LASER MODES AND BEAMS

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Invariance of mode transformation by an astigmatic $\pi/2$ converter upon the input-beam displacement and tilt

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Abstract. **It is shown theoretically and experimentally that the transformation of modes by an astigmatic $\pi/2$ converter is invariant with respect to the input-beam displacement and tilt. The possibility is considered of using this property for manipulating microobjects and simultaneous generation of Laguerre – Gaussian modes of different orders with the help of the same astigmatic $\pi/2$ converter.**

Keywords: *astigmatic $\pi/2$ converter, Laguerre – Gaussian modes, invariance of the fractional Fourier transform.*

1. Introduction

It was shown in the last decade that the use of Laguerre – Gaussian (LG) laser beams opens up new possibilities in a variety of fields in science and technology – from contactless manipulating microobjects and biological cells to quantum cryptography and experiments with a Bose – Einstein condensate.

LG beams are usually obtained by diffraction methods, which differ mainly in the type of a phase element. These can be both rather intricate kinoforms, which concentrate the incident radiation energy to a fixed diffraction order corresponding to a certain distribution of an LG beam [1], and diffraction elements, which produce simultaneously many different LG modes, each of them being located in its own diffraction order [2]. Also, spiral phase plates are used [3 – 5], whose optical delay provides the angular dependence of the phase of type $\exp(\mathrm{i}l\varphi)$, as for the corresponding LG mode. Most often are used computer-generated holograms [6, 7], which are analogous to the interference pattern obtained upon coherent summation of an LG mode with a reference wave of one type or another.

As a whole, most of the diffraction methods allow the simultaneous generation of a set of modes with the same angular index l and different values of the radial index p ($p \geqslant 0$) [3, 8]. In the best case, the transformation efficiency does not exceed 60 % – 70 % [1, 9], and, in addition, the radiation power is substantially limited (except maybe spiral phase plates, whose production technology was recently considerably improved [10]). A disadvantage of these methods is also that a rather accurate adjustment of the position of the incident beam with respect to a diffraction element is required to achieve the optimal transformation efficiency [1, 9], which is especially important for phase elements with a singular point (spiral plate and hologram).

An alternative to diffraction methods of generating LG modes is the use of an astigmatic $\pi/2$ converter [11] containing a set of spherical and cylindrical lenses. The initial radiation for the $\pi/2$ converter are Hermite – Gaussian (HG) modes, whose parameters (wave front, beam radius, and the orientation of the field-distribution axes) should be properly matched with the parameters of optical elements of the converter [11]. It was shown [12] that the astigmatic optics of the converter performs the Fourier transforms (FTs) in two mutually orthogonal planes, whose number differs by unity. In the general case, these FTs can be characterised by a fractional (including irrational) order, which allows a sufficiently broad choice of converter designs [13, 14], which differ from the configuration proposed in [11]. In addition, it is possible to change the $\pi/2$ converter parameters using the same set of optical components [15]. Another advantage is the possibility of generating LG modes with different angular indices $\pm l$ with the help of the same converter using different HG modes (u_{0m}^{HG} or u_{n0}^{HG}). However, there remains the question, which was not discussed in [11 – 15]: How sensitive is the scheme of the astigmatic $\pi/2$ converter to the displacement and tilt of the axis of the input HG beam with respect to the optical axis of the converter? This question is all the more of interest in view of the above-mentioned requirements on the adjustment of diffraction devices used for generating LG modes.

2. Fractional Fourier transform of a displaced and tilted beam

Consider the general case of using fractional FTs (FFTs) of the orders a (xz plane) and $a+1$ (yz plane) in the astigmatic $\pi/2$ converter. For an optical system performing FFT of the order a for the function $u(x)$, we have the integral representation

$$\mathscr{F}(\xi) = \mathscr{F}_{\xi}^{a}[u(x)] = \frac{\exp(\mathrm{i}\psi/2)}{(\mathrm{i}\lambda f \sin\psi)^{1/2}} \int_{-\infty}^{+\infty} u(x) \times \exp\left[\mathrm{i}\pi\,\frac{(x^2+\psi^2)\cos\psi - 2\xi x}{\lambda f \sin\psi}\right]\mathrm{d}x, \qquad (1)$$

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where x and ξ are the coordinates of the input and output reference planes; $\psi = a\pi/2$; λ is the radiation wavelength; and f is a scaling coefficient characterising the optical system, which is related to the beam radius w by the expression $f = \pi w^2/\lambda$. To obtain the usual FT (of the first order), we substitute $a = 1$ into (1). Let us denote $\sin\psi$ by S and $\cos\psi$ by C, and omit the insignificant factor in front of the integral in (1).

Because the distribution of the field of the HG $u_{nm}^{\rm HG}(x,y)$ mode can be represented as a product of independent functions of x and y, we restrict our consideration only by the xz plane, in which the beam displacement and tilt can be written as

$$u = u(x - \delta x)\exp\left[{\rm i}\,\frac{2\pi x\vartheta_x}{\lambda}\right]. \tag{2}$$

After substitution into (1) and transformations, we have

$$\mathscr{F}(\xi) = \exp\left[{\rm i}\,\frac{2\pi}{\lambda}\,\xi\left(C\vartheta_x - \frac{S\delta x}{f}\right)\right]$$
$$\times\exp\left[{\rm i}\,\frac{2\pi}{\lambda}\left(CS\,\frac{\delta x^2 - f^2\vartheta_x^2}{2f} + S^2\delta x\vartheta_x\right)\right]$$
$$\times\int_{-\infty}^{+\infty} u(x - \delta x)\exp\left\{{\rm i}\pi\,\frac{\left[(x-\delta x)^2 + (\xi - \delta xC - Sf\vartheta_x)^2\right]C}{\lambda fS}\right.$$
$$\left. - \frac{2(x-\delta x)(\xi - \delta xC - Sf\vartheta_x)}{\lambda fS}\right\}{\rm d}x. \tag{3}$$

Analysis shows that the integrand in (3) is the FFT of the displaced field distribution $u = u(x - \delta x)$ in the coordinates at the optical scheme output, which are changed in accordance with the matrix transformation

$$\begin{pmatrix}\xi'\\ \vartheta'_\xi\end{pmatrix} = \begin{pmatrix}\xi\\ 0\end{pmatrix} + \begin{pmatrix}C & Sf\\ -S/f & C\end{pmatrix}\begin{pmatrix}-\delta x\\ \vartheta_x\end{pmatrix}$$
$$= \begin{pmatrix}\xi - C\delta x - Sf\vartheta_x\\ -\delta xS/f + C\vartheta_x\end{pmatrix}, \tag{4}$$

where

$$\begin{pmatrix}C & Sf\\ -S/f & C\end{pmatrix}$$

is the $ABCD$ matrix of the optical system in the xz plane. In this case, the output field propagates at an angle of $C\vartheta_x - S\delta x/f$ to the optical axis (the first exponential factor in front of the integral) and acquires an additional phase shift depending on the displacement and the tilt angle at the input (the second exponential cofactor).*

According to (2)–(4), the result for the yz plane of the $\pi/2$ converter is obvious. A similar change in the coordinates, the tilt and additional [independent of coordinates and the specific form of the field $u_{nm}^{\rm HG}(x,y)$] phase shift will correspond to the output displacement and tilt of the input beam in this plane. The only difference from the xz plane is the different FFT order equal to $a + 1$ in the yz plane, i.e., the replacements $S \to -C$ and $C \to S$ should be made in (3) and (4).

Therefore, the additional phase shift appearing upon the displacement and tilt of the incident beam with respect to the $\pi/2$ converter axis retains invariant the relation between the accumulated Gouy phases of the $u_{nm}^{\rm HG}(x,y)$ modes in planes xz and yz, which still have the form [13]

$$\mathscr{F}_x^{\,a}\{\mathscr{F}_y^{\,a+1}\left[u_{nm}^{\rm HG}(x,y)\right]\}$$
$$= \exp\left(-\,{\rm i}Na\,\frac{\pi}{2} - {\rm i}m\,\frac{\pi}{2}\right)u_{nm}^{\rm HG}(x,y), \tag{5}$$

where $N = m + n$ is the mode order.

3. Use of the invariance property of the astigmatic $\pi/2$ converter

The invariance of transformation of the HG modes by the astigmatic $\pi/2$ converter allows a simple implementation of a displacement of LG beams at the converter output. It is sufficient to change the position or tilt of the input beam. Unlike this, in a number of optical 'tweezer' schemes, either a captured object (a cell with microparticles) [17, 18] or a beam after the converter [19] is displaced.

Note that, when one FT is used in the xz plane ($a = 1$), we have $C \equiv 0$, and displacement is not observed at the converter input. No corresponding tilts of the beam axis were observed in the yz plane after two FTs ($S \equiv 0$). Therefore, to displace the LG beam over the entire operation field of the objective placed at the converter output, it is most convenient to use the $\pi/2$ converter with the FTT of the half-integer order ($a = 0.5$, $\psi = \pi/4$). If the distance L from the point of changing of the input-beam angle to the converter is much greater than the focal distance F of the objective, one can easily show that

$$dx \sim -\frac{\gamma_x L}{F}, \quad dy \sim \frac{\gamma_y L}{F}, \tag{6}$$

where $\gamma_{x,y}$ are the tilt angles of the beam in the corresponding planes at the converter input. Therefore, we obtain the displacements dx and dy in the focal plane of the objective, which have close absolute values.

We demonstrated experimentally the control of the beam position in the focusing objective plane by changing the beam-tilt angle using the scheme shown in Fig. 1. We employed a laser generating the TEM_{01} mode, whose waist ($w_0 \approx 0.7$ mm) was located on the plane output mirror of

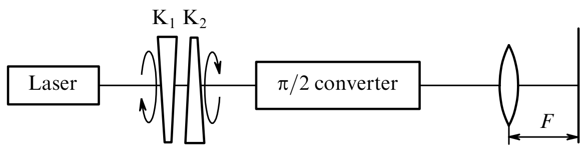

Figure 1. Scheme of the experiment on controlling the position of the LG mode beam in the focal plane of a lens by changing the tilt and displacement of the HG beam at the input of the $\pi/2$ converter (K_1 and K_2 are optical wedges).

* Expression (15) in [16], describing the effect of multiplying the function by the complex exponential on the FFT result (beam tilt in our terms), contains an error in the phase term in front of the integral.

the laser, and a simplified $\pi/2$ converter [20]. The coordinate and angle at the input plane of the converter were changed by rotating two optical wedges (angle $5'$). Figure 2 shows the positions of an LG mode for different rotations of one of the wedges around the optical axis of the converter (with a step of $\sim 50^\circ$). The ellipticity of the intensity distributions of LG beams and of the pattern as a whole is explained by the absence of matching optics between the output mirror of the laser and the converter, resulting in a spherical wave front at the converter input rather than the plane one.

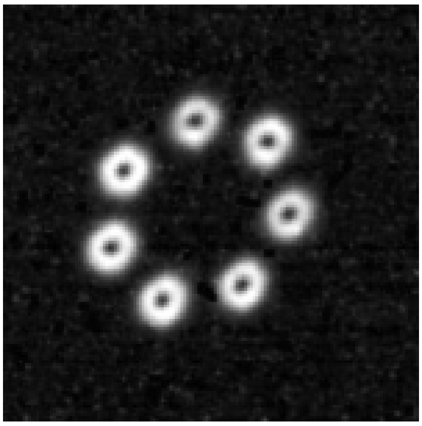

Figure 2. Positions of an LG mode recorded in the focal plane of a lens by rotating one of the wedges (Fig. 1) around the optical axis (with a step of $\sim 50^\circ$).

The above-described invariance property can be also used to obtain simultaneously two and more independently controllable LG beams by employing a Mach–Zehnder interferometer at the converter input. In this case, variants both with non-interfering and interfering beams can be realised. The first variant can be used for the independent control of motion of two particles (or groups), and the second one in the scheme of accumulation or sorting of particles, when the overlap of two LG $u_{01}^{\rm LG}$ modes, having fields of the opposite signs at the different ends of the diameter, results in the destructive interference (quenching) of the intersecting regions of the beams (Fig. 3). It is obvious that in this case an additional control of the optical paths of the interacting beams is required.

In such a scheme, several lasers generating HG modes of different orders can be also used, and hence LG modes with different angular orbital moments can be simultaneously obtained. For example, the TEM_{00} modes can be simultaneously used for manipulating particles with a high refractive index.

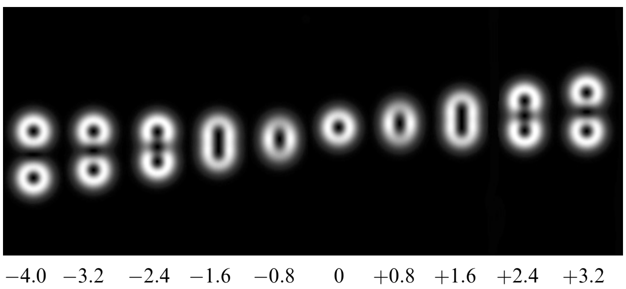

Figure 3. Interference of two LG mode beams with the same polarisations (calculation). The distance between the beams is in fractions of w_0.

4. Conclusions

We have shown that the transformation of the HG modes to the LG modes by the astigmatic $\pi/2$ converter is insensitive to the tilt and displacement of the input beam due to the invariance of the fractional Fourier transform. This gives the additional possibility for controlling laser beams in schemes where the LG modes are used for optical contactless manipulation of microobjects. The $\pi/2$ converters based on the fractional FT of the half-integer orders (1/2 or 3/2) are most convenient for this purpose.

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