop is also formed near the point of bifurcation of the symmetry breaking; this loop is not connected with any one of the steady states existing for the given values of the incident field. Both cases (Figs. 6a and b) correspond to an incident field higher than the instability threshold of the symmetric branch. This condensation of trajectories may be a precursor to the intersection of the symmetric and asymmetric branches of the steady states, which appears upon a further weakening of the field. The shape of the pulsations also changes, and intervals of slow motion for the system in the vicinity of small cycles and of fast motion over the large cycle are formed.

A further decrease in the incident field leads to a stabilisation of the symmetric equilibrium state. The cycles lose their stability and are broken, as shown in Fig. 6c with a highly magnified phase portrait. Apart from the symmetric steady state, the figure also shows additional asymmetric unstable steady states appearing due to intersection of symmetric and asymmetric branches (see Fig. 2b). The system trajectory tends to the region lying between one of these asymmetric states and the symmetric solution. The size of the small loop decreases until it degenerates and the system 'falls' into a stable symmetric steady state. Figures 6d–f show the field variation in one of the films corresponding to the above regimes.

Thus, pulsations lose stability when the symmetric state is stable. Upon an increase in the delay time τ and the α-factor, the dynamics of the system remains similar to that considered above, with the only difference that the point of destabilisation of the limiting cycle is displaced towards smaller values of e_{in} relative to the point of symmetry breaking.

6. Conclusions

We have studied a system consisting of two thin bistable semiconductor films. The equations for such a system, written with an adiabatically eliminated polarisation of the

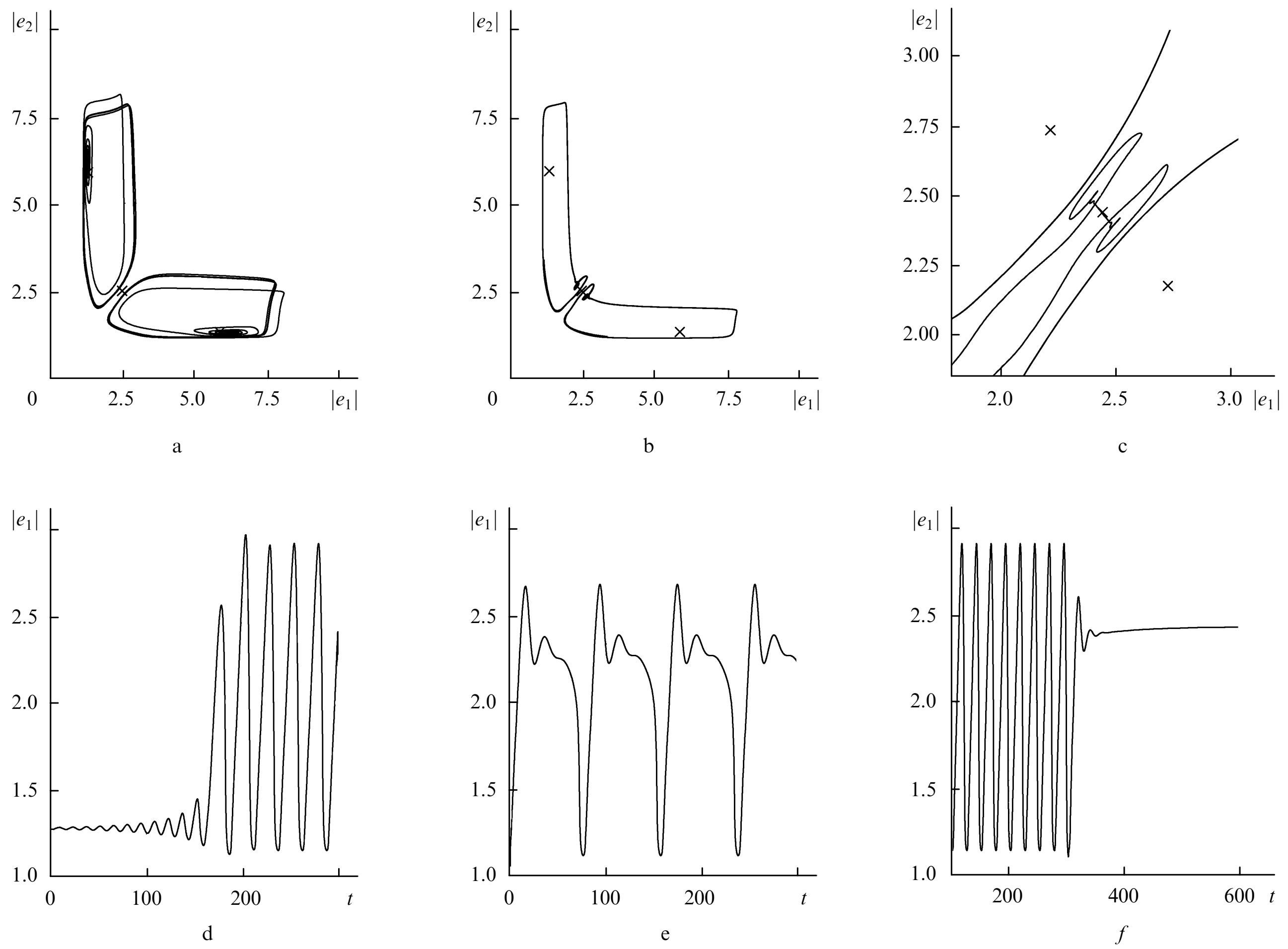

Figure 6. Time evolution of fields in the films for $e_{\text{in}} = 12.8$ (a, d), 12.92 (b, e), 12.94 (c, f) and $\tau = 3$. Curves in Figs a–c are the phase portraits of the system in the e_1e_2 plane (crosses indicate the steady states, two of which are asymmetric and one is symmetric), while Figs d, e show the time dynamics of the field in the first film.

nonlinear medium, take into account the effect of the α-factor describing the dynamic phase relation between the field and the charge carrier density, and form a system of differential equations with a delay.

Owing to a high sensitivity of such a system to phase relations for the field in the feedback loop, the inclusion of the α-factor affects the stability of the system. In contrast to the laser systems in which this effect is manifested mainly in dynamic regimes like self-pulsation, the presence of amplitude–phase relation in a system consisting of two films affects the appearance and disappearance of steady states. In particular, an increase in the α-factor is qualitatively equivalent to a simple increase in the system nonlinearity and leads to a broadening of the range of existence of asymmetric solutions and bistability.

Moreover, the system under consideration contains regions in which dynamic regimes (for a system with a small aperture) and spatial structures (for a system with a large aperture) may appear. These regions lie on the asymmetric branch of the steady-state curve. The effect of the α-factor leads to an increase in the size of the instability region, as well as to a decrease in the lower threshold of the Hopf instability in τ. On the other hand, the regions of instability to static perturbations with $\boldsymbol{k}_t \neq 0$, which lead to the formation of spatial structures, simply undergo a size variation without any displacement. In other words, the characteristic size of spatial structures formed in such systems remains the same.

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