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Femtosecond optical clock

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***Abstract*. New advances in the field of synthesis of optical frequencies and the development of a new generation of optical clocks are considered. The use of mode-locked femtosecond lasers and fibre emission-spectrum stretchers allows the synthesis of any frequencies (from radio-frequencies to the UV region) and drastically simplifies the structure of an optical clock. The schemes of femtosecond optical clock are presented and the application of tapered optical fibres in them is described.**

Keywords: *femtosecond laser, frequency synthesis, optical fibres, nonlinear optics, supercontinuum.*

1. Introduction

One of the fundamental achievements of laser physics was the creation of optical quantum frequency standards whose frequency characteristics surpass those of microwave quantum standards. The important results should be noted which were obtained in the development of optical frequency standards (OFSs) on the basis of now conventional methods of saturated absorption [1] and two-photon Doppler-free absorption [2]. Recently, prominent results were achieved in the development of new optical references by using laser cooling and trapping of particles [3]. First of all, these are the references based on neutral atoms of calcium, silver, strontium, etc. confined in a magnetic trap [3, 4]. Frequency references based on ions in radio-frequency quadrupole traps and Penning traps are also used [5]. These include references on mercury, indium, ytterbium, strontium, barium, and calcium ions [3, 5]. In traps of both types, temperatures at the level of a few millikelvin were achieved, which provides the almost complete absence of Doppler broadening. Such references are used in new frequency standards with unique frequency characteristics.

The development of optical standards with the long-term frequency stability no worse than that of microwave standards and substantially surpassing them in the short-term stability allowed the solution of a number of problems, including the creation of frequency and time standards – optical clocks (OCs) in which a period of highly stable optical oscillations is used as a time scale. As a result, the time unit, a second, is determined directly from the number of optical oscillation cycles.

After the development of a radio-optical bridge [6] and the first OCs [7], drastic changes occurred. Due to the advances in the development of small frequency standards and new systems for the transfer of frequency characteristics of standards to the radio-frequency range with the help of femtosecond lasers, it became possible to create small OCs instead of cumbersome and very complex systems. A femtosecond optical clock (FOC) allows the synthesis and measurement of frequencies from the radio-frequency to UV range with the accuracy determined by the frequency characteristics of the optical standard.

In this paper, we report the experimental and theoretical studies devoted to the creation of a FOC and synthesisers, which were performed at the Institute of Laser Physics, Siberian Branch, RAS.

2. Principle of the OC construction

In the development of an OC, it is important, along with creation of the optical standard, to provide the transfer of its frequency characteristics to the radio-frequency and other ranges without the loss of accuracy. The simplest principal scheme of the OC is shown in Fig. 1. The OC consists of six basic units: the OFS, the system for transfer of frequency characteristics (STFC) to the radio-frequency

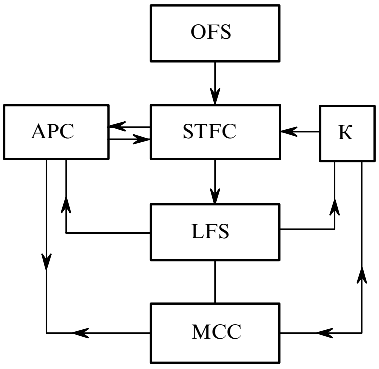

Figure 1. Block scheme of an OC. K: klystron; MCC: measuring and computing complex.

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range, the low-frequency synthesiser (LFS) for generating signals at standard and reference frequencies, the fast automatic phase offset lock (POL) unit for locking the frequencies of lasers used in the STFC and nonlinear elements. The transfer of frequency characteristics of the OFS in the first OC was achieved by dividing the frequency of the optical He – Ne/CH_4 standard down to the radio-frequency range through a system of lower-frequency lasers coupled with each other with the help of harmonics and phase-locked with the standard frequency [7]. A chain of the devices entering the STFC in an OC can be represented as:

He – Ne/CH_4 laser → He – Ne laser → CO_2 laser (first) → CO_2 laser (second) → CH_3OH laser → HCOOH laser → klystron → LFS.

The formula for synthesis of the OC frequency for one STFC unit has the form

$$\nu = n\nu_{\mathrm{las}} \pm mf_{\mathrm{UHF}} \pm f_{\mathrm{if}}, \tag{1}$$

where m and n are integers; ν_{las} is the laser frequency; f_{UHF} is the microwave generator frequency (usually a few tens of gigahertz); and f_{if} is the intermediate frequency.

Point diodes of the metal – insulator – metal type and Schottky diodes were used as fast nonlinear elements for laser frequency conversion and high-harmonic generation. The absolute measurements of the He – Ne/CH_4-laser frequency were performed with an accuracy of ~0.05 kHz with the help of radio-frequency bridges [6, 8] and the OC [7]. However, these devices did not find wide applications because of their complexity and unreliability.

The search for new ways of frequency synthesis resulted, as is known, in the use of femtosecond lasers, which became possible because they can generate a comb of many equidistant modes. The equidistant modes are generated due to the process of mode locking itself with an accuracy no worse than 10^{-16} [9]. Already the first study of stabilisation of the intermode frequency of a He – Ne/Ne-laser self-mode-locking to the hydrogen standard frequency showed that the frequency characteristics of the standard are transferred with a high accuracy to the entire mode spectrum [10]. Further studies were extended to solid-state lasers, in which the self-mode-locking was achieved for a few thousand modes with the same high accuracy [11, 12]. The intermode frequency of a Ti : sapphire laser was first used for frequency synthesis in [12]. We also performed studies devoted to the development of the FOC [13]. As in the first OC [7], a He – Ne/CH_4 laser was used in the FOC as an OFS, and the frequency characteristics of the FOC were transferred to the radio-frequency range by a self-mode-locked Ti : sapphire laser, which has the broadest gain band (above 100 THz). In this case, the frequency synthesis process is fundamentally different than that in papers [6 – 8]. The frequency characteristics of the standard are transferred by means of phase locking to a very broad frequency range containing many equidistant intermode intervals, and the expression for frequency synthesis takes the simplest form

$$\nu_{\mathrm{OFS}} = N\Delta\nu \pm f_{\mathrm{if}}, \tag{2}$$

where N is the number of modes entering the frequency interval ν_{OFS}, and $\Delta\nu$ is the intermode frequency.

The physical principle of the FOC construction based on the He – Ne/CH_4 laser is shown in Fig. 2. Let the frequency of the mode spectrum of the femtosecond laser is greater than that of the He – Ne laser. Then, according to (2), we can obtain the equality $\nu_1 - \nu_2 - \nu_{\mathrm{He-Ne}} = 0$ and, by using phase locking, to stabilise the entire mode comb by the optical standard frequency. As a result, the difference frequency $\nu_1 - \nu_2$, i.e., the frequency $N\Delta\nu$ and, hence, the intermode frequency $\Delta\nu$ acquire the frequency characteristics of the OFS, i.e., the frequency characteristics of the He – Ne/CH_4 standard are transferred from the optical to radio-frequency region. In the case under study (when the He – Ne laser is used), the width of the spectrum of the Ti : sapphire laser should be ~200 nm. Note that in the general case, instead of the He – Ne/CH_4 standard or the Ti : sapphire laser, another OFS or femtosecond laser can be used depending on the problem being solved.

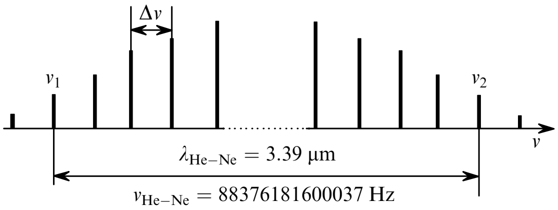

Figure 2. Physical principle of a FOC.

3. The FOC construction

Figure 3 shows the block scheme of the FOC based on the principle described above. The OFS (He – Ne/CH_4 laser) had the following parameters: the long-term stability $\sim 10^{-15}$ for 100 s, the sort-term stability 3×10^{-14} for 1 s, and the output power ~1 mW. The output power of the laser is insufficient to obtain phase locking of the difference frequency $\nu_1 - \nu_2 = N\Delta\nu$ of the mode comb of a Ti : sapphire laser to the OFS frequency. The output power was increased with the help of an additional 0.15-mW He – Ne laser whose frequency was phase-locked to the He – Ne/CH_4 standard frequency. The ~500-mW femtosecond Ti : sapphire laser used in experiments emitted 40-fs pulses of width ~20 nm (~10 THz) at a pulse repetition rate of 100 MHz (the intermode interval).

We paid a special attention to the problem of obtaining a sufficiently broad mode spectrum for synthesis of the optical standard frequency. This problem was solved by using a tapered [14 – 16] or a holey [17, 18] fibre allowing the broadening of the mode spectrum by an octave. Tapered optical fibres were fabricated at the University of Bath (Great Britain) and the Novosibirsk State University from a standard Corning SMF-28 telecommunication fibre with a core diameter of ~9 μm. The cut-off wavelength of the fibre was 1.25 μm. The emission spectrum broadened in the fibre should cover the range from 1.064 to 0.81 μm. It is undesirable to have the spectrum of width strongly exceeding this range because in this case the power of spectral components decreases. For this reason, we used a fibre with a waist diameter of 2.5 μm. The holey fibres were fabricated at the University of Bath. The power of each of the modes in the broadened spectrum was $\sim 10^{-7}$ W. It is virtually impossible to use such low powers for frequency summing or subtracting. We solved the problem of increasing power of modes at frequencies ν_1 and ν_2 by phase locking the

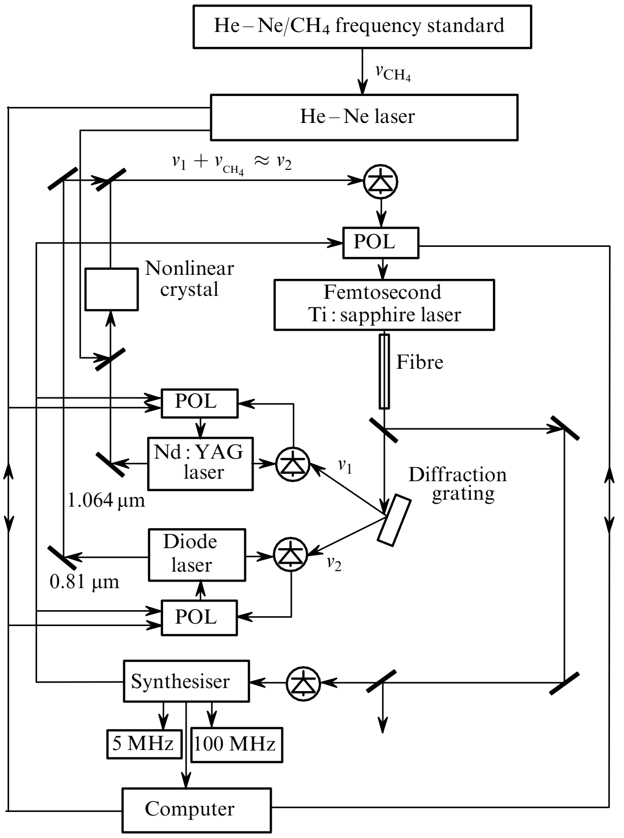

Figure 3. Scheme of the FOC based on the He – Ne/CH_4 frequency standard.

frequencies of a 1.064-µm, 100-mW Nd : YAG laser and a 0.81-µm, 10-mW diode laser to the frequencies of the corresponding modes of the Ti : sapphire laser. Single-frequency tunable Nd : YAG and diode lasers were manufactured at the Institute of Laser Physics, Siberian Branch, RAS. The tuning range of the diode-pumped Nd : YAG laser was 300 GHz, and that of the diode laser – up to 150 GHz (30 nm).

Frequencies of the Nd : YAG and He – Ne lasers were summed in a nonlinear $AgGaS_2$ crystal to obtain ~1 µW emission at the sum frequency. The beat signal between the sum frequency and the diode laser frequency was fed to the POL unit, and the output error signal from this unit was fed to the piezoelectric ceramics of the Ti : sapphire laser to control the intermode interval. As a result, the intermode interval of the Ti : sapphire laser was stabilised by the frequency of the He – Ne/CH_4 standard. Therefore, the characteristics of the He – Ne/CH_4 standard were transferred from the optical to radio-frequency region in one stage. Note that this scheme is autonomous, i.e., it does not require the external reference frequencies.

Because the intermode frequency lies in the radio-frequency range, it can be measured with an accuracy of $10^{-14} - 10^{-15}$ with respect to another radio-frequency standard (for example, the hydrogen maser). Note that in the scheme described above, only the intermode interval of the femtosecond laser is stabilised. When the clock is used for absolute frequency measurements, the absolute frequency of the components of the emission spectrum of the femtosecond laser should be stabilised by using POL by the OFS.

Therefore, OCs of a new generation – FOCs – were developed. Unlike similar systems of a previous generation with multistage frequency division, which were complex stationary setups, a new system can be made transportable.

The OC system was investigated stage by stage. It was very important to study the possibility of stabilisation of the mode comb as a whole by the intermode frequency. For this purpose, the 100-MHz intermode frequency was phase locked to the hydrogen standard frequency. Our experiments showed that the frequency characteristics of the hydrogen standard are transferred with high accuracy to the entire frequency comb and, therefore, the Ti : sapphire laser generates in this case highly stable femtosecond pulses [11].

At the next stage, we studied successfully the phase locking of the diode laser frequency to the frequency of modes with the aim of enhancing the power of the latter [19]. We also developed the measuring and computing complex to control the POL state and to measure absolute frequencies, including the optical standard frequency.

Great efforts were devoted to the development of physical foundations of the broadening of the mode spectrum in fibreoptic systems, and much attention was also paid to the study of the nonlinearity of fibres, determining the broadening of the spectrum, and of parameters of radiation transmitted through the fibre. The most important is the question whether the stability of the intermode frequency is preserved after the propagation of a highly stable femtosecond pulse through the fibreoptic system. Another question of interest is: What are the amplitude – frequency characteristics of the output radiation or, in other words, what are the relative intensities of the spectral components of the transformed radiation?

We have designed and manufactured a precision measuring unit for studying the effect of a tapered fibre on the stability of the intermode frequency during the propagation of a continuous train of femtosecond pulses from a Ti : sapphire laser through the fibre and performed these studies [20]. The stability of the intermode interval was investigated in the following way. We measured very carefully the stability of the ~100-MHz intermode frequency with respect to the hydrogen standard at the fibre input and them at its output. We found that during averaging for 1 s the fibre deteriorates the stability by half, while upon averaging for 1000 s the stability almost is not impaired. One can assume that the processes of radiation conversion to the long- and short-wavelength regions are different. To verify this assumption, we divided the output emission spectrum into two regions with the help of filters and then performed measurements. The results of the measurements for the averaging time of 100 s are presented in Table 1. One can see from the table that the noise increases during the frequency conversion, and this increase is asymmetric: the noise in the long-wavelength region

Table 1.

Measurement conditions	Number of measurements	Root-mean-square frequency deviation from the mean value$/10^{-5}$ Hz
In front of the fibre	98	5.738
Behind the fibre ($\lambda = 0.7 - 1.1$ µm)	83	8.458
Behind the fibre ($\lambda = 0.4 - 0.65$ µm)	92	6.940

becomes greater than that in the short-wavelength one. Therefore, we have shown that tapered optical fibres do not deteriorate strongly the stability of the intermode frequencies and can be used in FOCs and synthesisers with respect to this parameter.

We studied experimentally and theoretically the evolution of the spectrum of a train of femtosecond pulses in tapered fibres of length 90 mm with the waist diameters 2, 2.5, and 3 μm. The Ti:sapphire laser had parameters presented above. ~30 % of the incident radiation power was coupled to the fibre. According to our estimate, the pulse duration in front of the waist was 160 fs, i.e., the pulse was strongly chirped due to the propagation of radiation through optical elements and the first (not tapered) part of the fibre. The problem of formation of the spectrum of a train of ultrashort pulses transmitted through the fibre can be considered both in the pulse (temporal) and frequency representation. We described theoretically the broadening of the experimental spectrum by using the first approach. The results of this study are described in detail in our paper [20]. We have shown that the theoretical and experimental results are in qualitative agreement.

We studied the low-frequency (1 – 10 Hz) instability of the envelope of the broadened spectrum and showed that the angular fluctuations of the input radiation cause the instability of the spectrum envelope if the initial part of the fibre is multimode. This instability prevents the use of tapered fibres in FOCs and synthesisers, and because of this we used microstructure holey fibres. At present, work is underway toward the elimination of this instability because the manufacturing technology of tapered fibres is rather simple and their cost is lower than that of microstructure fibres.

4. FOCs based on other optical standards and lasers

The FOCs considered above use the IR optical standard (a 3.39-μm He – Ne/CH$_4$ laser). This system has substantial disadvantages, one of which is the complicated FOC stabilisation by the optical frequency. Only intermode frequencies are stabilised, while the absolute frequencies of the modes remain unstable. Another drawback of the FOC is a large size of the OFS, preventing the construction of a small FOC. The dimensions of a transportable He – Ne/CH$_4$ laser described in [21] are made minimal, however the laser remains quite large.

These problems were solved to a great extent in our FOCs based on the use of the Nd : YAG/I$_2$ standard. The block scheme of such a FOC is shown in Fig. 4. The operation principle of this FOC differs substantially from that described above (Fig. 2) and is determined by the properties of the Nd : YAG/I$_2$ laser. The essence of the operation principle is that the reference I$_2$ line lies in the visible region at 0.532 μm, i.e., it corresponds to the second harmonic of the Nd : YAG laser. We developed the Nd : YAG/I$_2$ standard [22] operating at the wavelengths 1.064 and 0.532 μm and having the frequency stability 10^{-14} during 200 s, the frequency reproducibility 3.3×10^{-13}, and the output power ~100 mW at 1.06 μm and 20 – 40 mW at 0.532 μm.

It follows from Fig. 4 that the frequency of the Ti : sapphire laser in this FOC is stabilised by phase locking of the corresponding frequencies of the spectrum broadened in the fibre to the optical frequencies ν_0 and $2\nu_0$ of the Nd : YAG/I$_2$ laser. This system contains no additional lasers for increasing the power at these frequencies because the radiation power at the frequencies ν_0 and $2\nu_0$ of the Nd : YAG/I$_2$ standard is sufficiently high. Because of this, radiation at the frequencies ν_1 and ν_2 of the Ti : sapphire laser and ν_0 and $2\nu_0$ of the Nd : YAG laser is directed on a detector, from which the mixed signals at intermediate frequencies are directed to the POL unit and then to the controlling piezoelectric elements of the Ti : sapphire laser. As a result, the absolute and intermediate frequencies are stabilised, and the FOC structure is considerably simplified.

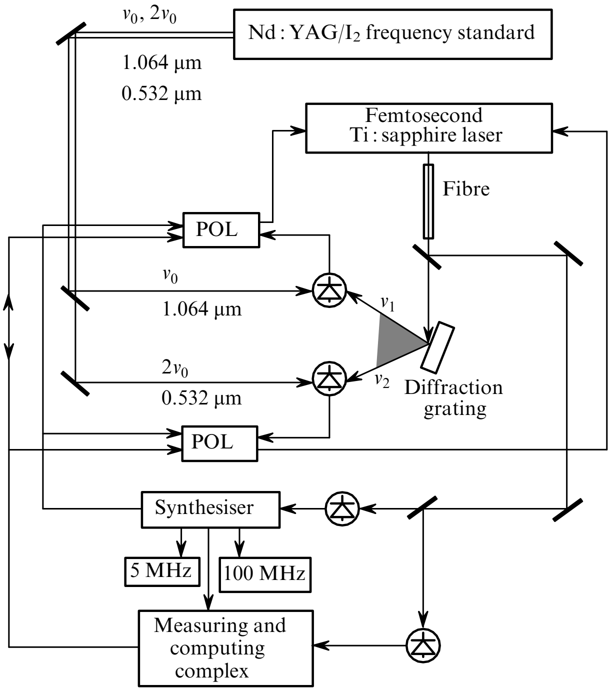

Figure 4. Scheme of the FOC based on the Nd:YAG/I$_2$ frequency standard.

At present, we are developing a small FOC based on the Yb : YAG/I$_2$ standard and a femtosecond diode-pumped Yb : KWG laser instead of the Ti : sapphire laser. The FOC structure remains similar to that presented in Fig. 4.

5. Conclusions

We have considered new advances in the field of synthesis of optical frequencies and the development of a new generation of OCs on this basis. The use of mode-locked femtosecond lasers and stretchers of the mode spectrum based on tapered or holey fibres allows the synthesis of any frequencies (from the radio-frequency to UV range), considerably simplifying the FOC design. As follows from studies [23], the accuracy of absolute frequency measurements with the help of FOCs can be as high as $\sim 10^{-19}$ and will be determined by the stability of the optical standard used in the FOC. An important property of the FOC is the high stability of optical and intermode radiation frequencies after the stretcher. It seems that the FOC will be used in the future for the development of unique frequency synthesisers for very precise frequency measurements in various spectral regions.

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