

NONLINEAR OPTICAL PHENOMENA

PACS numbers: 42.65.Ky; 42.70.Mp; 06.60.Jn
DOI: 10.1070/QE2004v034n09ABEH002745

Effect of nonlinear crystal thickness on the parameters of the autocorrelator of femtosecond light pulses

A.V. Masalov, A.V. Chudnovsky

Abstract. **It is shown that the finite thickness of the second-harmonic crystal distorts the results of measurements in nonlinear autocorrelators intended for measuring the durations and fields of femtosecond light pulses mainly due to dispersive broadening (or compression) of the pulses being measured, as well as due to the group velocity mismatch between the fundamental and sum-frequency pulses. The refractive index dispersion of the crystal, scaled by half its thickness, distorts the pulse duration to a certain extent depending on its initial chirp and thus determines the width of the energy distribution recorded in the autocorrelator. As the crystal thickness increases, the group velocity mismatch leads to a transformation of the recorded distribution from the correlation function of intensity to the squared modulus of the field correlation function. In the case of Gaussian pulses, such a transformation does not affect significantly the recorded distribution. Errors of pulse duration measurements are estimated.**

Keywords: *femtosecond light pulses, chirp, ultrashort pulse duration measurement, autocorrelator, nonlinear crystal dispersion.*

1. Introduction

An autocorrelator using the sum-frequency generation in a nonlinear crystal, which was first proposed in Ref. [1], is the basic element in a number of devices for measuring the duration and phase modulation of femtosecond light pulses (see, for example, the method of femtosecond pulse duration measurement by the SHG-FROG autocorrelator with spectral resolution [2] and the method of the SPRINT interference autocorrelator with a spectral–space resolution of the signal [3]). The thickness of the crystal used in these devices determines their sensitivity and the accuracy of reproduction of the correlation function: as the crystal thickness increases, the signal intensity increases, but also the dispersive distortions of the recorded energy distribution become greater. Although, autocorrelators are widely used for measuring the duration of light pulses, only scant quantitative data are available on the distortions of the correlation function caused by the thickness of a nonlinear crystal. The dispersive distortions of the correlation functions were calculated in detail in Ref. [4], where it was concluded that the group velocity mismatch between the fundamental and the second-harmonic pulses plays a dominating role. In the period that has elapsed since the publication of [4], the progress made in the femtosecond light pulse generation technique has led to a decrease in their duration by an order of magnitude. This requires a more accurate calculation of the dispersion effects in an autocorrelator by including factors like the chirp of the initial pulses that were neglected in Ref. [4].

In this work, we performed such calculations and presented the relations describing the effect of the thickness of a nonlinear crystal on the profile of the spatial distribution recorded in an autocorrelator based on sum-frequency generation. The results of calculations showed that the dispersion broadening (compression) of the initial pulses is the most important reason for distortions. The obtained relations are used for quantitative estimates of the measuring errors.

Calculations are based on the following measuring scheme for the autocorrelation intensity function [1]. Two copies of the initial pulse are directed to the crystal which sums the spectral components ω_1 and ω_2 of each of the pulses at a small angle α (Fig. 1). The spatial distribution of the radiation energy at the sum-frequency ω, which is interpreted as the autocorrelation function of the pulse intensity

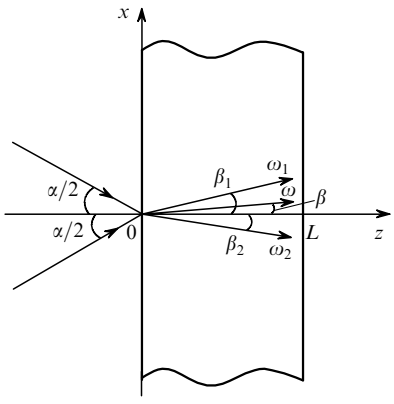

Figure 1. Scheme of noncollinear sum-frequency generation.

A.V. Masalov, A.V. Chudnovsky P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninskii prosp. 53, 119991 Moscow, Russia; e-mail: masalov@mail1.lebedev.ru, a_v_ch@mail.ru

Received 26 April 2004
Kvantovaya Elektronika **34** (9) 812–818 (2004)
Translated by Ram Wadhwa

$$\int I\left(t-\frac{x}{c}\sin\frac{\alpha}{2}\right)I\left(t+\frac{x}{c}\sin\frac{\alpha}{2}\right)\mathrm{d}t$$

is recorded at the output from the crystal. However, because of the broadening of pulses propagating through the crystal, the limited spectral phase-matching and some other effects, the recorded distribution differs from this autocorrelation function.

In this work, we calculated the field and energy distributions recorded in the autocorrelator taking into account four mechanisms of the crystal-thickness effect: (1) limited spectral width of phase-matching; (2) dispersive broadening (or compression) of pulses due to the dispersion of the nonlinear crystal; (3) angular dispersion of the crystal manifested during noncollinear propagation of the beams; and (4) dispersion of nonlinear susceptibility of the crystal during sum-frequency generation.

Note that only the first of these factors was considered in Ref. [4]. In paper [5], where this computation was generalised to the case of summation of pulses with different frequencies, only the group velocity mismatch was taken into account.

We integrated the equation describing the sum-frequency by taking into account the higher-order dispersive effects compared to the group-velocity mismatch. This allows us to use the obtained results for studying pulses of extremely small duration (4–5 fs). Because we obtained expressions not only for the energy distribution, but also for the sum-frequency field, the results can be employed for analysis of images recorded by the SHG-FROG and SPRINT methods.

2. Equation for the sum-frequency field

The recorded radiation consists of sum-frequency waves generated by each elementary layer $(z', z'+\mathrm{d}z')$ of the crystal. Emission at sum-frequency ω is due to polarisation of matter, proportional to nonlinear susceptibility $\chi(\omega_1, \omega_2; \omega=\omega_1+\omega_2)$ and spectral amplitudes of incident pulses,

$$\mathrm{d}^3P(\omega,x,z') = \chi(\omega_1,\omega_2;\omega=\omega_1+\omega_2) \times s(\omega_1)\exp[-\mathrm{i}\omega_1 t+\mathrm{i}k_1(x\sin\beta_1+z'\cos\beta_1)]s(\omega_2) \times \exp[-\mathrm{i}\omega_2 t+\mathrm{i}k_2(-x\sin\beta_2+z'\cos\beta_2)]\mathrm{d}z'\mathrm{d}\omega_1\mathrm{d}\omega_2, \quad (1)$$

where $s(\omega_1)\exp[-\mathrm{i}\omega_1 t+\mathrm{i}k_1(x\sin\beta_1+z'\cos\beta_1)]\mathrm{d}\omega_1$ and $s(\omega_2)\exp[-\mathrm{i}\omega_2 t+\mathrm{i}k_2(-x\sin\beta_2+z'\cos\beta_2)]\mathrm{d}\omega_2$ are spectral components of the waves being summed in a given layer of the crystal (Fig. 1 shows the coordinate axes and angles). Polarisation of a given layer generates in the surrounding space a plane monochromatic wave with a spectral amplitude

$$\mathrm{d}^3s_{\mathrm{SF}}(\omega,x,z,z') = \frac{2\pi\mathrm{i}\omega}{cn(\omega)}\chi(\omega_1,\omega_2;\omega)s(\omega_1)s(\omega_2) \times \exp[\mathrm{i}k(x\sin\beta+z\cos\beta)+\mathrm{i}\varphi(z')]\mathrm{d}z'\mathrm{d}\omega_1\mathrm{d}\omega_2, \quad (2)$$

where the factor $2\pi\mathrm{i}\omega/cn(\omega)$ appears due to the transition from polarisation to the field. The direction of wave (2) and its phase are determined by the condition of matching of spatial structures of wave (2) and polarisation (1) at $z=z'$, leading to the equations

$$\varphi(z') = (k_1\cos\beta_1+k_2\cos\beta_2-k\cos\beta)z' \equiv \Delta k z', \quad (3)$$

$$k\sin\beta = k_1\sin\beta_1-k_2\sin\beta_2 \equiv \frac{\omega_1-\omega_2}{c}\sin\frac{\alpha}{2}. \quad (4)$$

These relations assume the type ooe phase-matching during sum-frequency generation, the refractive index for the extraordinary wave stands for $n(\omega)$, while the ordinary refractive index stands for $n(\omega_1)$ and $n(\omega_2)$. The walk-off of the sum-frequency beam, which is typical of extraordinary waves, is not taken into account here because it occurs along the y axis and does not affect significantly the distribution along the x axis. Expressions for other types of phase matching can be written in an identical manner without any further complication. After integration of expression (2) over the crystal thickness L, we arrive at the following expression for the sum-frequency field at the output face of the crystal ($z=L$) under the assumption that the energy of incident pulses are not depleted:

$$\mathrm{d}^2s_{\mathrm{SF}}(\omega,x,L) \propto \frac{\sin\Delta kL/2}{\Delta k}\exp\left(\frac{\mathrm{i}\Delta kL}{2}\right) \times\frac{\omega}{n(\omega)}\chi(\omega_1,\omega_2;\omega)s(\omega_1)s(\omega_2)\exp(\mathrm{i}k_1x\sin\beta_1 -\mathrm{i}k_2x\sin\beta_2+\mathrm{i}kL\cos\beta)\mathrm{d}\omega_1\mathrm{d}\omega_2. \quad (5)$$

This expression coincides with the analogous expression from [6] where the effect of crystal thickness was not analysed. The right-hand side of (5) can be factorised with a good accuracy into frequency multipliers, which simplifies subsequent integration. Let us factorise the frequency dependence of the nonlinear polarisability using the Miller rule:

$$\chi(\omega_1,\omega_2;\omega) \propto [n^2(\omega_1)-1][n^2(\omega_2)-1][n^2(\omega)-1]. \quad (6)$$

Then

$$\mathrm{d}^2s_{\mathrm{SF}}(\omega,x,L) \propto \frac{\omega}{n(\omega)}[n^2(\omega)-1]\frac{\sin\Delta kL/2}{\Delta k} \times\exp\left(\mathrm{i}k\frac{L}{2}\cos\beta\right)[n^2(\omega_1)-1]s(\omega_1) \times\exp\left(\mathrm{i}k_1x\sin\beta_1+\mathrm{i}k_1\frac{L}{2}\cos\beta_1\right)[n^2(\omega_2)-1]s(\omega_2) \times\exp\left(-\mathrm{i}k_2x\sin\beta_2+\mathrm{i}k_2\frac{L}{2}\cos\beta_2\right)\mathrm{d}\omega_1\mathrm{d}\omega_2. \quad (7)$$

It will be shown below that the frequencies of incident waves appear in the expression for the mismatch Δk mainly in the form of a sum which, after replacement of $\omega_1+\omega_2$ by ω, leads to the independence of mismatch from frequencies. Because the angle β characterising the direction of the spectral component of the sum-frequency wave is small, we can put $\cos\beta=1$. Under these conditions, the spectral

component of the sum-frequency field (7) can be represented in the form of three factors:

the spectrum

$$\tilde{s}(\omega_1) = [n^2(\omega_1) - 1]s(\omega_1)\exp\left(\mathrm{i}k_1\frac{L}{2}\cos\beta_1\right) \tag{8}$$

of the first initial pulse with factors responsible for dispersion of half (!) the crystal thickness (exponential factor) and for dispersion of nonlinearity of the medium (factor $[n^2(\omega_1) - 1]$);

the spectrum

$$\tilde{s}(\omega_2) = [n^2(\omega_2) - 1]s(\omega_2)\exp\left(\mathrm{i}k_2\frac{L}{2}\cos\beta_2\right) \tag{9}$$

of the second initial pulse with analogous factors, and the function $\mathrm{sinc}(\Delta kL/2) \equiv (2/\Delta kL)\sin(\Delta kL/2)$ with factors describing the effect of dispersion of half thickness of the crystal on the sum-frequency wave [the factor $\exp(\mathrm{i}kL/2)$], and the nonlinear dispersion [factor $\omega[n^2(\omega) - 1]/n(\omega)$]:

$$g(\omega) = \frac{\omega}{n(\omega)}[n^2(\omega) - 1]\mathrm{sinc}\left(\Delta k\frac{L}{2}\right)\exp\left(\mathrm{i}k\frac{L}{2}\right). \tag{10}$$

The separation of the dependences on three frequency functions allows us to write expressions for the resultant field. To calculate the total spectral component of the resultant field, expression (5) should be integrated with respect to the spectral components $\mathrm{d}\omega_1$ and $\mathrm{d}\omega_2$ under the condition $\omega_1 + \omega_2 = \omega$:

$$\begin{aligned} s_{\mathrm{SF}}(\omega,\tau) &\propto g(\omega)\exp(-\mathrm{i}\omega\tau/2) \\ &\times\int\tilde{s}(\omega_1)\tilde{s}(\omega-\omega_1)\exp(\mathrm{i}\omega_1\tau)\mathrm{d}\omega_1 \\ &\equiv g(\omega)\int\tilde{s}\left(\frac{\omega}{2}-\Omega\right)\tilde{s}\left(\frac{\omega}{2}+\Omega\right)\exp(-\mathrm{i}\Omega\tau)\mathrm{d}\Omega, \end{aligned} \tag{11}$$

where we use the exact relation $k_{1,2}\sin\beta_{1,2} \equiv \omega_{1,2} \times \sin(\alpha/2)/c$ taking into account the fact that all the spectral components of the initial pulses at the crystal input have the same incidence angle $\alpha/2$ and that the transverse coordinate x is expressed in the form of delay time $\tau = (2x/c)\sin(\alpha/2)$. By using the Fourier transform, we obtain from (11) the time representation of the field at the crystal output:

$$\begin{aligned} E_{\mathrm{SF}}(t,\tau) &\propto \exp(-2\mathrm{i}\omega_0 t) \\ &\times\int h(t-t')\tilde{A}\left(t'-\frac{\tau}{2}\right)\tilde{A}\left(t'+\frac{\tau}{2}\right)\mathrm{d}t', \end{aligned} \tag{12}$$

where the frequency corresponding to the exact phase-matching is used as the central frequency ω_0. Here we have introduced the amplitude envelope of the pulse field $\tilde{A}(t)$ relative to the central frequency ω_0, which is distorted by the crystal dispersion at *half* its thickness and by the nonlinearity dispersion:

$$\begin{aligned} \tilde{A}(t) &= \int\tilde{s}(\omega)\exp[-\mathrm{i}(\omega-\omega_0)t]\mathrm{d}\omega \\ &= \int[n^2(\omega_0+\Omega) - 1]s(\omega_0+\Omega) \\ &\times\exp\left(-\mathrm{i}\Omega t + \mathrm{i}k_{1,2}\frac{L}{2}\cos\beta_{1,2}\right)\mathrm{d}\Omega, \end{aligned} \tag{13}$$

where the frequency dependences of the wave vector and the angles of propagation are assumed to be the same for both input pulses. Moreover, the notation $h(t)$ is introduced in (12) for the amplitude envelope of a certain fictitious pulse characterising the mismatch and the dispersion properties of the medium in the sum-frequency region:

$$\begin{aligned} h(t) &= \int g(\omega)\exp[-\mathrm{i}(\omega-2\omega_0)t]\mathrm{d}\omega \\ &= \int\frac{\omega}{n(\omega)}[n^2(\omega) - 1]\mathrm{sinc}\left(\Delta k\frac{L}{2}\right) \\ &\times\exp\left(\mathrm{i}k\frac{L}{2}\right)\exp[-\mathrm{i}(\omega-2\omega_0)t]\mathrm{d}\omega. \end{aligned} \tag{14}$$

The physical meaning of the pulse $h(t)$ will be described below. Its shape is determined by the Fourier transform of the function $g(\omega)$ in which the mismatch $\mathrm{sinc}(\Delta k \times L/2)$ plays the main role. Hence the pulse $h(t)$ is nearly rectangular and has a duration $\sim L\mathrm{d}(\Delta k)/\mathrm{d}\omega$.

Relations (11) and (12) for the sum-frequency field in the spectral and time representations are of fundamental importance in the subsequent analysis of the effects related to the thickness of the nonlinear crystal and arise during the application of one autocorrelation scheme or the other. Thus, the energy of the spectrally resolved sum-frequency field, i.e., the square of the modulus of the spectral amplitude (11), is recorded in the SHG-FROG scheme. The same field participates in the formation of the interference pattern in the SPRINT scheme. The transformation of relations (11) and (12) to the spatial frequency region (i.e., with respect to the variable τ) through the Fourier transform leads to the expressions for the sum-frequency field in the far-field zone (in spectral or time representations), which generalise the angular distributions obtained in Ref. [8]. In the present context, however, we are interested in the energy distribution of the sum-frequency radiation recorded at the crystal output, which is obtained by integrating the square of the modulus of field (12) with respect to time:

$$\begin{aligned} Q_{\mathrm{SF}}(\tau) &\propto \int\int H(t''-t')\tilde{A}\left(t'-\frac{\tau}{2}\right) \\ &\times\tilde{A}\left(t'+\frac{\tau}{2}\right)\tilde{A}^*\left(t''-\frac{\tau}{2}\right)\tilde{A}^*\left(t''+\frac{\tau}{2}\right)\mathrm{d}t'\mathrm{d}t'', \end{aligned} \tag{15}$$

where $H(t) = \int h(t'+t)h^*(t')\mathrm{d}t'$. In order to analyse the character of the distribution (15), let us specify the profile of the pulse $h(t)$.

The properties of the pulse $h(t)$ are mainly determined by the frequency dependence of the function $\mathrm{sinc}(\Delta kL/2)$. Let us estimate this dependence in the first approximation from a series expansion of mismatch in frequency around $2\omega_0$. In this approximation, we have

$$\Delta k(\omega) \approx (\omega - 2\omega_0)\left[\frac{\cos\beta_0}{v(\omega_0)} - \frac{1}{v(2\omega_0)} + \frac{\omega_0}{c}\left(\frac{\mathrm{d}n}{\mathrm{d}\omega}\right)_{\omega_0} \times\right.$$

$$\times \frac{\sin^2\beta_0}{\cos\beta_0}\Bigg] = \frac{\omega - 2\omega_0}{c}\left[\left(\lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)_{\lambda_0/2} - \left(\lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)_{\lambda_0}\frac{1}{\cos\beta_0}\right]. \quad (16)$$

Here, $v(\omega_0) = 1/k'(\omega_0)$ and $v(2\omega_0) = 1/k'(2\omega_0)$ are the group velocities of the radiation pulses at the corresponding frequencies. Moreover, it is assumed that the exact phase-matching condition $k(2\omega_0) = 2k_{1,2}(\omega_0)\cos\beta_0$ is satisfied for the central emission frequency ω_0. The first equality in (16) shows the mismatch structure: the main contribution comes from the difference between the group velocity $v(\omega_0)/\cos\beta_0$ of the fundamental radiation (the factor $\cos\beta_0$, where β_0 is the mean angle of propagation of the fundamental sum-frequency radiation in the crystal, has been introduced due to the oblique propagation of the input pulses) and the group velocity of the sum-frequency radiation. The third term with the frequency derivative of the refractive index is the correction associated with the angular dispersion of the fundamental radiation. The second equality in (16) is written to simplify the practical estimates. One can see that, in this approximation, the mismatch is independent of frequencies of incident waves. This is due to the fact that these frequencies appear in expression (16) as a sum and are hence replaced by ω. In this case, the pulse $h(t)$ acquires a nearly rectangular shape with a duration T equal to the difference in group propagation times for the fundamental and sum-frequency radiation through the crystal:

$$T = \frac{L}{c}\left[\left(\lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)_{\lambda_0/2} - \left(\lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)_{\lambda_0}\frac{1}{\cos\beta_0}\right]. \quad (17)$$

The shape of this pulse differs slightly from rectangular and matches the shape of the pulse with central frequency $2\omega_0$ experiencing nonlinearity dispersion and ordinary dispersion (14) at half the crystal thickness. The physical meaning of the pulse $h(t)$ can be seen from expression (12). If we consider the collinear propagation of initial pulses ($\tau = 0$) of small duration (much smaller than T), $h(t)$ will obviously describe the form of the resulting second harmonic field under the conditions of a strong influence of the mismatch. The authors of [7] observed just such a rectangular second-harmonic pulse under identical conditions.

Note that in the second order of the frequency expansion of the mismatch Δk, its dependence on the frequencies of incident waves turns out to be quite weak. This is due to the fact that in the second term of the expansion, these frequencies also appear mainly in the form of a sum and are omitted from the expression after their replacement by ω, while the dependence of the mismatch on other combinations of the initial frequencies is not significant. This statement is illustrated in Fig. 2 showing the zero lines of the function $\sin[\Delta k(\omega_1;\omega_2)L/2]$ in the frequency range 0.375 PHz (800 nm) for a 1-mm thick KDP crystal. The frequency range of the diagram corresponds to the triple spectral width of the incident pulses of duration 15 fs. One can see that the departure of the calculated dependences from the lines corresponding to the situation in which Δk is independent of the frequencies of incident waves is quite insignificant. Hence, we shall ignore in the following analysis the dependence of Δk on ω_1 and ω_2.

3. Effects depending on the crystal thickness

The above expressions make it possible to estimate quantitatively the effect of factors associated with the crystal thickness on the formation of the output field in the autocorrelator. The angular dispersion and the nonlinearity dispersion play insignificant roles in this case.

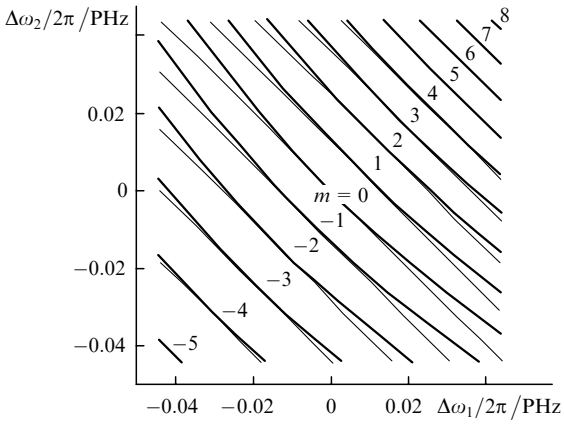

Figure 2. Dependence of the mismatch on $\Delta\omega_1$ and $\Delta\omega_2$ in the form of lines along which $\Delta k(\omega_1;\omega_2)L/2 = m\pi$, where $\Delta\omega_1$ and $\Delta\omega_2$ are deviations of frequencies from the exact phase-matching frequency. Thin lines illustrate the approximation (11) obtained by taking into account the dependence of Δk only on the sum-frequency, while the thick lines correspond to calculations using all second-order terms in the frequency expansion of Δk and are given for comparison.

3.1 Angular dispersion

The angular dispersion of the crystal affects the mismatch (16) through the factor $1/\cos\beta_0$, and the dispersion of $k_{1,2}$ distorting the pulse (13) propagating in the crystal, through the factor $\cos\beta_{1,2}$. One can see that angular dispersion does not independently affect the resultant field, but just quantitatively modifies the ordinary dispersion and the nonlinearity dispersion effects. Moreover, the corresponding cosinusoidal factors differ insignificantly from unity (by no more than 1 %) for small beam convergence angles (not exceeding 10°). Hence, angular dispersion can be treated only as an insignificant correction to the effects considered below.

3.2 Nonlinearity dispersion

The dispersion of nonlinear transformation factorised in accordance with the Miller rule appears in the form of the factor $\omega[n^2(\omega) - 1]/n(\omega)$ in the integrand in expression (14) for $h(t)$, and in the form of the factor $[n^2(\omega) - 1]$ in expression (13) for transformed fields of the pulses being measured. Simple estimates show that for moderate thicknesses of nonlinear crystals ($L > 10$ μm), nonlinearity dispersion plays a much less significant role than ordinary dispersion. Hence, we neglect the nonlinearity dispersion effects here.

3.3 Phase-matching width

Phase-matching in a nonlinear crystal is violated due to first-order dispersion effects (in the frequency expansion of the wave vector) and is reduced to the difference T in the group delay times. We shall estimate this effect in two limiting cases when the duration of the initial pulses is much longer than, and much shorter than T, i.e., in the case of thin and thick crystals

For a thin crystal, the functions $h(t)$ and $H(t)$ are delta functions in expressions (12) and (15), and the recorded

distribution is transformed into the known relation with the correlation function of pulse intensities:

$$Q_{\mathrm{SF}}(\tau) \propto \int \tilde{I}\left(t-\frac{\tau}{2}\right)\tilde{I}\left(t+\frac{\tau}{2}\right)\mathrm{d}t, \tag{18}$$

where $\tilde{I}(t) = |\tilde{A}(t)|^2$.

For a thick crystal, the time variations of light fields occur against the background of a weakly varying function $H(t)$, i.e., the role of the delta function will now be played by the pulses being measured. Consequently, we can take the function $H(t)$ out of the integration sign, and the recorded distribution will have the form

$$Q_{\mathrm{SF}}(\tau) \propto \left|\int \tilde{A}\left(t-\frac{\tau}{2}\right)\tilde{A}\left(t-\frac{\tau}{2}\right)\mathrm{d}t\right|^2. \tag{19}$$

Formally, this distribution is quite a different correlation function as compared to (18), which now has the form of the square of the modulus of *field* correlation. Such a modification of the observed distribution (19) was not detected in Refs [4–6] devoted to autocorrelation measurements. However, the very first examples show that there is no significant quantitative difference between distributions (18) and (19).

Indeed, in the case of a Gaussian pulse with a linear chirp b appearing in the amplitude $\exp(-\mathrm{i}bt^2/2) \times \exp[-2(t/\Delta t_0)^2 \ln 2]$ of the Gaussian pulse, these distributions are exactly identical. The expressions for a noise pulse representing the Gaussian noise of the light field, inscribed in a relatively slow pulse envelope, are also identical. In the case of a hyperbolic pulse with the amplitude $\exp(-\mathrm{i}bt^2/2)/\cosh[2(t/\Delta t_0)\ln(1+\sqrt{2})]$, the correlation functions being compared are slightly different (Fig. 3), the difference between them being determined by the chirp b. For zero chirp, the distribution width of the function corresponding to a thick crystal is about 10 % larger than the correlation function width for a thin crystal. The situation is reversed in the case of a large chirp. The correlation function width for a thick crystal is smaller by about the same amount than the correlation function width for a thin crystal. Only for a rectangular light pulse (which is quite hard to realise in the femtosecond range) the intensity correlation has a triangular distribution, while the distribution (19) is the square of the triangular distribution.

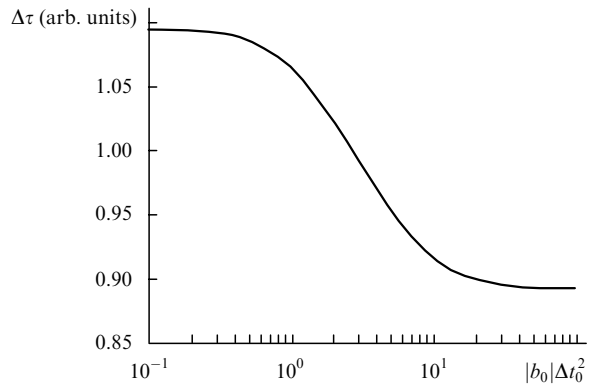

Figure 3. Dependence of the FWHM of the square of the modulus of the 'hyperbolic' pulse field correlation function on the initial chirp. The width of the intensity correlation function for the same pulse is taken as the unit width.

Thus, it can be concluded that the effect of the crystal thickness, which is taken into account as the group velocity mismatch between the fundamental radiation and the sum-frequency pulses, is not the main factor responsible for distortion of the distribution recorded in the correlator in most cases of practical importance. Under these conditions, the crystal thickness is manifested through dispersive broadening of the pulses being measured and propagating in the bulk of the crystal.

3.4 Dispersive broadening

Finally, let us consider the crystal dispersion effects described by exponential factors of the type $\exp[\mathrm{i}k_{1,2}(L/2)\cos\beta_{1,2}]$ in expression (13) and $\exp(\mathrm{i}kL/2)$ in expression (14). In accordance with the approach used in linear optics, we shall analyse the role of dispersion through the series expansion of wave vectors at the central frequency. In this case, the zero-order terms (in frequency mismatch) contribute insignificant constant phase shifts to the expression (12) for the field. The first terms appear in the form of the group delay times: the resultant sum-frequency field is delayed by the group time $L/v(2\omega_0)$, the incident pulses will be delayed by their group time, while the pulse $h(t)$ will be in the interval between 0 and T. These delays do not introduce any significant variations in the form of the resultant field (12). Nontrivial contribution should be expected from the second-order (and probably higher-order) expansion terms of the wave vectors. These terms lead to a broadening (or compression) of pulses. We shall not analyse the broadening of the fictitious pulse $h(t)$ because it follows from Section 3.3 that the peculiarities of the pulse shape affect the energy distribution (15) quite weakly. Conversely, a change in the shape of the initial pulses is directly reflected in the recorded distribution and hence calls for a quantitative estimate.

Thus, the distribution recorded in the autocorrelator is defined by the pulse shape in the middle of the crystal. Because of dispersion of the crystal, the pulse in the middle of the crystal may be broadened or compressed, which depends qualitatively and quantitatively on the magnitude of the initial pulse chirp. During the propagation of the pulse in the medium, its duration Δt and chirp b change. In the first nonvanishing approximation in the dispersion contribution, the combined variation of the duration and chirp of the pulse at the depth z of the medium are described by the relation

$$b\Delta t^2 \approx b_0\Delta t_0^2 + k''(\omega_0)z\Delta\omega^2, \tag{20}$$

where b_0 and Δt_0 are respectively the chirp and duration of the pulse at the crystal input; and $\Delta\omega$ is the spectral width of the pulse. By solving Eqn (20), we must take into account the combined contribution of chirp and pulse duration to the spectral width

$$(\Delta\omega\Delta t)^2 \approx (\Delta\omega\Delta t)^2_{\mathrm{min}} + (b\Delta t^2)^2, \tag{21}$$

where $(\Delta\omega\Delta t)_{\mathrm{min}}$ is the minimum possible dimensionless number for the pulse shape under consideration. Recall that this value is $\sim 2\pi \times 0.44$ for a Gaussian pulse, while for hyperbolic pulses it is $\sim 2\pi \times 0.31$. Approximate expressions (20) and (21) are satisfied exactly for Gaussian pulses with a

linear chirp. The solution of these equations gives the pulse duration:

$$\Delta t^2 \approx \Delta t_0^2 + 2b_0\Delta t_0^2 k''(\omega_0)z + [k''(\omega_0)z\Delta\omega]^2. \quad (22)$$

A distinguishing feature of expressions (20)–(22) is that the duration and spectral width of the pulse must be defined in an identical manner (for example, FWHM in both cases). Figure 4 shows dependence (22) of the pulse duration on the depth of its penetration into the medium. One can see that at the initial stage of penetration into the medium, when the main contribution to (22) comes from the term linear in z, the magnitude and sign of the chirp determine the rate of variation of the pulse duration. In this case, the term quadratic in z is significant only for zero chirp of the initial pulse, and the broadening of the pulse at the input is insignificant. It is this small contribution from the quadratic term (for a small penetration depth of the pulse with a zero chirp) that serves as a false argument for neglecting the effect of dispersion broadening of pulses. In the general case, the linear term associated with the chirp plays a dominating role.

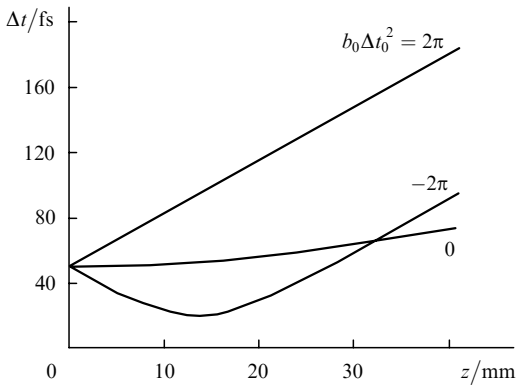

Figure 4. Dependence of the pulse duration on the depth of its penetration into the crystal for three values of the initial chirp. Calculations are made for a KDP crystal and pulses with initial duration 50 fs at the central wavelength 0.8 μm.

To estimate the pulse duration at the initial stage of its penetration into the medium (when the relative variation of the pulse duration is insignificant), we can use instead of (22) the relation

$$\frac{\Delta t - \Delta t_0}{\Delta t_0} \approx z b_0 k''(\omega_0) + \frac{z^2}{2}\left[\frac{\Delta\omega k''(\omega_0)}{\Delta t_0}\right]^2. \quad (23)$$

When the duration of a pulse is measured with the help of an autocorrelator, its chirp is not known as a rule. However, its magnitude can be determined from expression (21) if the emission spectrum is measured in addition to the pulse duration. In this case, the chirp is estimated from the difference between the product $\Delta\omega\Delta t$ and the minimum possible value of this product by using the relation $|b_0| \approx [(\Delta\omega\Delta t_0)^2 - (\Delta\omega\Delta t)_{\min}^2]^{1/2}/\Delta t_0^2$.

For a penetration depth into the medium equal to half the crystal thickness, i.e., $z = (L/2)\cos\beta_0 \approx L/2$, formula (23) can be treated as an estimate of the systematic error in the measurement of the pulse duration caused by the crystal dispersion:

$$\frac{\delta t}{\Delta t_0} \approx \frac{L}{2} b k''(\omega_0) + \frac{L^2}{8}\left[\frac{\Delta\omega k''(\omega_0)}{\Delta t_0}\right]^2. \quad (24)$$

The second derivative of the wave vector for traditional nonlinear crystals is equal to tens of $fs^2\ mm^{-1}$. To estimate the pulse broadening, we use the value $k'' = 27\ fs^2\ mm^{-1}$ characteristic of a KDP crystal ($\lambda = 800$ nm). Consider two examples.

Example 1. We assume that the pulse duration is measured in an autocorrelator with a 1-mm thick KDP crystal, the resulting pulse duration Δt_{obs} at the wavelength 800 nm is ~ 50 fs, and the shape of the autocorrelation function is nearly Gaussian. Let us also suppose that independent measurements of the spectral width show a substantial value of the pulse chirp since $\Delta\omega\Delta t_{obs} \approx 1.6(\Delta\omega\Delta t)_{\min}$. This leads to the following estimate for the chirp in expression (21): $|b| \approx 1.4 \times 10^{-3}\ fs^{-2}$. In this case, the error is estimated by retaining the first term in (24) which gives $|\delta t/\Delta t| \approx 0.02$ (the second term is about two orders of magnitude smaller). Thus, it can be assumed that the above value of the crystal thickness introduces an error ~ 1 fs in the measured value of pulse duration (the sign of the error is not known and is determined only by the sign of the chirp).

Example 2. We assume that measurements with another laser source using the same autocorrelator lead to a pulse duration of 30 fs, the shape of the autocorrelator function is close to hyperbolic, and independent measurements of the spectral width give $\Delta\omega\Delta t_{obs} \approx 1.1 \times (\Delta\omega\Delta t)_{\min}$. In this case, $|b| \approx 10^{-3}\ fs^{-2}$. In spite of a decrease in the contribution from the chirp to the product $\Delta\omega\Delta t$, its contribution to the error of measurement of the pulse duration (22) continues to play a dominant role $|\delta t|/\Delta t \approx 0.014$. This leads to a measuring error of 0.4 fs for the pulse duration. Let us also estimate the difference in group delay times for the second harmonic pulse and the fundamental frequency pulse: $T \approx 77$ fs. This estimate shows that the results of measurements of 30-fs pulses in this autocorrelator correspond probably to the case of a thick crystal. If the pulse shape is close to hyperbolic, the effect of limited phase-matching width on the results of measurements must be quite significant. Estimating the dimensionless chirp as $|b|\Delta t^2 = 0.9$, we note in accordance with Fig. 3 that the measured duration may be 6 %–7 % (i.e., about 2 fs) longer than the real pulse duration. Thus, it can be assumed that in this case, the use of an autocorrelator with a 1-mm thick KDP crystal may increase the pulse duration by 2 fs and introduce an error of ~ 0.4 fs (due to uncertainty in the sign of the chirp).

For an unknown pulse chirp, the error associated with the dispersive distortion of pulses in the medium of the crystal may also turn out to be indefinite. However, this is a systematic error of the autocorrelator and can be eliminated. If a medium with opposite sign of k'' is placed in front of the crystal and its thickness is chosen in such a manner that the product $k''l$ is modulo equal to the quantity $k''L/2$ for the crystal, the pulse returns to its initial duration in the middle of the crystal irrespective of the magnitude of the chirp, and the dispersive distortion in the crystal will not affect the form of the distribution being registered. Standard transparent materials can hardly be used for such an additional medium, and a compressor (say, a pair of prisms) with a preset dispersion may perhaps be suitable for this purpose.

4. Conclusions

Thus, we have used the analysis of the generation of sum-frequency radiation in the crystal of an autocorrelator to obtain expressions (12) and (15) respectively for the field and the recorded second-harmonic energy distribution. These expressions determine the dependence of both distributions on the crystal thickness. The effect of crystal thickness is mainly determined by two factors: a limited phase-matching width (group velocity mismatch between the fundamental frequency pulse and the second harmonic pulse) and the dispersive broadening of the initial pulses. The limited phase-matching width is manifested when the difference T between the group delay times (17) exceeds the pulse duration and leads to a transformation of the recorded distribution function from correlation intensities function (18) to the square of the modulus of the correlation field function (19).The latter distribution differs slightly from the former distribution, but the two expressions coincide for Gaussian pulses with a linear chirp. Hence, the second factor, the dispersive broadening of pulses at half the crystal thickness, may acquire a greater significance, especially due to the fact that the pulse broadening increases indefinitely with the crystal thickness. The dispersive distortion of the duration being measured can be estimated from expression (24) in which the term proportional to the chirp of the pulses being measured plays a significant role. A knowledge of the chirp value during autocorrelation measurements of the pulse duration [which can be determined, for example, from the spectral data with the help of (21)] plays a significant role in the estimation of the measuring error.

Acknowledgements. This work was supported by the Federal Specific Programme 'Integration' (Project No. B0049).

References

1. Jansky J., Corradi G., Gyuzalian R.N. *Opt. Commun.,* **23**, 293 (1979).
2. DeLong K.W., Trebino R., Hunter J., White W.E. *J. Opt. Soc. Am. B,* **11**, 2206 (1994).
3. Masalov A., Nikitin S., Qiang Fu. *Tech. Dig. IQEC-2002* (Moscow, 2002) p. 448.
4. Weiner A.M. *IEEE J. Quantum Electron.,* **19**, 1276 (1983).
5. Baronavski A.P., Ladouceur H.D., Shaw J.K. *IEEE J. Quantum Electron.,* **29**, 580; 2928 (1993).
6. Baltuska A., Pshenichnikov M.S., Wiersma D.A. *IEEE J. Quantum Electron.,* **35**, 459 (1999).
7. Noordam L.D., Bakker H.J., De Boer M.P., Van Linden van den Heuvell H.B. *Opt. Lett.*, **15**, 1464 (1990).
8. Kabelka V., Masalov A.V. *Opt. Commun.*, **100**, 482 (1993).