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Polynomial model of the frequency characteristic of a slowly rotating vibrating laser gyro with differently amplified counterpropagating waves

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Abstract. **Expressions are obtained for calculating the four first coefficients of a polynomial model of the frequency characteristic of a slowly rotating laser gyro on a vibration stage. The expressions are valid when the gyro operates at the gain line centre and currents in its arms are balanced, however, the amplifications of counterpropagating waves are slightly different due to slightly different Q factors of the resonator for these waves. Analysis of the expressions suggests that the difference in the amplification of counterpropagating waves of the laser gyro leads to the zero shift and produces the component of the output signal that does not commute with the angular velocity.**

Keywords: *laser gyro, vibration stage, frequency characteristic.*

1. Introduction

Among the main types of laser gyros (LGs) widely used in practice, a gyro based on a ring gas He – Ne laser (^{20}Ne : ^{22}Ne = 1 : 1) with a flat N-mirror ($N \geqslant 3$) resonator emitting radiation linearly polarised in the sagittal plane can be distinguished. The laser emitting, as a rule, at 0.6328 μm is pumped by a dc discharge using a symmetric circuit consisting of one cathode and two anodes [1, 2].

LGs of this type mounted on a vibrating stage can be used, for example, as sensitive elements in a stageless inertial navigation system (SINS) for large airplanes in civil aviation performing maneuvering at small angular velocities [3].

To project the SINS and simulate its operation, it is necessary to have a mathematical model of the output signal of a vibrating LG. One of the components of this model is an analytic expression for the output characteristic of the device, which is called the dynamic frequency characteristic [4] or simply frequency characteristic.

The frequency characteristic of a vibrating LG is determined by the expression

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{k_f}{2\pi}\,\omega_{\mathrm{beat}}, \qquad (1)$$

where k_f is the frequency multiplication coefficient ($k_f = 1$, 2, 4, ...), which is realised by means of the optoelectronic system for data retrieving and processing; $\mathrm{d}N/\mathrm{d}t$ is the repetition rate of information pulses N from the LG output; and ω_{beat} is the circular beat frequency of counterpropagating waves averaged over the oscillation period. The frequency characteristic (1) is determined if the expression for ω_{beat} is known as a function of the angular rotation velocity Ω of the LG, of its internal parameters, and parameters of torsional vibrations of a monoblock.

The LG includes necessarily automated systems to stabilise the resonator perimeter and discharge currents. The first of them provides lasing in the LG at the emission line centre, while the second one ensures stable and identical currents in the discharge arms. The combined operation of these systems provides the specified lasing regime of the LG and eliminates the effect of undesirable factors such as the frequency detuning and current unbalance on the LG accuracy.

However, other, irremovable factors yet remain, which can give rise to additional errors in the LG operation upon multiplicative interaction. These factors are backscattering and absorption of radiation in optical elements of the resonator and the difference in the gains for counterpropagating waves due to different Q factors of the resonator for these waves. The consideration of these two factors and the quantitative analysis of their influence on the output signal of LGs is the object of this paper.

For a slowly rotating LG, under the necessary condition that the automated system of vibrational frequency separation contained in the LG produces with the help of a special algorithm the noise of the vibration amplitude of the monoblock [2, 3, 5], thereby suppressing the dynamic synchronisation of counterpropagating waves [4, 6 – 8], it is expedient to represent the dependence $\omega_{\mathrm{beat}} = \omega_{\mathrm{beat}}(\Omega)$ in the form of the polynomial

$$\omega_{\mathrm{beat}} = K_0 + (1 + K_1)M\Omega + \sum_{j=2}^{J} K_j M^j \Omega^j. \qquad (2)$$

Here, K_0, K_1, and K_j ($j = 2, \ldots, J$) are the coefficients of the polynomial model of the frequency characteristic of a vibrating LG, which are caused by coupling between counterpropagating waves through backscattering from the resonator mirrors and by losses inhomogeneously distributed along the axial contour; and M is a scale factor. In a perfect LG, where counterpropagating waves are uncoupled, the coefficients K_0, K_1, and K_j in (2) are zero and $\omega_{\mathrm{beat}} = M\Omega$.

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Expressions for calculating odd coefficients K_1, K_3, K_5, ... of polynomial (2) can be obtained from the results presented in section 3.6 of paper [2] devoted to the analysis of the nonlinearity of the frequency characteristic of a vibrating LG with a resonator with identical Q factors. The specific feature of these expressions is that they contain the coupling coefficients for the waves not only in the second power but also in higher powers (see section 5).

The author has failed to find expressions for even coefficients K_0, K_2, K_4, ... of polynomial (2) in the literature available.

The aim of this paper is, by considering the case $J = 3$, to obtain in the second order of smallness in the coupling coefficients the expressions for calculating the four first coefficients K_0, K_1, K_2, and K_3 of polynomial model (2) of the frequency characteristic (1) of a vibrating LG under the condition that the LG systems for stabilisation of the resonator perimeter and discharge currents operate perfectly, but the gains for counterpropagating waves are slightly different due to different Q factors of the resonator for them. The reasons for the latter are not specified.

2. Basic relations

According to Ref. [9], the expression for the beat frequency ω_{beat} of a uniformly rotating LG of the type under study has the form

$$\omega_{\text{beat}} = \left[1 - \frac{r_{\text{p}}^2}{2\omega^2} + \frac{r_{\text{m}}^2}{2(\alpha_{\text{m}}^2 + \omega^2)}\right]\omega + D(r_2^2 - r_1^2)$$
$$\times\left\{-\frac{1}{\omega^2} + \frac{1}{\alpha_{\text{m}}^2 + \omega^2}\left[1 + \frac{\alpha_{\text{m}}(\alpha_{\text{p}} + \alpha_{\text{m}})}{2(\alpha_{\text{p}}^2 + \omega^2)}\right]\right\}\omega$$
$$+ D\alpha_{\text{m}} r_1 r_2 \sin\varepsilon_{12}\,\frac{\alpha_{\text{p}}\alpha_{\text{m}} - \omega^2}{(\alpha_{\text{p}}^2 + \omega^2)(\alpha_{\text{m}}^2 + \omega^2)} \quad (\omega = M\Omega). \quad (3)$$

The first and second terms in the right-hand side of (3) are known from papers [10, 11] [expressions (10) and (23), respectively].

Expression (3) was obtained by solving, in the weak coupling approximation for counterpropagating waves, the system of differential equations describing the dynamics of the LG, which, using relations (6.45)–(6.47) from [12] or (5.55)–(5.57) from [2] (by generalising the latter to the case of different gains for counterpropagating waves), can be written in the known form [expressions (7)–(9) in Ref. [13]]

$$\dot{I}_1 = (\alpha_1 - \beta I_1 - \theta I_2)I_1 - 2r_2(I_1 I_2)^{1/2}\cos(\psi + \varepsilon_2),$$
$$\dot{I}_2 = (\alpha_2 - \beta I_2 - \theta I_1)I_2 - 2r_1(I_1 I_2)^{1/2}\cos(\psi - \varepsilon_1), \quad (4)$$
$$\dot{\psi} = \omega + r_2(I_2/I_1)^{1/2}\sin(\psi + \varepsilon_2) + r_1(I_1/I_2)^{1/2}\sin(\psi - \varepsilon_1),$$

where

$$\omega = M_{\text{g}}\Omega + \sigma_2 - \sigma_1 = M\Omega; \quad M_{\text{g}} = \frac{8\pi A_0}{\lambda L}; \quad (5)$$

$I_{1,2}$, ψ, and $\dot{\psi}$ are the dimensionless intensities, the instantaneous phase difference, and the instantaneous circular beat frequency of counterpropagating waves, respectively; $\alpha_{1,2}$, β, and θ are the Lamb coefficients characterising the excess of the linear gain over losses for each of the counterpropagating waves, their self-saturation, and mutual saturation, respectively; $r_{1,2}$ and $\varepsilon_{1,2}$ are the moduli and arguments of the complex integral coefficients of coupling between counterpropagating waves through backscattering; ω is the splitting of circular frequencies of counterpropagating waves caused by the LG rotation in the inertial space at the angular velocity Ω and calculated by neglecting coupling between the waves; M_{g} is the geometrical scale factor of the LG; A_0 is the area encompassed by the axial contour; L is the perimeter of the axial contour; λ is the lasing wavelength; $\sigma_{1,2}$ are the Lamb coefficients determining a small correction to the geometrical scale factor; and M is the scale factor of the LG taking the effect of the active medium into account.

The coefficients $\alpha_{1,2}$ in equations for $I_{1,2}$ are determined by the expressions

$$\alpha_{1,2} = \alpha \mp \delta, \quad (6)$$

from which it follows that $\alpha = (\alpha_2 + \alpha_1)/2$ and $\delta = (\alpha_2 - \alpha_1)/2$.

The parameters α_{p} and α_{m} in (3) are calculated from the expressions

$$\alpha_{\text{p}} = \alpha, \quad \alpha_{\text{m}} = \frac{\alpha_{\text{p}}(1 - h)}{1 + h}, \quad (7)$$

where $h = \theta/\beta$, and represent the inverse relaxation times of the sum and difference of the intensities of counterpropagating waves, respectively, i.e.,

$$\alpha_{\text{p}} = \frac{1}{T_{\alpha_{\text{p}}}}, \quad \alpha_{\text{m}} = \frac{1}{T_{\alpha_{\text{m}}}}, \quad (8)$$

where $T_{\alpha_{\text{p}}}$ and $T_{\alpha_{\text{m}}}$ are the relaxation times.

The small dimensionless parameter D in (3) characterises the difference in the gains for counterpropagating waves. It is defined by the relation

$$D = \frac{\delta}{\alpha_{\text{m}}} \quad (|D| \ll 1), \quad (9)$$

and when this difference is caused by the different Q factors of the resonator for counterpropagating waves (characterised by the quantity $\Delta Q/Q$), this parameter is calculated from the expression

$$D = \frac{\Delta Q}{Q}\,\frac{1 + h}{1 - h}. \quad (10)$$

Expression (3) also contains parameters r_{p} and r_{m} representing combinations of the coupling coefficients for counterpropagating waves. They can be calculated from expressions

$$r_{\text{p}} = (r_1^2 + r_2^2 + 2r_1 r_2\cos\varepsilon_{12})^{1/2},$$
$$r_{\text{m}} = (r_1^2 + r_2^2 - 2r_1 r_2\cos\varepsilon_{12})^{1/2}, \quad (11)$$

where $\varepsilon_{12} = \varepsilon_1 + \varepsilon_2$.

3. Formulation of the problem

Let us assume that the law of torsional vibrations of the LG monoblock has the form

$$\vartheta(t) = A \sin \nu t, \tag{12}$$

where A and ν are the amplitude and circular frequency, respectively, averaged over the vibration period.

According to (12), the angular velocity $\Omega_{\text{rel}}(t)$ of vibrations of the LG monoblock is described by the expression

$$\Omega_{\text{rel}}(t) = W \cos \nu t, \tag{13}$$

where $W = \nu A$ is the amplitude of the relative angular velocity of vibrations of the LG monoblock.

To solve the formulated problem, it is necessary to obtain from (3), taking (13) into account, the expressions for calculating the four first coefficients K_0, K_1, K_2, and K_3 of polynomial model (2) for Ω small compared to the amplitude W.

4. Brief description of the calculation method and the result obtained

By using identical transformations, expression (3) for ω_{beat} can be reduced to the form

$$\omega_{\text{beat}} = \left[1 - \frac{2r_1 r_2 \cos \varepsilon_{12}}{\alpha_{\text{m}}^2 + \omega^2} - \frac{\alpha_{\text{m}}^2 R_{\text{p}}^2}{2\omega^2 (\alpha_{\text{m}}^2 + \omega^2)} + \frac{D\alpha_{\text{m}}(\alpha_{\text{p}} + \alpha_{\text{m}})(r_2^2 - r_1^2)}{2(\alpha_{\text{p}}^2 + \omega^2)(\alpha_{\text{m}}^2 + \omega^2)}\right]\omega + D\alpha_{\text{m}} r_1 r_2 \sin \varepsilon_{12} \frac{\alpha_{\text{p}}\alpha_{\text{m}} - \omega^2}{(\alpha_{\text{p}}^2 + \omega^2)(\alpha_{\text{m}}^2 + \omega^2)}, \tag{14}$$

where $R_{\text{p}}^2 = r_{\text{p}}^2 + 2D(r_2^2 - r_1^2)$; $\omega = M\Omega$.

The second, third, and fourth terms in square brackets in (14) characterise corrections to the scale factor of the uniformly rotating LG caused by coupling between counter-propagating waves. These terms are grouped in the diminishing order according to their significance. For large Ω, the second term dominates because it contains Ω^2 in the denominator. The third and fourth terms contain Ω^4 in the denominator and, therefore, their contributions are much smaller. An LG on a vibration stage typically rotates at large angular velocities. Taking this into account, we simplify expression (14) by excluding small quantities from it to obtain

$$\omega_{\text{beat}} = \left(1 - \frac{2r_1 r_2 \cos \varepsilon_{12}}{\alpha_{\text{m}}^2 + \omega^2}\right)\omega + D\alpha_{\text{m}} r_1 r_2 \sin \varepsilon_{12} \frac{\alpha_{\text{p}}\alpha_{\text{m}} - \omega^2}{(\alpha_{\text{p}}^2 + \omega^2)(\alpha_{\text{m}}^2 + \omega^2)}. \tag{15}$$

Note here that this expression for $D = 0$ and a large frequency splitting $\omega = M\Omega$ gives the well-known [expression (16) in Ref. [14]] asymptotic representation

$$\omega_{\text{beat}} = \omega - \frac{2r_1 r_2 \cos \varepsilon_{12}}{\omega}, \tag{16}$$

which was verified experimentally for a broad range of values of r_1 and r_2 and, which is essential, for ε_{12}.

Following the method proposed in Ref. [2], we will solve the formulated problem by using a quasi-static approach. This approach is approximate and uses the expression for ω_{beat} that is valid for the uniform rotation of the LG, which is, however, then averaged over the vibration period $\tau = 2\pi/\nu$ of the LG monoblock. This approach is intuitively based on the condition of smallness of relaxation times $T_{\alpha_{\text{p}}}$ and $T_{\alpha_{\text{m}}}$ compared to the vibration period τ. When this condition is satisfied, the laser system will have time to follow an external perturbation. Note that the possibility of using the quasi-static approach for a slowly rotating LG on a vibration stage was also pointed out in Ref. [4] [integral relation (57)].

Let us make a substitution in (15)

$$\omega = M\Omega + w \cos \nu t, \tag{17}$$

where $w = MW = M\nu A$ is the amplitude value of the frequency splitting for counterpropagating waves in the LG caused by the angular vibration of the monoblock with respect to the LG housing.

Consider now the case of small angular velocities and expand (15) as a power series in Ω restricting ourselves to the four first terms of the series, which we will average over the period τ. As a result, we obtain

$$\omega_{\text{beat}} = K_0 + (1 + K_1)M\Omega + K_2 M^2 \Omega^2 + K_3 M^3 \Omega^3, \tag{18}$$

where

$$K_0 = \frac{D\alpha_{\text{m}} r_1 r_2 \sin \varepsilon_{12}}{\alpha_{\text{p}} - \alpha_{\text{m}}}\left[\frac{1}{(\alpha_{\text{m}}^2 + w^2)^{1/2}} - \frac{1}{(\alpha_{\text{p}}^2 + w^2)^{1/2}}\right]; \tag{19}$$

$$K_1 = -\frac{2\alpha_{\text{m}} r_1 r_2 \cos \varepsilon_{12}}{(\alpha_{\text{m}}^2 + w^2)^{3/2}}; \tag{20}$$

$$K_2 = \frac{w^6 D\alpha_{\text{m}} r_1 r_2 \sin \varepsilon_{12}}{2(\alpha_{\text{p}} - \alpha_{\text{m}})(\alpha_{\text{p}}^2 + w^2)^2 (\alpha_{\text{m}}^2 + w^2)^2} \times \left[\frac{N_{\text{m}}}{(\alpha_{\text{m}}^2 + w^2)^{1/2}} - \frac{N_{\text{p}}}{(\alpha_{\text{p}}^2 + w^2)^{1/2}}\right]; \tag{21}$$

$$K_3 = -\frac{(3w^2 - 2\alpha_{\text{m}}^2)\alpha_{\text{m}} r_1 r_2 \cos \varepsilon_{12}}{(\alpha_{\text{m}}^2 + w^2)^{7/2}}. \tag{22}$$

The parameters N_{m} and N_{p} in the right-hand side of (21) can be calculated from expressions

$$N_{\text{m}} = 1 + 2(\alpha_{\text{p}}^2 - \alpha_{\text{m}}^2)w^{-2} + \alpha_{\text{p}}^2(\alpha_{\text{p}}^2 - 4\alpha_{\text{m}}^2)w^{-4} - 2\alpha_{\text{p}}^4\alpha_{\text{m}}^2 w^{-6}, \tag{23}$$

$$N_{\text{p}} = 1 + 2(\alpha_{\text{m}}^2 - \alpha_{\text{p}}^2)w^{-2} + \alpha_{\text{m}}^2(\alpha_{\text{m}}^2 - 4\alpha_{\text{p}}^2)w^{-4} - 2\alpha_{\text{m}}^4\alpha_{\text{p}}^2 w^{-6}. \tag{24}$$

The coefficient K_0 in expression (18) characterises the shift of the LG zero, K_1 determines the correction to the scale factor, and K_2 gives the component of the beat frequency, which depends quadratically on Ω and does not commute with the angular velocity. The coefficient K_3 characterises the component of ω_{beat} proportional to Ω^3, which commutes with the angular velocity.

The even coefficients K_0 and K_2 of polynomial (18) are caused by the multiplicative interaction of the factors of different gains for counterpropagating waves and their coupling through backscattering. The odd coefficients K_1 and K_3 are caused only by the latter factor.

Therefore, expressions (19)–(24) for calculating the four first coefficients K_0, K_1, K_2, and K_3 of polynomial model (2) of the frequency characteristic (1) of the vibrating LG are the result of the solution of the problem formulated above.

5. Comparative analysis of the obtained results and known data

The results obtained above can be compared with the known data presented explicitly in Ref. [2] only for the case of a slowly rotating ($\Omega \ll W$) gyro with the same ($D = 0$) gain of counterpropagating waves.

Expression (6.4) for the coefficient $S_{\rm nl}(\Omega)$ of the relative nonlinearity of the frequency characteristic of the LG on a vibration stage presented in Ref. [2] is valid for all values of Ω (except the point $\Omega = 0$) and was obtained by averaging the expression

$$\omega_{\rm beat} = \left[1 - \frac{r_{\rm p}^2}{2(r_{\rm p}^2 + \omega^2)} + \frac{r_{\rm m}^2}{2(\alpha_{\rm m}^2 + \omega^2)}\right]\omega \quad (\omega = M\Omega) \quad (25)$$

over the vibration period τ.

By using our notation, the expression for $S_{\rm nl}(\Omega)$ can be written in the form

$$S_{\rm nl}(\Omega) = S_{\rm nl}^{(-)}(\Omega) + S_{\rm nl}^{(+)}(\Omega), \quad (26)$$

where

$$S_{\rm nl}^{(-)}(\Omega) = -\frac{\sqrt{2}}{4} r_{\rm p}^2 \frac{\left[U_{1(-)} + \left(U_{1(-)}^2 + U_{2(-)}^2\right)^{1/2}\right]^{1/2}}{M(\Omega^2)^{1/2}\left(U_{1(-)}^2 + U_{2(-)}^2\right)^{1/2}}; \quad (27)$$

$$S_{\rm nl}^{(+)}(\Omega) = \frac{\sqrt{2}}{4} r_{\rm m}^2 \frac{\left[U_{1(+)} + \left(U_{1(+)}^2 + U_{2(+)}^2\right)^{1/2}\right]^{1/2}}{M(\Omega^2)^{1/2}\left(U_{1(+)}^2 + U_{2(+)}^2\right)^{1/2}}; \quad (28)$$

$$U_{1(-)} = M^2\Omega^2 w^2 - r_{\rm p}^2; \quad U_{2(-)} = 2r_{\rm p}M\Omega; \quad (29)$$

$$U_{1(+)} = M^2\Omega^2 w^2 - \alpha_{\rm m}^2; \quad U_{2(+)} = 2\alpha_{\rm m}M\Omega. \quad (30)$$

By approximating (26) by the expression

$$S_{\rm nl}(\Omega) = S_1 + S_3 M^2\Omega^2 \quad (31)$$

and retaining the terms of no higher than the third order of smallness over the coupling coefficients, we obtain the expressions for S_1 and S_3

$$S_1 = -\frac{r_{\rm p}^3}{2(r_{\rm p}^2 + w^2)^{3/2}} + \frac{\alpha_{\rm m} r_{\rm m}^2}{2(\alpha_{\rm m}^2 + w^2)^{3/2}}, \quad (32)$$

$$S_3 = -\frac{3w^2 r_{\rm p}^3}{4(r_{\rm p}^2 + w^2)^{7/2}} + \frac{(3w^2 - 2\alpha_{\rm m}^2)\alpha_{\rm m} r_{\rm m}^2}{4(\alpha_{\rm m}^2 + w^2)^{7/2}}. \quad (33)$$

Let us compare in pairs expressions (20) and (32) for K_1 and S_1 and also expressions (22) and (33) for K_3 and S_3. Analysis of these relations shows that they are equivalent only if $r_1 = r_2 = r$, $\varepsilon_{12} = \pi$, when $\cos\varepsilon_{12} = -1$, $r_{\rm p} = 0$, and $r_{\rm m} = 2r$. Under these conditions,

$$K_1 = S_1 = \frac{2\alpha_{\rm m} r^2}{(\alpha_{\rm m}^2 + w^2)^{3/2}},$$
$$K_3 = S_3 = \frac{(3w^2 - 2\alpha_{\rm m}^2)\alpha_{\rm m} r^2}{(\alpha_{\rm m}^2 + w^2)^{7/2}}. \quad (34)$$

The discrepancy between the results of calculations obtained from the expressions analysed increases when ε_{12} decreases from π to zero.

Consider now the result obtained in paper [4]. Assuming that $r_{\rm p} = 0$ in (25), we obtain

$$\omega_{\rm beat} = \left[1 + \frac{r_{\rm m}^2}{2(\alpha_{\rm m}^2 + \omega^2)}\right]\omega \quad (\omega = M\Omega). \quad (35)$$

It is this expression for $\omega_{\rm beat}$ that was used in Ref. [4] [integral relation (57)] for determining $S_{\rm nl}(\Omega)$. By applying the above method to (35), we find

$$S_1 = \frac{\alpha_{\rm m} r_{\rm m}^2}{2(\alpha_{\rm m}^2 + w^2)^{3/2}}, \quad S_3 = \frac{(3w^2 - 2\alpha_{\rm m}^2)\alpha_{\rm m} r_{\rm m}^2}{4(\alpha_{\rm m}^2 + w^2)^{7/2}}. \quad (36)$$

It follows from these relations that, under the condition $3w^2 > 2\alpha_{\rm m}^2$, the quantities S_1 and S_3 are positive for any values of the parameter ε_{12}. In a particular case $r_1 = r_2 = r$, $\varepsilon_{12} = \pi$, expressions (36) take the form (34).

6. Numerical example

Let us estimate numerically the coefficients K_0, K_1, K_2, and K_3 of the polynomial model of the frequency characteristic of the vibrating LG and analyse quantitatively their influence on the output signal.

Consider the LG that has been studied theoretically and experimentally in Ref. [15]. The resonator of the gyro has the form of an equilateral triangle with the nominal perimeter $L = 210$ mm. We will assume below, however, that L is equal to 215.5 mm. In this case, the calculated arc scale division of the LG pulse for $k_f = 1$ will be equal to 3.147″, in accordance with Ref. [15]. In addition, we will neglect dispersion coefficients $\sigma_{1,2}$ in (5), thereby assuming that $M = M_{\rm g}$, which introduces only a small error in calculations. Then, for $L = 215.5$ mm, the scale factor is $M = 411793$.

Let us first calculate the parameters $\alpha_{\rm p}$ and $\alpha_{\rm m}$. According to Ref. [15], $\alpha_{\rm p} = (c/L)\gamma(N_{\rm rel} - 1)$, where γ is the average losses per transit for counterpropagating waves and $N_{\rm rel}$ is the relative excess of the pump over the threshold. Let $N_{\rm rel} = 1.45$ and $\gamma = 1.8 \times 10^{-3}$. Then, $\alpha_{\rm p} = 2\pi \times 179465$ s^{-1} (which gives the angular velocity $\Omega_{\alpha_{\rm p}} = \alpha_{\rm p}/M = 156.9$ deg s^{-1}). For the LG under study, the estimated ratio $h = \theta/\beta$ is $1.564/2.228 = 0.702$, which gives $(1-h)\times(1+h)^{-1} = 0.175$. Therefore, $\alpha_{\rm m} = 2\pi \times 31425$ s^{-1} ($\Omega_{\alpha_{\rm m}} = 27.5$ deg s^{-1}). These values of $\alpha_{\rm p}$ and $\alpha_{\rm m}$ yield the relaxation times $T_{\alpha_{\rm p}} = 8.9 \times 10^{-7}$ s and $T_{\alpha_{\rm m}} = 5.1 \times 10^{-6}$ s.

Let us calculate the parameter D. For example, for $\Delta Q/Q = 10^{-2}$, we have $D = 0.057$.

Let us now specify the values of coupling parameters for counterpropagating waves. According to Ref. [15], $(L/c)r_1 = (L/c)r_2 = 3 \times 10^{-6}$, which gives $r_1 = r_2 = 2\pi \times 665$ s^{-1}

($\Omega_{r_1} = \Omega_{r_2} = 0.58$ deg s^{-1}). We will take the value of the parameter ε_{12} so as to obtain the half-width $\Omega_s = r_p/M$ of the static synchronisation region of the LG equal to 0.05 deg s^{-1}. This condition is satisfied for $\varepsilon_{12} = 175°$. Then, $\cos\varepsilon_{12} = -0.996$, $\sin\varepsilon_{12} = 0.087$, and $r_p = 2\pi\times$ 58 s^{-1}.

Let us specify finally the parameters of torsional vibrations of the LG monoblock assuming approximately that $A = 3'$ and $\nu = 2\pi \times 500$ s^{-1}. In this case, the amplitude W of the relative angular velocity of vibrations is 157.1 deg s^{-1} and the period τ is 2×10^{-3} s.

Then, using (19)–(24), we obtain the following estimates: $K_0 = 4.53 \times 10^{-3}$ s^{-1}, $K_1 = 4.56 \times 10^{-6}$, $K_2 = 6.71 \times 10^{-15}$ s, and $K_3 = 4.95 \times 10^{-18}$ s^2.

These values of the coefficients allow us to estimate now all the components of the output signal of the LG. For this purpose, we rewrite expression (18) for $\omega_{\rm beat}$ in the form

$$\omega_{\rm beat} = \omega + \omega_0 + \omega_1 + \omega_2 + \omega_3. \quad (37)$$

Here, $\omega = M\Omega$ is the main component of the beat frequency of counterpropagating waves neglecting their coupling through backscattering (approximation of a perfect LG); $\omega_0 = K_0$, $\omega_1 = K_1 M\Omega$, $\omega_2 = K_2(M\Omega)^2$, and $\omega_3 = K_3(M\Omega)^3$ are the corrections to the beat frequency caused by coupling between the waves and their different gains. Let, for example, the LG rotate in the inertial space at the angular velocity $\Omega = 30$ deg s^{-1}. In this case, $\omega = 2\pi \times 34316$ s^{-1}, $\omega_0 = 2\pi \times 7.21 \times 10^{-4}$ s^{-1}, $\omega_1 = 2\pi \times 1.56 \times 10^{-1}$ s^{-1}, $\omega_2 = 2\pi \times 4.96 \times 10^{-5}$ s^{-1}, and $\omega_3 = 2\pi \times 7.89 \times 10^{-3}$ s^{-1}. Let us estimate the relative contribution of each of the corrections to $\omega_{\rm beat}$ with respect to the main component ω. We obtain from the expression $\rho_i = \omega_i/\omega$ that $\rho_0 = 2.10 \times 10^{-8}$, $\rho_1 = 4.56 \times 10^{-6}$, $\rho_2 = 1.45 \times 10^{-9}$, and $\rho_3 = 2.30 \times 10^{-7}$.

7. Conclusions

We have obtained expressions (19)–(24) for calculating the four first coefficients K_0, K_1, K_2, and K_3 of the polynomial model of the frequency characteristic of a slowly rotating LG on a vibration stage. The expressions are valid when the LG operates at the emission line centre and currents in the discharge arms are balanced, however, the gains for counterpropagating waves are slightly different due to different Q factors of the resonator for the waves.

Analysis of the expressions has shown that the non-identity factor of the gains of counterpropagating waves in the LG interacts multiplicatively with their coupling factor through backscattering, leads to the shift of zero, and produces the component of the output signal which does not commute with the angular velocity. The latter circumstance can introduce an additional error in the case of low-frequency local angular vibrations of the LG stage.

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