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Estimate of the angular dimensions of objects and reconstruction of their shapes from the parameters of the fourth-order radiation correlation function

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***Abstract*. It is shown that, when the angular resolution of a receiving optical system is insufficient, the angular dimensions of a located object can be estimated and its shape can be reconstructed by estimating the parameters of the fourth-order correlation function (CF) of scattered coherent radiation. The reliability of the estimates of CF counts obtained by the method of a discrete spatial convolution of the intensity-field counts, the possibility of estimating the CF profile counts by the method of one-dimensional convolution of intensity counts, and the applicability of the method for reconstructing the object shape are confirmed experimentally.**

Keywords: *intensity correlation, object reconstruction, optical location.*

1. Introduction

A number of problems of optical location require information on the transverse dimensions and profile of a located object when the angular resolution of a receiving optical system is insufficient for object imaging. It is known that the parameters of the correlation function (CF) of detected radiation are related to the spatial distribution of the coherent radiation intensity scattered by the object surface Ω. Therefore, by analysing the CF parameters, one can estimate the angular size of an axially symmetric object [1]. It was shown in [2, 3] that the fourth-order CF counts $\hat{\Gamma}^{(2,2)}(\delta\boldsymbol{r},\tau)$ can be obtained by the method of spatial convolution of the intensity field

$$\hat{\Gamma}_S^{(2,2)}(\delta\boldsymbol{r},\tau) = \frac{1}{S}\int_S I(\boldsymbol{r},t)I(\boldsymbol{r}+\delta\boldsymbol{r},t+\tau)\mathrm{d}\boldsymbol{r}, \qquad (1)$$

where S is the observation region chosen in the plane parallel to a plane object surface and located at a distance of R from the object; $(\boldsymbol{r},\delta\boldsymbol{r})\in S$; t is the current time; $\tau=\delta r/c$; and c is the speed of light in a medium. Compared to the classical intensity correlation method of Hanbery, Brown, and Twiss [4], this method is weakly sensitive to the field instability and provides reliable estimates of $\hat{\Gamma}^{(2,2)}(\delta\boldsymbol{r},\tau)$ using a small number of object location cycles. Although the applicability of expression (1) for the location of objects with arbitrary surface profiles has not been strictly mathematically proved so far, numerical simulations showed that the angular dimensions of an object can be determined from the coordinates of the first local minimum of $\hat{\Gamma}^{(2,2)}(\delta\boldsymbol{r},\tau)$ found from (1) and then the object profile can be reconstructed.

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2. Estimate of the parameters of the fourth-order CF of radiation scattered by a plane object of an arbitrary shape

Consider a plane object in a rectangular coordinate system (CS) (Ξ, H). Let us introduce the left Cartesian observation CS (X, Y, Z) whose axis Z specifies the direction of the observation vector $\boldsymbol{v}$ and is perpendicular to the ΞH plane located at a distance of R from the XY plane and $\boldsymbol{n}_X \| \boldsymbol{n}_\Xi$. It follows from the CF definition that [1]

$$\Gamma^{(2,2)}(\boldsymbol{r}_1,\boldsymbol{r}_2,0) = \dot{J}(\boldsymbol{r}_1,\boldsymbol{r}_2)\dot{J}^*(\boldsymbol{r}_1,\boldsymbol{r}_2),$$

where $\dot{J}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is the function of mutual intensity determined at point with coordinates $(x_1,y_1)=\boldsymbol{r}_1$ and $(x_2,y_2)=\boldsymbol{r}_2$ of the observation region S, which belongs to the XY plane. According to the theorem of van Cittert – Zernicke, we obtain

$$\dot{J}(x_1,y_1;x_2,y_2) = \frac{\zeta\exp(-\mathrm{i}\psi)}{(\lambda R)^2} \times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(\xi,\eta)\exp\left[\mathrm{i}\,\frac{2\pi}{\lambda R}(\Delta x\xi+\Delta y\eta)\right]\mathrm{d}\xi\mathrm{d}\eta, \qquad (2)$$

where $\zeta=\lambda^2/\pi$; λ is the radiation wavelength; $\psi=\pi(\lambda R)^{-1}\times[(x_2^2+y_2^2)-(x_1^2+y_1^2)]$ is the phase factor; $I(\xi,\eta)$ is the scattered radiation intensity in the object vicinity; $\Delta x = x_2 - x_1$; and $\Delta y = y_2 - y_1$. We will assume that the plane object is a set of $N_\xi \times N_\eta$ square scatterers touching by the adjacent sides of size a, with positions of their centres in the CS (Ξ, H) specified by the coordinates $\xi_0 + a(k+0.5)$, $\eta_0 + a(l+0.5)$, and $1<k<N_\xi$, $1<l<N_\eta$. The intensity I_{kl} of scattered coherent radiation in the vicinity of each point on the scattered surface is assumed constant and scattering is assumed diffuse. Then,

$$I(\xi,\eta) = \sum_{k=1}^{N_\xi}\sum_{l=1}^{N_\eta} I_{kl}\,\mathrm{rect}[\xi-\xi_0-a(k+0.5),\eta-\eta_0 -a(l+0.5),a], \qquad (3)$$

where

$$\mathrm{rect}(\xi,\eta,a)=\begin{cases}1, & \xi\in[-0.5a,0.5a], \quad \eta\in[-0.5a,0.5a],\\ 0, & \xi\notin[-0.5a,0.5a], \quad \eta\notin[-0.5a,0.5a].\end{cases}$$

Taking the sphericity of the light-wave front into account in (2), we obtain

$$\dot{J}(x_1,y_1;x_2,y_2)=\frac{\zeta}{(\lambda R)^2}\{[\cos(\psi)a_J(x_1,y_1;x_2,y_2)+\sin(\psi)b_J(x_1,y_1;x_2,y_2)]+\mathrm{i}[\cos(\psi)b_J(x_1,y_1;x_2,y_2)-\sin(\psi)a_J(x_1,y_1;x_2,y_2)]\},$$

where

$$a_J(x_1,y_1;x_2,y_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(\xi,\eta)\cos[2\mu(\Delta x\xi+\Delta y\eta)]\mathrm{d}\xi\mathrm{d}\eta;$$

$$b_J(x_1,y_1;x_2,y_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(\xi,\eta)\sin[2\mu(\Delta x\xi+\Delta y\eta)]\mathrm{d}\xi\mathrm{d}\eta;$$

$\mu=\pi/\lambda R$. By using (3) and integrating, we have

$$a_J(x_1,y_1;x_2,y_2)=a^2\,\frac{\sin(\mu a\Delta x)}{\mu a\Delta x}\,\frac{\sin(\mu a\Delta y)}{\mu a\Delta y}\times\sum_{k=1}^{N_\xi}\sum_{l=1}^{N_\eta}I_{kl}\cos\{\mu a[(2k+1)\Delta x+(2l+1)\Delta y]\}.$$

The function $b_J(x_1,y_1;x_2,y_2)$ is determined similarly. By setting $x_2=0$ and $y_2=0$, we obtain

$$\tilde{\Gamma}^{(2,2)}(x,y,0)=\frac{\zeta}{(\lambda R)^2}\left[a_J^2(x,y;0,0)+b_J^2(x,y;0,0)\right]. \quad (4)$$

By comparing in pairs the analytically calculated CFs $\Gamma^{(2,2)}(x,y,0)$ and CFs of radiation diffusively scattered by plane objects with elliptic contours found from (4), we found that deviations of the corresponding counts of $\tilde{\Gamma}^{(2,2)}(x,y,0)$ from true values $\Gamma^{(2,2)}(x,y,0)$ do not exceed 1 %. These results were obtained for $N_\xi=N_\eta=300$ and the maximal linear size of each object equal to 1/3 of aN_ξ. We can assume that, for the above relations between a, N_ξ, and N_η, expression (4) is valid for the calculation of the CF of radiation scattered by a plane object with an arbitrary surface shape. This assumption is confirmed by the coincidence between calculations of $\Gamma^{(2,2)}(x,y,0)$ and estimates of CF counts obtained by the method of spatial convolution [2].

Because the simulation of spatial realisations of the intensity-field counts by the quadrature method is complicated, the fourth-order CF counts were determined by the method of spatial convolution of experimentally recorded realisations of intensity counts $I(i\Delta\tilde{x},j\Delta\tilde{y})$:

$$\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x},n\Delta\tilde{y},0)=\frac{1}{N_x}\frac{1}{N_y}\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}I(i\Delta\tilde{x},j\Delta\tilde{y})\times I[(i+m)\Delta\tilde{x},(j+n)\Delta\tilde{y}], \quad (5)$$

where $(m\Delta\tilde{x},n\Delta\tilde{y})$ are the coordinates of CF counts; $m=0,1,\ldots,L_x-N_x$; $n=0,1,\ldots,L_y-N_y$; L_x and L_y are the numbers of photodetectors (PDs) located at the nodes of a rectangular grid; $\Delta\tilde{x}$ and $\Delta\tilde{y}$ are the corresponding distances between adjacent PDs in the CS observation directions $\boldsymbol{n}_X$ and $\boldsymbol{n}_Y$; and $N_x\times N_y$ is the number of counts of the two-dimensional correlated sampling.

Radiation formed by the physical model of a diffusively scattering object described in [2] was detected with a CCD array of a Mintron-OS65D video camera containing 795×596 pixels. We found that the dispersion of mathematical expectations of CF counts $\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x},n\Delta\tilde{y},0)$ obtained over 500 independent realisations of radiation fields was less than 1 % for $N_x=L_x/3$ and $N_y=L_y/3$ and weakly depended on the object shape. This fact confirms the correctness of the conditions chosen for the observation of the object model and radiation detection.

Our studies have shown that the estimates of the fourth-order CF counts obtained by numerical simulation and experimentally are in good agreement: the difference between the values of the corresponding CF counts does not exceed 5 % of their arithmetic mean (Fig. 1). This means that the fourth-order CF counts of radiation scattered by a plane asymmetric object can be estimated by the method of discrete spatial convolution of intensity-field counts.

The method of one-dimensional discrete convolution of radiation-field intensity counts allows one to estimate the values $\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x},n\Delta\tilde{y},0)$ of counts of the CF section by the plane Ψ containing the Z axis and making the angle φ with the XZ plane if PDs are located on the intersection line of planes Ψ and XY:

$$\hat{\Gamma}^{(2,2)}(k\Delta x,0)=\frac{1}{N}\sum_{i=1}^{N}I_iI_{i+k}, \quad (6)$$

where Δx is the distance between adjacent PDs; $k=0,1,\ldots,L-N$; L is the number of PDs; and N is the number of counts of the correlated sampling. The difference of the values of corresponding counts for arbitrarily taken sections $\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x},n\Delta\tilde{y},0)$ and $\hat{\Gamma}^{(2,2)}(k\Delta x,0)$ (Fig. 1i) does not exceed 1 % of their arithmetic mean for asymmetric objects (Fig. 2). Therefore, the detection of radiation with a PD linear array rotated in a plane perpendicular to the observation Z axis provides the reliable estimates of $\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x},n\Delta\tilde{y},0)$ and substantially simplifies the design of a location system.

3. Reconstruction of the object shape from estimates of its angular dimensions

The method for determining the angular size of an object from the parameters of the fourth-order CF of scattered radiation can be used for reconstructing of the profile L_l of the projection S_l of the surface Ω of a located object on the ΞH plane perpendicular to the observation vector $\boldsymbol{v}$ [5]. The reconstruction is based on the possibility of measuring angular dimensions $\hat{\gamma}(\varphi)$ of various sections of the profile L_l (Fig. 3). The image $\hat{S}_l$ of the projection S_l can be constructed by finding the intersections N_φ of bands of width $b\hat{\gamma}(\varphi_i)$, where $i=1,\ldots,N_\varphi$ (b is the scaling factor) with axial lines $y'=y'_{0i}+\tan\varphi_i(x'-x'_{0i})$ on the model $X'Y'$ plane. The intersection of two bands, for which the condition

$$\hat{\gamma}(\varphi_k)\geqslant\hat{\gamma}(\varphi_j)\geqslant\max[\hat{\gamma}(\varphi_1),\ldots,\hat{\gamma}(\varphi_{k-1}),\hat{\gamma}(\varphi_{k+1}),\ldots,\hat{\gamma}(\varphi_{j-1}),\hat{\gamma}(\varphi_{j+1}),\ldots,\hat{\gamma}(\varphi_{N_\varphi})],$$

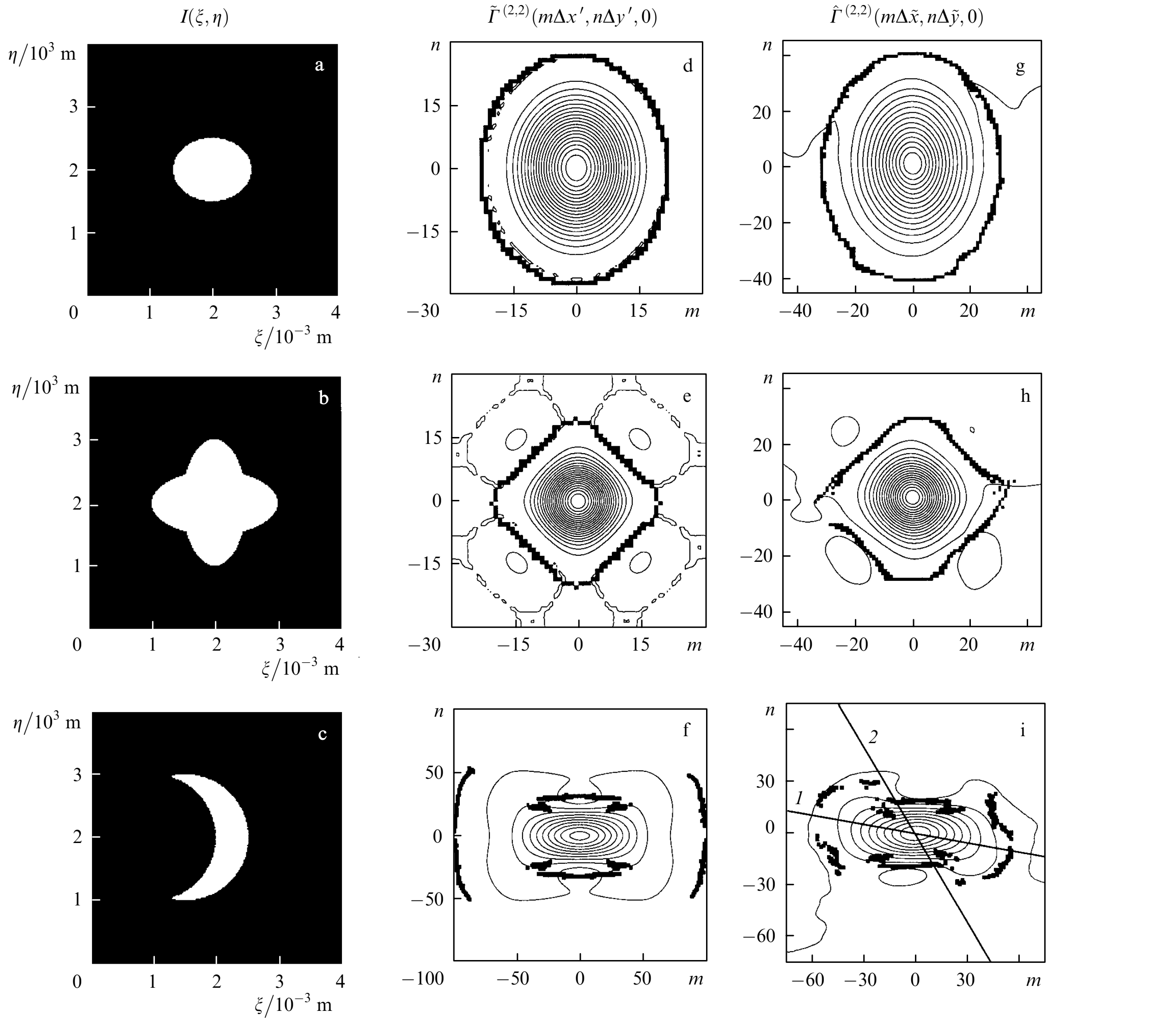

Figure 1. Contours of physical models of objects (a, b, c), results of simulations of $\tilde{\Gamma}^{(2,2)}(m\Delta x', n\Delta y', 0)$ (d, e, f), and estimates of $\hat{\Gamma}^{(2,2)}(m\Delta\tilde{x}, n\Delta\tilde{y}, 0)$ (g, h, i,) obtained from (5) using experimentally recorded realisations of intensity-field counts. The thick lines indicate positions of the first local minima of CFs.

is fulfilled for $|\varphi_k - \varphi_j] \to \pi/2$, determines the region $\tilde{S}_l$ of the possible solution. The parameters x'_{0i} and y'_{0i}, which are nonzero in the general case, can be found by the variation method [6] for the other $N_\varphi - 2$ bands by satisfying the condition that all the intersection points of the band boundaries belong to the region $\tilde{S}_l$. As a result, we find the image of the profile $\hat{L}_l$ representing a convex polyhedron

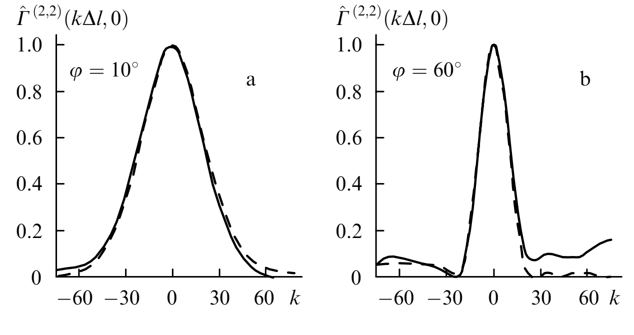

Figure 2. Correspondence between CF counts, obtained by the method of one-dimensional spatial convolution (solid curves), and the CF section counts obtained by the method of discrete spatial convolution (dashed curves) in cases when the intersecting plane makes the angle $\varphi = 10^\circ$ (a) and 60° (b) with the X' axis. The corresponding projections of CF sections are indicated by straight lines (*1*) and (*2*) in Fig. 1i.

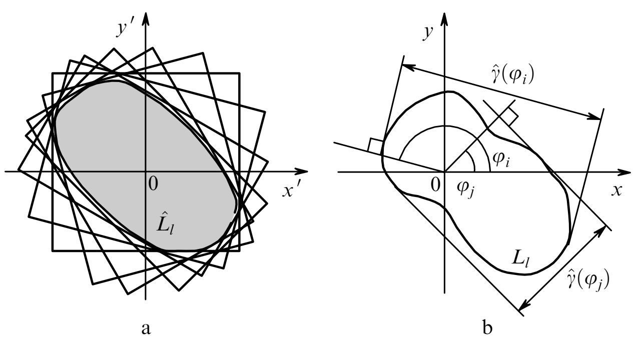

Figure 3. Reconstruction of the object contour (a) from the parameters of the fourth-order CF (b).

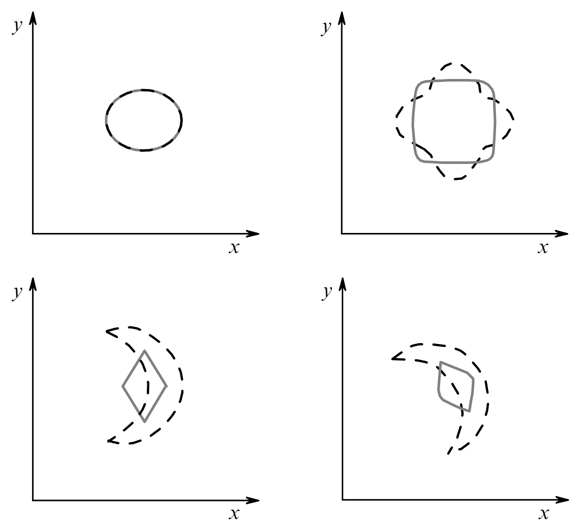

Figure 4. Reconstructions (solid curves) of contours of plane objects (dashed lines).

having principally at least one symmetry axis, whose angular dimensions coincide with accuracy to a constant with the angular dimensions of the located object.

We have found that the contour $\hat{L}_l$ reconstructed from the line of the first minimum of the CF $\Gamma^{(2,2)}(x, y, 0)$ approximates L_l with a small error when L_l is a circle or an ellipse. In other cases, errors appear in measuring the contour $\hat{L}_l$ and its orientation with respect to the XY CS (Fig. 4). This means that, when S_l does not have the axial symmetry, a comparison of the reconstructed contour $\hat{L}_l$ with a set $\{L_l'\}$ of contours calculated for a set $\{S_l\}$ of object models allows one to determine a subset $\{S_l\}_R$ of models (and the corresponding set $\{L_l'\}_R$ of contours) whose radiation CFs can be considered corresponding to the found function $\hat{\Gamma}^{(2,2)}(x, y, 0)$. If *a priori* information on the object-surface shape is absent, a change in the object orientation with respect to the observation CS can be estimated from a change in the orientation of $\hat{L}_l$ by performing successive location cycles.

4. Conclusions

By detecting with one or a few linear PD arrays the counts of coherent radiation intensity scattered by a located object with an elliptic contour, one can determine the coordinates of counts of the line of the first minimum of the fourth-order CF with good accuracy and reconstruct the contour of this object. Errors in the reconstruction of the contour of the located object with an arbitrary shape of the surface can be large; however, the information obtained allows one to identify the object given *a priori* information on the object-surface geometry, or to estimate the dynamics of a change in the object position with respect to a laser ranging system. The orientation of the located object and its shape can be determined more accurately by analysing the parameters of the sixth-order radiation CF containing information on the phase of scattered radiation at different spatial points.

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