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Effect of intracavity back reflections on the accuracy and stability of optical frequency standards

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***Abstract*. The shifts of centres of nonlinear absorption and dispersion resonances in lasers with an internal absorbing cell are described. The shifts are caused by parasitic reflection of light from an intracavity plate and oscillate depending on the plate position. It is shown that this reflection leads to losses oscillating depending on the plate position. The main reasons of shifts in this case are frequency modulation used for laser frequency stabilisation and the curvature of the gain line in the two-mode regime. The results of a model experiment agree with theoretical estimates.**

Keywords: *optical frequency standards, saturated absorption resonances, stability, intracavity reflections.*

1. Introduction

Optical frequency standards (OFSs) based on the nonlinear dispersion and absorption resonances in He – Ne lasers with an intracavity methane cell ($\lambda = 3.39$ μm) have been used over many years as one of the most precision standards in the optical spectral region, which provide the frequency stability and reproducibility $10^{-14} - 10^{-15}$ and $\sim 10^{-14}$, respectively (see, for example, Refs [1 – 3]). Recently they were employed in small mobile devices [4, 5] whose stability and precision is an order of magnitude inferior to record values achieved in stationary devices [2, 6] but are, nevertheless, sufficient for the solution of many problems. For example, these frequency standards can be used for the development of highly stable master (reference) oscillators with a narrow emission spectrum ($\Delta\omega/\omega \sim 10^{-14} - 10^{-15}$) in the optical and microwave regions.

Due to the femtosecond technology of coupling between the optical and microwave spectral ranges developed in recent years [7, 8], a high frequency stability of the methane reference can be rather simply transferred both to the visible and microwave spectral ranges. Therefore, methane standards can be used in some problems instead of hydrogen masers, which are now employed as master oscillators in high-precision clocks and frequency standards. In the optical region, they can be employed as sources of extremely narrow spectral lines to develop a new generation of frequency standards based on cooled atoms and ions captured in traps (predicted precision is $10^{-17} - 10^{-18}$) and to calibrate other secondary OFSs.

In this paper, we study the factors limiting the stability of such OFSs with the aim of improving their stability. One of the important factors at present, which is difficult to control in experiments, is parasitic back reflection from surfaces inside the resonator and the possible instability of positions of these surfaces. The effect of weak reflected fields (mainly from external sources) on the operation of a laser with a linear Fabry – Perot resonator has been long discussed in the literature (see, for example, Refs [9 – 11]). However, the mechanisms of manifestation of this effect in frequency standards with gas cells and its detailed theoretical description are presented for the first time in this paper.

Such effects were observed experimentally in OFSs by correlations between the oscillations of a stabilised frequency and temperature during long-term frequency measurements with respect to the cesium frequency reference. Figure 1 shows a typical example of absolute frequency measurements performed in December 2000 for a mobile PTB radio-optical bridge OFS (Germany) with respect to the primary cesium frequency standard [12]. The laser was stabilised against the central component of the resolved hyperfine structure of the $F_2^{(2)}$ line of methane. Stabilisation was performed using the saturated dispersion resonance with the 4.5-kHz HWHM. Figure 1 demonstrates characteristic frequency oscillations of the standard with the amplitude up to 10 Hz, which cannot be explained by the signal-to-noise ratio of the reference used.

Based on the self-consistent theory [13, 14] of a filled resonator (taking the gain and absorbing media into account), which is close to a plane one and has losses at

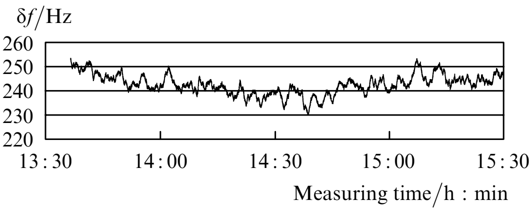

Figure 1. Typical absolute frequency measurements (the absolute frequency is 88 376 181 000 000 Hz) with respect to the cesium frequency standard performed on the PTB radio-optical bridge (Germany) in December 2000 (the averaging time is 100 s).

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mirrors, we describe in section 2 the model of a resonator containing a plate with weak back reflection (in experiments, such an effect can be also related to scattering). As a result, we obtained the equation for the field in the resonator, in which the fraction of losses (caused by back reflection from the plate) oscillates depending on the position of this plate in the resonator. Note here that no violation of the energy conservation law occurs in this case. Simply, after insertion of even one plate with a low reflectivity, the resonator is formally divided into two, but the method described in Ref. [13] allows us to reduce equations for the output frequency and field of the laser to the equations for a conventional two-mirror resonator. Therefore, it seems that no real significance should be attached to the 'oscillating' losses in this reduced equation. In a certain sense, this is simply the energy captured inside a complex (containing more than two mirrors) resonator.

The next step was to find the reasons how the 'oscillating' losses cause the shift of the stabilised laser frequency. At present we found two reasons that substantially affect the laser frequency shift.

The first one is the inevitable use of frequency modulation to stabilise the OFS frequency against the nonlinear resonance centre. This requires the modulation of the resonator length and, therefore, of the position of parasitic reflecting surfaces, resulting in the modulation of losses. Such a modulation of losses leads to the shift of the stabilised frequency from the maximum of any nonlinear (absorption or dispersion) resonance against which the frequency is stabilised.

The second important reason, which is typical for a two-mode OFS, is the frequency shift caused by the curvature of the linear gain profile [15], which is proportional to the coupling coefficient between modes and to the excess over the threshold for a give mode. A change in losses due to the drift of reflecting surfaces leads to the corresponding drift of the stabilised frequency.

In section 3, we describe the model experiment performed with the help of an external reflecting source to compare qualitatively the experimental data with the theoretical estimates obtained.

2. Theory

2.1 Field in the cavity with an intracavity reflecting plate

The only self-consistent method describing the intensity and frequency resonances in lasers with internal and external absorbing cells was described in Refs [13, 14]. This method was developed for resonators close to a plane resonator [$(L/R)^{1/2} \ll 1$, where L is the length and R is the radius of resonator mirrors] and it allows one to include into the final equation the boundary conditions on all the surfaces for the field amplitude E_0 averaged over a round trip in the resonator (namely, due to the resonator round trip, i.e., integration over the resonator length). Only in this case, it is possible to avoid the contradiction between the expansion in modes of an 'empty' resonator (always used in the literature) and the boundary conditions with losses, to which these modes never can satisfy. In this case, losses at mirrors are taken into account in the same way as a linear gain, and they can be further considered together with the gain within the framework of the perturbation theory to describe nonlinear absorption and dispersion resonances.

The equation for the field amplitude E_0 integrated along the entire resonator has the form

$$\left[\mathrm{i}\,\frac{L}{2k}\,\Delta_r - \mathrm{i}\,\frac{kr^2}{R} + \mathrm{i}\varphi_0 - \delta + g_+ - g_-\right] E_0(\boldsymbol{r}) =$$

$$= g_+\beta_+(\omega)|E_0|^2 E_0 - g_-\beta_-(\omega)|E_0|^2 E_0. \quad (1)$$

Here, $k = \omega/c$; ω is the laser field frequency; r is the radial coordinate; $\varphi_0 = kL - \pi n$; n is an integer; $|\varphi_0| \ll 1$; $1/R = \frac{1}{2}(1/R_1 + 1/R_2)$; and $\delta = \frac{1}{2}(\delta_1 + \delta_2)$ are the average inverse radius and mirror losses, respectively; g_+ and g_- are the linear gain and absorption coefficient per round trip in the resonator, respectively; and $\beta_\pm(\omega)$ is the saturation parameter for the amplifying and absorbing media [13, 14].

The effect of surfaces of interest to us, which introduce additional reflection in the resonator, can be considered using a simple model of a plane plate located at a distance of l from the resonator centre and having the amplitude back reflectivity r_0 and the transmission coefficient $1 - q$. In this case, the boundary conditions on the plate (Fig. 2) have the form

$$E_-^{\mathrm{L}} = E_-^{\mathrm{R}}(1-q) + r_0 E_+^{\mathrm{L}}, \quad E_+^{\mathrm{R}} = E_+^{\mathrm{L}}(1-q) + r_0 E_-^{\mathrm{R}}.$$

As a result, the total resonator losses in Eq. (1) have the form

$$\bar{\delta} = \delta + q - r_0 \cos[2k(l - z_0)], \quad (2)$$

where z_0 is the coordinate of the resonator centre.

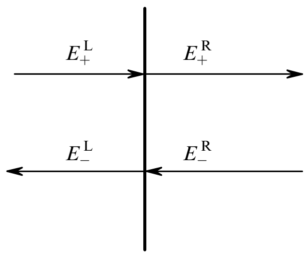

Figure 2. Notation of forward and backward waves on the plate surface.

In this case, it is difficult to speak directly about losses 'oscillating' depending on the coordinate l. They oscillate only for the field amplitude integrated over the entire resonator (1). On the other hand, only in this case we can speak about a parameter common for the entire resonator (irrespective of the number of boundary conditions encountered on the way) and also write an absolutely adequate equation for this parameter, which allows us to describe both nonlinear absorption and dispersion resonances. The next question is: How do these 'oscillations' of losses depending on the position of a reflecting surface affect the accuracy of measuring the nonlinear resonance frequency?

2.2 Shift caused by the frequency modulation of the resonator

At present we can distinguish two most important problems. The first one is related to the necessity of using frequency modulation to stabilise the resonance frequency. The resonator length of a stabilised laser is modulated at the frequency Ω_0,

$$L = \bar{L}(1 + x\cos\Omega_0 t).$$

Therefore, other parameters of the resonance (for example, its losses and intensity) also are modulated due to the presence of losses depending on the position of surfaces inside the resonator:

$$\omega = \bar{\omega}(1 - x\cos\Omega_0 t), \quad \delta = \bar{\delta} + \alpha\bar{\omega}x\cos\Omega_0 t,$$

$$\alpha = \frac{r_0}{c/\bar{L}}\left[1 - \frac{2(l - z_0)}{\bar{L}}\right]\sin[2k(l - z_0)].$$

The detection of the signal S at the nth harmonic frequency at small modulation amplitudes ($|x| \ll 1$) is formally reduced to the resonance derivatives

$$\frac{1}{T}\int_0^T \mathrm{d}t\cos(n\Omega_0 t)S(\omega,\delta) = \frac{(-1)^n}{n!!}x^n\bar{\omega}^n \times\left[\frac{\partial^n}{\partial\omega^n} - \alpha n\frac{\partial^{n-1}}{\partial\omega^{n-1}}\frac{\partial}{\partial\delta} + \alpha^2\ldots\right]S|_{x=0}. \quad (3)$$

Taking into account that a change in δ mainly affects the nonlinear dispersion or absorption resonance through the intensity, which is proportional to the threshold excess $\eta = g_+ - g_- - \delta$, we can replace the derivative $\partial S/\partial\delta$ in (3) by S/η. Therefore, we obtain already here in a general form that such 'oscillating' losses cause systematic shifts of nonlinear resonances when the laser is stabilised using frequency modulation. These losses depend only on the position of reflecting surfaces and resonance parameters, and can be reduced only by decreasing reflection from these surfaces.

For example, the frequency shift of the absorption resonance in the single-mode regime in the Doppler limit $[\beta_- \sim 1 + \gamma/(\gamma + \mathrm{i}\Delta\omega)]$ is

$$\Delta_\mathrm{a} = \frac{r_0}{\eta}\frac{\gamma^2}{c/L_0}K\left[1 - \frac{2(l - z_0)}{L_0}\right]\sin[2k(l - z_0)],$$

where γ is the homogeneous width of the absorption line in methane and $K^{-1} = g_-\beta_-(0)/g_+\beta_+(0)$ is the resonance contrast. The expression for the shift of the centre of the first derivative of dispersion is similar, but without the contrast coefficient,

$$\Delta_\mathrm{d} = \frac{r_0}{\eta}\frac{\gamma^2}{c/L_0}\left[1 - \frac{2(l - z_0)}{L_0}\right]\sin[2k(l - z_0)].$$

In the case of the two-mode regime, additional coefficients appear in the expressions due to coupling between the modes, but the values of frequency shifts remain of the same order.

2.3 Shifts in the two-mode regime

The second factor, which is sensitive to 'oscillating' losses, is the frequency shift observed in the two-mode regime used to separate resonances in OFSs [3, 4]. The shift is related to the curvature of the gain profile and is proportional to the coupling coefficient between the modes [15]. It also depends on many parameters of the nonlinear resonance, including losses.

By using the method described in Ref. [13], we expand the field in the resonator in the modes of an empty resonator

$$E_0(\boldsymbol{r}) = \sum_{n,m} a_{nm}\Psi_{nm}(\boldsymbol{r}),$$

which are determined by the equation

$$\Delta_r\Psi_{nm}(\boldsymbol{r}) - \frac{2k^2}{LR}r^2\Psi_{nm} = -\mu_{nm}\Psi_{nm}.$$

Then, by introducing the coupling coefficient $\theta_\pm^{(1,2)}$ between the modes, we obtain in the two-mode regime the equations

$$W^2\left[-\mathrm{i}\left(\varphi_{1,2} - \frac{L}{2k}\mu_0^{(1,2)}\right) + \eta_{1,2}\right] = \left(g_+\beta_+^{(1,2)} - g_-\beta_-^{(1,2)}\right)\left|a_0^{(1,2)}\right|^2 + \left(g_+\theta_+^{(1,2)} - g_-\theta_-^{(1,2)}\right)\left|a_0^{(2,1)}\right|^2,$$

for the frequencies $\varphi_{1,2}$ and amplitudes $a_{1,2}$ of the zero modes (of the orthogonal polarisation), where $\eta_{1,2} = \eta'_{1,2} + \mathrm{i}\eta''_{1,2} = g_+ - g_- - \delta^{(1,2)}$; $\varphi_{1,2} = \omega_{1,2}c^{-1}L - \pi n$; and W is the laser beam width.

The mode intensities are described in this case by the expressions

$$I_{1,2} = \left|a_0^{(1,2)}\right|^2 = K\frac{W^2}{g_0\beta_0} \times\frac{\eta'_{1,2}\left(K - \beta'_-(\Delta\omega_{2,1})/\beta_-^{(0)}\right) - \eta'_{2,1}\left(K\sigma - \theta'_-/\beta_-^{(0)}\right)}{g_0\beta_0\left(K - \beta'_-(\Delta\omega_1)/\beta_-^{(0)}\right)\left(K - \beta'_-(\Delta\omega_2)/\beta_-^{(0)}\right) - \left(K\sigma - \theta'_-/\beta_-^{(0)}\right)^2}$$

[where $\beta_-^{(0)} = \beta_-(0)$; $g_0 = g_+(0)$; $\sigma = \theta_0/\beta_0$; $\beta_0 = \beta_+(0)$; $\theta_0 = \theta_+(0)$; $\Delta\omega_i = \omega_i - \omega_0$; and ω_0 is the molecular transition frequency] and take the form

$$I_1 = \frac{W^2}{g_0\beta_0(1 - \sigma^2)}\left(\eta'_1 - \eta'_2\sigma\right)\left[1 + A\beta'_-(\Delta\omega_1)\right],$$

$$I_2 = \frac{W^2}{g_0\beta_0(1 - \sigma^2)}\left[\left(\eta'_2 - \eta'_1\sigma\right) - A\beta'_-(\Delta\omega_1)\sigma\left(\eta'_1 - \eta'_2\sigma\right)\right],$$

$$A = \frac{g_-}{g_0\beta_0}\left(1 - \sigma^2\right)^{-1}$$

in the vicinity of one of the resonances ($\Delta\omega \approx 0$).

The nonlinear resonance of the difference frequency between the two modes is described by the equation

$$-W^2\frac{L}{c}(\omega_1 - \omega_2 + \Omega_{12}) = W^2\left(\eta''_1 - \eta''_2\right) + I_1\left[g_0\beta''_0(\Delta\omega_1) - g_-\beta''_-(\Delta\omega_1)\right] - I_2\left[g_0\beta''_0(\Delta\omega_2) - g_-\beta''_-(\Delta\omega_2)\right] + \left(g_0\theta''_+ - g_-\theta''_-\right)(I_1 - I_2).$$

By introducing the dependence for the dispersion part of polarisation and coupling coefficients between modes in the amplifying medium

$$\beta''_+(\Delta\omega_{1,2}) = \beta_0\alpha_0\frac{\Delta\omega_{1,2} + \Omega}{\Gamma_+}, \ \alpha_0 \sim 1, \ \Omega = \omega_0^{(-)} - \omega_0^{(+)},$$

$$\theta''_{+} = \theta_0 \frac{\Delta\omega_1^{(+)} + \Delta\omega_2^{(+)}}{2\Gamma_+} = \theta_0 \frac{\Delta\omega_1 + \Omega - \Omega_{12}/2}{\Gamma_+}$$

(where Γ_+ is the absorption line width), we obtain the nonlinear dispersion resonance in the intermode frequency

$$\omega_1 - \omega_2 + \Omega_{12} = \ldots + B\left[-\beta''_{-}(\Delta\omega_1) + \beta'_{-}(\Delta\omega_1)D\right], \quad (4)$$

$$B = \left(\eta'_1 - \eta'_2\sigma\right)\frac{c}{L}\frac{1}{g_0\beta_0(1-\sigma^2)},$$

$$D = \frac{1}{1-\sigma^2}\frac{\Omega}{\Gamma_+}(1+\sigma)(\alpha_0+\sigma) - \frac{\Omega_{12}}{\Gamma_+}\sigma\left(\alpha_0 + \frac{1+\sigma}{2}\right).$$

We have omitted in (4) the terms linearly proportional to the frequency and constants that vanish after the first differentiation. In addition, we assume at present that the frequency shift related to the coefficient D in (4) is insignificant. However, the curvature of the gain profile (in the threshold excess η) pointed out in Ref. [15] should be taken into account, and, according to our estimates, it leads to strong instabilities caused by internal reflecting surfaces. Let us represent it as

$$\eta'_1 - \sigma\eta'_2 = -\delta_1 + \sigma\delta_2 + g_0\left[1 - \left(\frac{\Delta\omega_1}{\Gamma_+}\right)^2\right]$$
$$- g_0\sigma\left[1 - \left(\frac{\Delta\omega_1 - \Omega_{12}}{\Gamma_+}\right)^2\right] \approx g_0\left[1 - \sigma\left(1 - \frac{\Omega_{12}^2}{\Gamma_+^2}\right)\right]$$
$$- \delta_1 + \sigma\delta_2 - g_0\sigma\frac{2\Omega_{12}\Delta\omega_1}{\Gamma_+^2}. \quad (5)$$

This leads to the appearance of a nonlinear resonance (both for dispersion and absorption) of the type $(1+\rho\Delta\omega)S(\Delta\omega)$, resulting in the Doppler limit in the shifts of absorption $[\Delta_a = \frac{1}{2}\rho\gamma^2K(1-\sigma^2)]$ and dispersion $(\Delta_d = -\frac{1}{3}\rho\gamma^2)$ resonances.

The coefficient ρ can be taken from experiments or estimated from (5) for approximately equal losses for the modes as

$$\rho \approx 2\sigma\frac{g_0}{\eta(1-\sigma)}\frac{\Omega_{12}}{\Gamma_+^2}.$$

This shift was indicated in Ref. [15]. The part of the shift of interest to us is determined by the expression for 'oscillating' losses contained in η_i. Therefore, to obtain the shift caused by the instability of positions of reflecting surfaces, the shift calculated in the absence of these losses should be multiplied by the coefficient

$$\frac{r_1^{(0)}\cos[2k_1(l-z_0)] - r_2^{(0)}\sigma\cos[2k_2(l-z_0)]}{\eta(1-\sigma)},$$

where $r_i^{(0)}$ are reflection losses introduced by the plate in the ith mode (in principle, they can be different). As a result, for example, the shift of the absorption resonance has the form

$$\Delta_a = \rho\gamma^2K(1+\sigma)$$
$$\times\frac{r_1^{(0)}\cos[2k_1(l-z_0)] - r_2^{(0)}\sigma\cos[2k_2(l-z_0)]}{2\eta}.$$

In the case of strongly coupled modes $(\sigma \to 1)$ and approximately equal reflection losses introduced by the plate $(\delta_1^{(0)} \approx \delta_2^{(0)})$, this shift can be written in the form

$$\Delta_a = \rho\gamma^2K\frac{r_0}{\eta}\frac{2\Omega_{12}}{c/L}\frac{l-z_0}{L}\sin[2k(l-z_0)].$$

3. Model experiment

Figure 3 shows the scheme of the model experiment performed to compare theoretical shifts with experimental shifts of the stabilised frequency oscillating depending on the position of the reflecting plate (external plate in this case). The description of the effect of an external reflecting plate is similar to that of an intracavity plate, and frequency shifts are described almost by identical expressions. Only 'oscillating' losses in this case are introduced as additional reflection from one of the mirrors with the phase shift $[\bar{\delta}_1 = r_0\exp(\mathrm{i}2kL_0)]$, where L_0 is the distance to the external reflector.

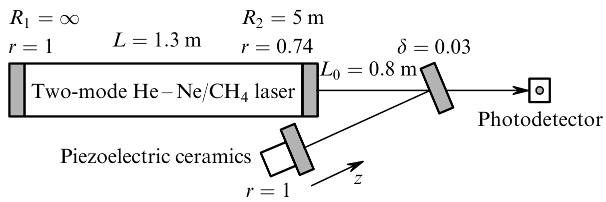

Figure 3. Scheme of the model experiment with an external weakly reflecting plate.

We used a two-mode He – Ne laser with a methane absorption cell, which was stabilised against the 3.39-μm nonlinear absorption resonance of methane. The HWHM of the amplitude resonance was 150 kHz (Fig. 4). A part of laser radiation was returned back by a deflecting mirror with a low reflectivity (3 % in intensity) and a mirror mounted on a piezoelectric ceramics. A change in the absolute frequency of the He – Ne laser after the insertion of parasitic back reflection into the resonator was detected at the beat frequency with another (reference) stabilised laser.

Figure 5 shows the typical oscillating dependence of the stabilised frequency upon translational motion of the piezoelectric ceramics. The amplitude of oscillations was

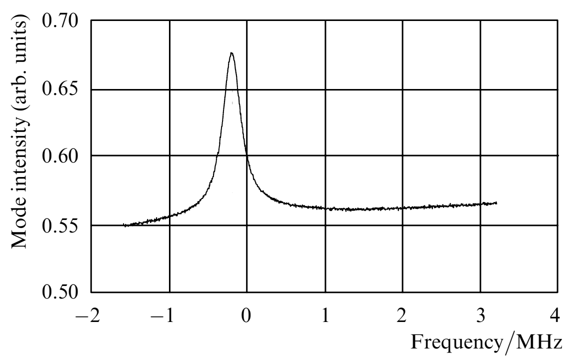

Figure 4. Saturated absorption resonance in methane in the two-mode laser under study observed for one of the modes. The laser frequency was stabilised by coupling to the resonance maximum.

$\sim$ 160 Hz. To compare this value with the above-mentioned shifts, it is necessary to estimate several values from experimental data. The amplitude r_0 of losses oscillating depending on the piezoelectric ceramics position with the transmission of the output resonator mirror (0.26 in intensity) and the coefficient of overlapping of the returned signal ($\sim$ 0.57 for the scheme in Fig. 2), is $r_0 = 0.0044$. The experimental value of the amplitude of oscillations of the quantity $\omega_1 - \omega_2$ during the ceramics motion [$\sim r_0 \Omega_{12} 2L_0/L$ (see above)] also confirms the correctness of r_0 calibration. Other coefficients obtained from experimental data are: $\rho \approx 2.5 \times 10^{-8}$ Hz^{-1}, $K \approx 50$, $2\eta \sim 0.3$, and $\Omega_{12} = 3.75$ MHz. Expressions for the external reflecting plate differ somewhat from those obtained above for the intracavity plate, but they can be derived by the same method. For the oscillating part of the shift of the absorption resonance in a two-mode laser, they have the form

$$\Delta\omega_{\rm s} = G\rho\sin(2kL_0), \quad G = \frac{r_0}{\eta} K \frac{\Omega_{12}}{c/L} \gamma^2 \frac{L_0}{L}$$

for the shift due to the slope of the gain and

$$\Delta\omega_{\rm m} = G \frac{1}{c/L} 2\left(\frac{L_0}{L} + 1\right) \cos(2kL_0)$$

for the shift due to modulation of the resonator length. As a result, we obtain that the amplitude of oscillations of the shift of the stabilised frequency due to frequency modulation is equal to $\sim$ 92 Hz and to $\sim$ 26 Hz for the shift caused by the slope of the linear gain (the amplitudes will be greater at lower values of η). This estimate of the oscillation amplitudes shows that they are of the same order of magnitude as in experiments.

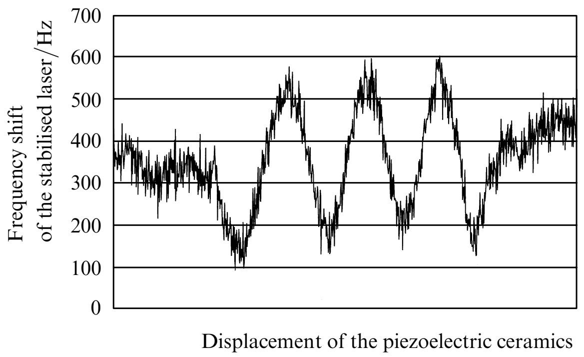

Figure 5. Dependence of the frequency shift of the stabilised two-mode laser on the distance between the laser and retrodirective mirror.

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