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Electrophysical and optical properties of spherical and cylindrical liquid-crystal optically addressed lenses

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Abstract. **New experimental results on direct measurements of the phase response of spherical liquid-crystal (LC) optically addressed lenses (OALs) are presented. A simple and easy-to-use formula for the focal length of an adaptive modal LC lens is obtained for the first time. The complex conductance is measured with an accuracy that is sufficient to find the aperture region contribution and the calculated conductance is in good qualitative agreement with the experiment. The possibility of using OALs for self-correction of defocusing and as adaptive laser-beam deflectors is proposed and demonstrated experimentally.**

Keywords: *liquid crystals, optically addressed spatial light modulators, adaptive lenses.*

1. Introduction

Liquid-crystal optically addressed lenses (LC OALs) proposed by one of the authors [1] are attractive because they can be used for the development of adaptive optics (AO) devices with a built-in optical feedback. Our previous experimental studies of LC OALs [2, 3] were devoted to the visualisation of wave fronts obtained with the help of spherical OALs. In this paper, we measured directly for the first time the shape of the wave front of spherical OALs using a wave-front (WF) sensor and studied the wave-front formation with the help of a cylindrical OAL. The main difference between OALs and adaptive LC lenses with a purely electric control [4] is the dependence of their focal lengths (WF curvature) on the incident radiation intensity which makes it possible to develop new OAL-based AO devices. In addition to the technique for power stabilisation at a diaphragm [3], we have also demonstrated experimentally two new schemes for automatic compensation of defocusing and a light-sensitive laser-beam deflector.

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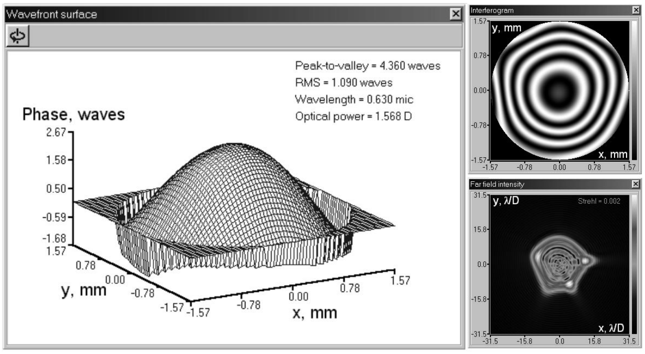
2. Formation of the laser radiation wave front using an OAL

Our experimental results on WF formation using a spherical OAL are published in Ref. [3]. A standard scheme for LC cells with crossed polarisers was used for visualisation. In this case, the spherically symmetric WF emerging from the device produces an interference pattern in the form of concentric rings. In the present work, we have measured directly for the first time the shape of the wave front generated by a spherical OAL. Measurements were made using a WF sensor developed at the Delft Technical University (the Netherlands). The precision of reconstruction of the wave front at a wavelength of 0.63 μm was about $\lambda/20$, while the procedure itself was associated with the expansion into a certain number of Zernike polynomials. Detailed investigations of the WF shape made it possible to measure quite precisely the dependence of the focal length of the lens on the controlling parameters and to estimate the quality of the obtained wave fronts.

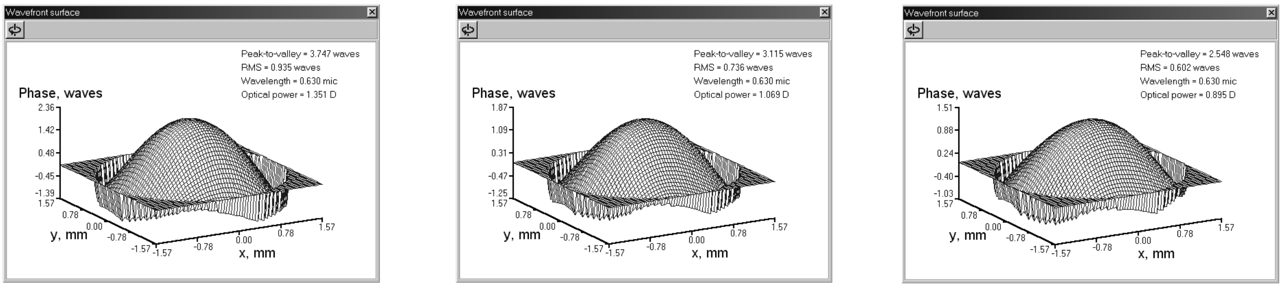
Figure 1 shows the typical shape of a reconstructed WF generated by an LC OAL illuminated by a homogeneous plane beam. Expansion in Zernike polynomials up to the seventh order (35 polynomials in all) inclusive was used. A number of the wave-front parameters, e.g., the phase deflection at the aperture, root-mean-square deviation from the plane wave front, optical power (inverse focal length), aberrations, etc. could be determined after wave-front reconstruction.

Figure 2 shows the variation in the WF behind a spherical LC OAL. As the incident-light intensity increases from 0.23 to 0.34 mW cm^{-2}, the phase deflection decreases



Peak-to-valley = 4.360 waves
RMS = 1.090 waves
Wavelength = 0.630 mic
Optical power = 1.568 D

Figure 1. Screen copy for the WF sensor software used with a spherical OAL. The intensity is 0.2 mW cm^{-2}.



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Peak-to-valley = 3.747 waves
RMS = 0.935 waves
Wavelength = 0.630 mic
Optical power = 1.351 D
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Peak-to-valley = 3.115 waves
RMS = 0.736 waves
Wavelength = 0.630 mic
Optical power = 1.069 D
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Peak-to-valley = 2.548 waves
RMS = 0.602 waves
Wavelength = 0.630 mic
Optical power = 0.895 D
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a b c

Figure 2. Wave-front shape obtained using spherical OALs for the incident-radiation intensities: (a) 0.23, (b) 0.29, (c) 0.34 mW cm^{-2}. The amplitude and frequency of the control voltage are 9 V and 100 Hz, respectively.

from 3.7λ to 2.5λ. These results demonstrate a considerable photosensitivity of the lens because the relative variations in the phase deflection are comparable with relative intensity variations. Moreover, one can see from Fig. 2 that the WF shape is close to parabolic at high intensities. Indeed, the aberrations in Figs 2a, b, c are 0.242λ, 0.219λ and 0.162λ, respectively. A decrease in aberrations with decreasing phase deflection is a common property of adaptive modal LC lenses [4].

As mentioned above, an LC OAL differs significantly from other lenses in that its focal length depends on the intensity of the radiation being focused. By using the theory of LC OALs [1–3], we can derive this important dependence analytically under the following assumptions: the linear segment of the phase–voltage characteristic of the LC is operative, the impedance of the LC layer is independent of the voltage applied to the layer (the so-called constant impedance approximation), and the voltage profile deflection at the aperture is small compared to the contact voltage.

The focal length F of the lens is related to the phase shift at its aperture by the expression

$$F = \frac{\pi l^2}{\lambda[\Delta\Phi(0) - \Delta\Phi(l)]}, \tag{1}$$

where l is the radius of the OAL aperture, and $\Delta\Phi(0) - \Delta\Phi(l)$ is the phase shift between the centre and the aperture edge. On the linear segment of the phase–voltage characteristic, we obtain the expression

$$\Delta\Phi(r) = \Delta\Phi_0 - \alpha|U(r)| \tag{2}$$

for the dependence of phase delay $\Delta\Phi$ introduced in the transmitted light wave by the LC layer. Here, $\Delta\Phi_0$ is the maximum phase delay introduced by the LC layer, α is the slope of the phase–voltage characteristic on the linear segment, and $U(r)$ is the effective voltage at a point with radial coordinate r on the aperture.

In turn, the voltage distribution over the OAL aperture is described by the modal corrector equation with appropriate boundary conditions. Within the framework of the constant impedance approximation, the solution of this equation has the form [1, 4]

$$U(r) = U_0 \frac{J_0(\mathrm{i}\chi r)}{J_0(\mathrm{i}\chi l)}, \tag{3}$$

where U_0 is the contact voltage and χ is the so-called modal parameter defined by the following relation for an LC OAL:

$$\chi^2 = \frac{g - \mathrm{i}\omega c}{h\sigma(I)}. \tag{4}$$

In this relation, g and c are respectively the conductance and capacitance of the LC; ω is the frequency of the control voltage; h is the photolayer thickness, and $\sigma(I)$ is the intensity dependence of the photolayer conductance. In the theory of modal LC correctors, the parameter χ plays an important role since its reciprocal has the meaning of the characteristic length of the local voltage drop at the contacts.

According to expressions (2) and (3), the deflections of the voltage distribution at the aperture are small for $|\chi l| \ll 1$. In this case, taking into account the asymptotic behaviour of the function J_0, for small values of the argument we obtain from (1)–(4) the following expression:

$$F = \frac{4\pi h}{\lambda\alpha U_0} \frac{\sigma(I)}{(g^2 + \omega^2 c^2)^{1/2}}. \tag{5}$$

This simple relation is quite convenient for analysing the properties of LC OALs. In particular, if the dependence $\sigma(I)$ is close to linear for the photosensitive layer, the focal length of the LC OALs must also increase linearly with incident-radiation intensity in accordance with expression (5), and the slope of the plot must decrease with increasing frequency. The results of measurements of the focal length of a real LC OAL as a function of intensity are presented in Fig. 3a. One can see that the dependences are indeed nearly linear, while their slopes (Fig. 3b) are almost inversely proportional to the frequency. Note that the quasilinear dependence of the focal length on the radiation intensity simplifies considerably the process of calibration of LC OALs for practical applications.

Cylindrical LC OALs differ from the spherical ones only in the geometry of the low-resistance contact forming the distribution of the control voltage over the aperture. The low-resistance contact in a cylindrical lens is formed by two parallel strips on the edges of a glass substrate. The distance between the strips determines the width of the lens aperture. We prepared a cylindrical LC OAL with the 15 × 5-mm aperture, the width of the aperture being equal to 5 mm. The remaining geometrical parameters are similar to those of a spherical LC OAL. In a scheme with crossed polaroids, the cylindrical fronts produce interference fringe patterns (Fig. 4). As the beam intensity increases, the surface

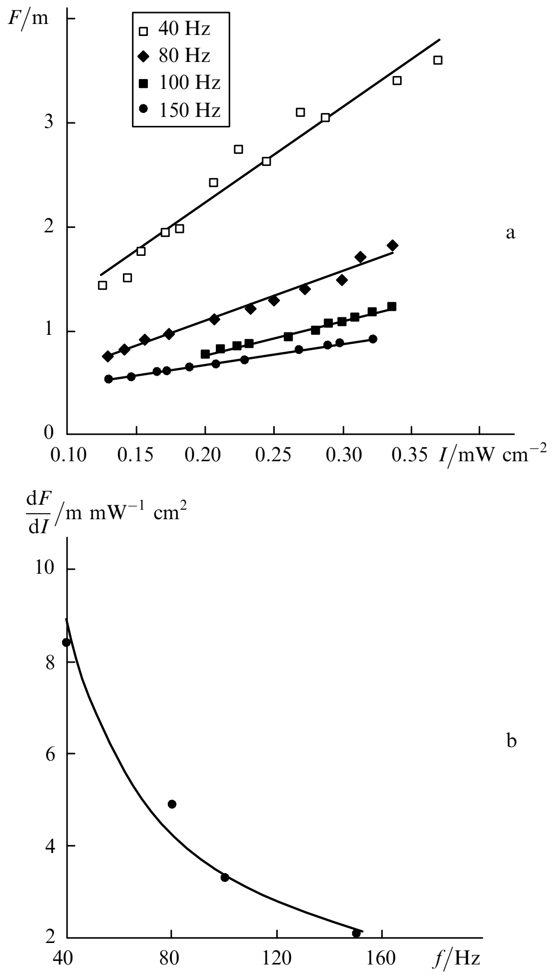

Figure 3. Dependences of the focal length of a spherical LC OAL on the intensity for various frequencies (a), and slopes of the curves (dark circles) and the hyperbola $1/f$ (solid curve) (b).

resistance of the OAL photolayer decreases, thus leading to a decrease in the deflection of the voltage distribution at the aperture and hence an increase in the separation between adjacent fringes (see Fig. 4a). On the other hand, an increase in the frequency of the applied voltage increases the phase distribution gradient, thus reducing the separation between fringes (see Fig. 4b).

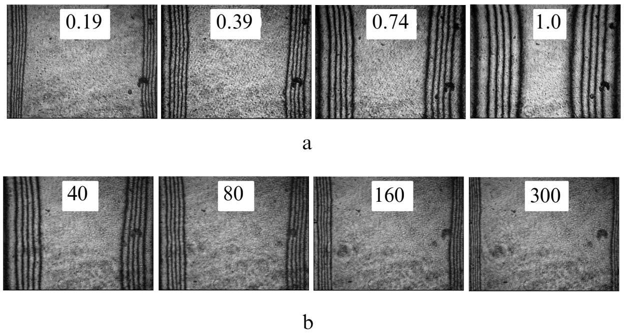

Figure 4. Interference patterns visualising the WF behind a cylindrical LC OAL (a) for a voltage 15 V and frequency 40 Hz (the figures on the patterns indicate the intensity of radiation incident on the lens in relative units, its maximum value being 2.5 mW cm^{-2}; (b) voltage 10 V, intensity 2.4 mW cm^{-2} [the figures on the patterns correspond to the frequency of the applied voltage (in Hz)].

3. Electrophysical properties of OALs

Although the main purpose of adaptive LC lenses is to control the shape of the wave front, they are two-terminal networks from the point of view of electrodynamics. The study of the complex impedance of such devices often provides valuable information on their functional properties [4, 5]. Moreover, the complex impedance of an LC layer plays an important role in the operation of modal LC correctors of which LC OALs are a particular case. For example, according to Eqn (5), the focal length of an LC OAL is determined directly by the modulus of the LC layer impedance. Note first of all that to study the modal LC devices, it is more convenient to use the concept of reciprocal impedance, i.e., the complex conductance (admittance) Z^{-1}, defined as the ratio of the complex amplitude of the current through a two-terminal network to the complex amplitude of the voltage across it. Knowing the complex conductance of the lens, we can determine its equivalent capacitance and equivalent resistance in a parallel substitution chain:

$$C = -\frac{1}{\omega}\operatorname{Im} Z^{-1}, \qquad R = \frac{1}{\operatorname{Re} Z^{-1}}. \tag{6}$$

We studied the equivalent RC-parameters of an LC OAL both experimentally and theoretically. The admittance of an adaptive LC lens was calculated for the first time in Ref. [6]. The following expressions were obtained for cylindrical and spherical lenses respectively:

$$Z_{\text{cyl}}^{-1} = \frac{2a(g - \mathrm{i}\omega c)}{\chi}\tanh(l\chi) + Z_1^{-1}, \qquad Z_{\text{sph}}^{-1} = \frac{2\pi l(g - \mathrm{i}\omega c)}{\mathrm{i}\chi}\frac{J_1(\mathrm{i}l\chi)}{J_0(\mathrm{i}l\chi)} + Z_1^{-1}, \tag{7}$$

where $Z_1^{-1} = S_{\text{cont}}(g - \mathrm{i}\omega c)$ is the contribution to the admittance from lens regions located below the low-resistance contact and having an area S_{cont}. For cylindrical lenses, the aperture is a rectangle of size $a \times 2l$, while for spherical lenses it is a circle of radius l. Since $g - \mathrm{i}\omega c$ is the specific complex conductance of the LC layer, it is convenient to introduce the effective area S_{eff} of the aperture of an adaptive LC lens using the relation

$$Z^{-1} = (g - \mathrm{i}\omega c)(S_{\text{eff}} + S_{\text{cont}}). \tag{8}$$

According to expression (7), the effective area of the aperture depends on the lens geometry and is defined by the following relations for cylindrical and spherical lenses, respectively:

$$S_{\text{eff cyl}} = \frac{2a}{\chi}\tanh(l\chi), \qquad S_{\text{eff sph}} = \frac{2\pi l}{\mathrm{i}\chi}\frac{J_1(\mathrm{i}l\chi)}{J_0(\mathrm{i}l\chi)}. \tag{9}$$

Expressions (6)–(9) can also be used for describing the RC-parameters of an LC OAL, the modal parameter χ being defined by expression (4). In terms of the modal parameter, the contribution to the complex conductance associated with the aperture depends not only on the frequency, but also on the light intensity. In the case of low intensities, we obtain from (9) the expressions $S_{\text{eff cyl}} = 2a\chi^{-1}$ and $S_{\text{eff sph}} = 2\pi l\chi^{-1}$. Because χ^{-1} characterises the voltage drop region in the vicinity of the contacts, both these relations have a clear physical meaning: from the point of view of conductance, the aperture consists of two strips of size $a \times \chi^{-1}$ adjoining the contacts (cylindrical lens) or a narrow ring adjacent to the aperture boundary. On the other hand, expressions (9) for large values of intensity give $S_{\text{eff cyl}} = 2al$ and $S_{\text{eff sph}} = \pi l^2$, i.e., the effective area of the aperture coincides with its geometrical area. This is an expected result, because at high intensities the voltage at all points of the aperture coincides with the voltage applied to the contact, i.e., the lens works in the cell mode. This analysis shows that the effective area of the lens monotonically increases with intensity, which leads to an increase in capacitance and a decrease in resistance according to expressions (6) and (8).

Note also that the contribution of the aperture to the conductance does not exceed $S_{\text{eff max}}/(S_{\text{eff max}} + S_{\text{cont}})$, which is about 47 % for cylindrical lenses and about 12 % for spherical lenses.

The complex impedance of LC OALs was measured by the bridge method, which we employed earlier for studying the impedance of a multichannel modal LC corrector [5]. Figure 5 shows the typical voltage dependences of the capacitance and resistance of a cylindrical lens. The dependences for a spherical LC OAL are similar. These data show that the dependence of the RC-parameters on voltage are mainly determined by the voltage dependence of the conductance and capacitance of an LC layer upon a reorientation of the LC molecules in an external field (Fredericksz transition). The threshold voltage for the transition in the LC used by us is about 1 V, while the initial orientation of the layer is planar (the molecules are oriented along the substrates). Hence, for lower voltages the LC layer parameters are determined by the complex permittivity tensor components measured at right angles to the electric field. As the voltage across an LC cell is increased, a transition to the corresponding components is observed, which are parallel to the field and whose magnitude is much larger. This circumstance accounts for the increase in the effective capacitance and a decrease in the equivalent resistance of the LC cells with voltage. One can see from Fig. 5 that this effect is also observed for a lens, but an increase, for example, in the capacitance with voltage is slower. This is due to the fact that the local voltage in the lens is equal to the applied voltage U_0 only in the vicinity of the contacts, and has a lower value in other regions of the aperture. Consequently, higher voltages are required for attaining the same values of R and C in the lens.

As was mentioned above, for an LC OAL, the dependence of its parameters on the light intensity (Fig. 6) is of considerable interest. The variation in the RC-parameters with increasing the intensity is in good agreement with the theoretical conclusions. For a voltage of 0.5 V, the capacitance is about one-third, and the resistance is about thrice the value for $U = 9$ V, which corresponds to a transition from the perpendicular components of the permittivity tensor to the parallel components. The interference patterns shown in the insets indicate a decrease in the voltage profile deflection with increasing the intensity, although the extreme right interference pattern indicates that a total uniformity of the voltage over the aperture (cell mode) at this frequency is not achieved even at the maximum intensity.

Thus, the local characteristics of an LC OAL (WF shape) and its integral parameters (conductance) demon-

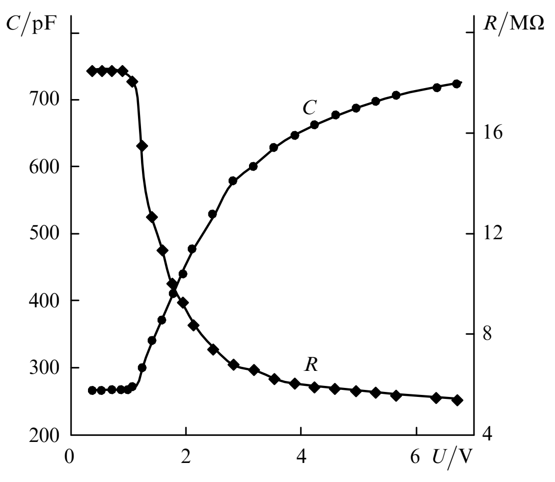

Figure 5. Dependences of the equivalent capacitance and equivalent resistance of a cylindrical LC OAL on the applied voltage (effective values) without illumination for a signal frequency 1 kHz.

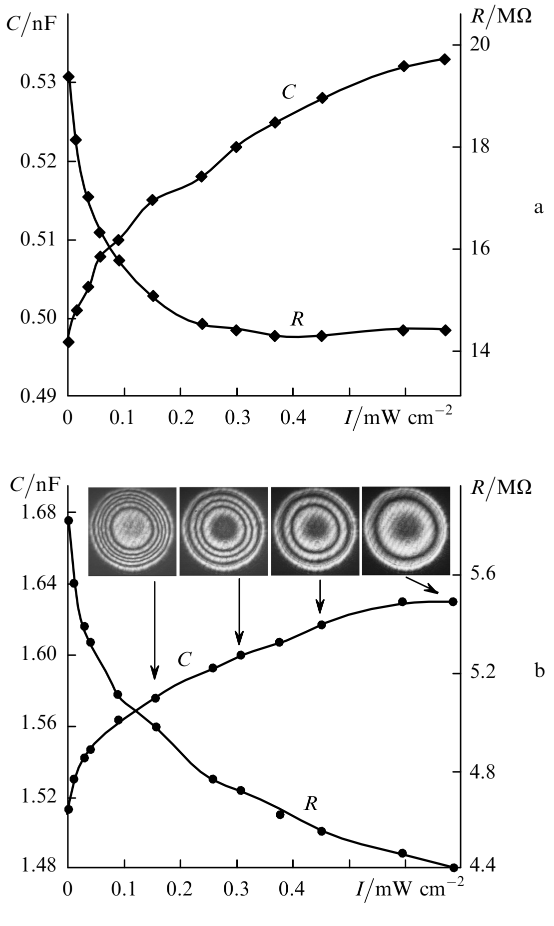

Figure 6. Dependences of the equivalent capacitance and equivalent resistance of a spherical LC OAL on the applied voltage (a) 0.5 V and (b) 9 V. The insets show the experimental interferograms obtained for the intensity values indicated by the arrows.

strate a high photosensitivity of the lenses, twhich makes them prospective for use in AO systems.

4. Possible applications of OALs for AO systems

4.1 System for automatic compensation for defocusing

Phase correction is one of the most important problems in adaptive optics. Lower order aberrations such as slope and defocusing often play the dominant role among various types of phase aberrations. Because AO systems without an electronic feedback loop are of considerable interest, we analyse here the simplest version of such a system based on an LC OAL (Fig. 7).

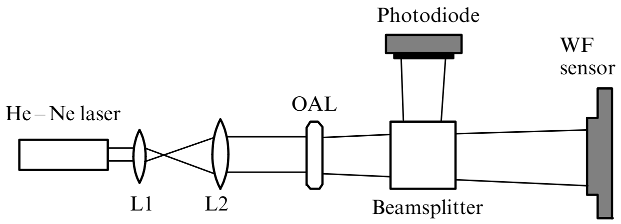

Figure 7. System for automatic compensation for defocusing based on a spherical LC OAL.

The system of lenses L1 and L2 played the role of a collimator and at the same time served as the controlled defocusing source. A beamsplitter and a photodiode were used to control the beam intensity directly behind the LC OAL. The need for such a control was dictated by the small intensity fluctuations arising during measurements. The beam emerging from the lens was analysed by a WF sensor (see Section 2) and the optical power D of the beam was measured.

If the distance between lenses L1 and L2 is equal to the sum of their focal lengths, the light wave behind lens L2 is a plane wave. If, however, lens L2 is displaced from this position by a small distance d, the wave behind lens L2 becomes divergent (L2 is displaced towards L1, $d < 0$) or convergent (L2 is displaced away from L1, $d > 0$). As a result, the WF shape and the beam intensity just in front of the LC OAL change. If the latter is switched off, the WF curvature remains unchanged by the lens. If the LC OAL is switched on, the intensity variations in front of this lens lead to a change in its focal length.

Consider, for example, the case $d < 0$, when a divergent beam is incident on an LC OAL. Because of the divergence, the intensity of the beam in front of the LC OAL will be lower than that behind L2 because only a part of the beam energy falls on the aperture of the LC OAL. The magnitude of this energy depends, in particular, on the beam divergence. Hence, an increase in the value of $|d|$ is accompanied by a simultaneous increase in the divergence and a decrease in the beam intensity at the LC OAL entrance. Both these factors influence the curvature of the beam emerging from the lens. Obviously, a decrease in the beam divergence leads to a corresponding increase in the beam divergence at the lens exit. On the other hand, a decrease in the intensity (see Section 2) reduces the focal length of the LC OAL (see Fig. 3), resulting in a decrease in the WF divergence. Thus, adaptive focusing with the help of an LC OAL partly compensates for the WF divergence at the lens entrance. The degree of compensation may be changed through an appropriate choice of the magnitude and frequency of the control voltage, as well as the distance s between the OAL and lens L2.

We measured the optical power D of the output beam as a function of the magnitude of displacement d (Fig. 8). One can see that the optical power D is negative when the LC OAL is switched off, and its value increases with d [Fig. 8, curve (*1*)]. However, the shape of the curves changes as a function of s if the LC OAL is switched on. This is due to the fact that the same variation in d leads to a stronger variation in the intensity in the aperture plane of the LC OAL with increasing the separation s. Therefore, it can be expected that the effect of the LC OAL will be stronger for higher values of s. As a matter of fact [see curve (*4*) in Fig. 8], this effect is so strong that an increase in the divergence of the input beam makes the output beam more convergent. A slight decrease in the value of s (to 50 cm in our case) allows a nearly total compensation for the variable beam divergence: instead of a linear variation of D from 0 to $-0.46\ \mathrm{m}^{-1}$, only slight fluctuations of D occur due to variations in the laser source intensity.

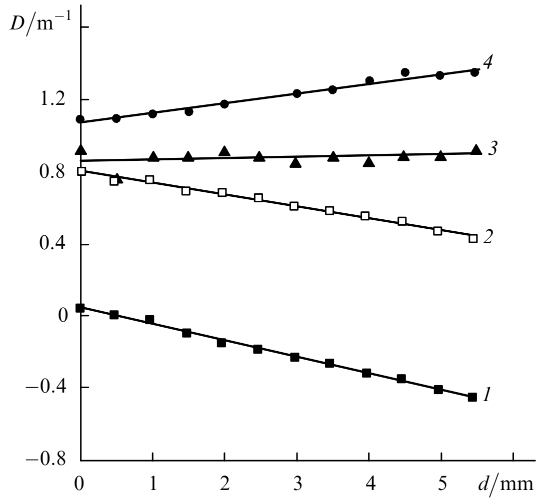

Figure 8. Adaptive compensation for defocusing with the help of an LC OAL for $d < 0$ (*1*) LC OAL is off; LC OAL is on (*2*) $s = 30$, (*3*) 50, and (*4*) 80 cm.

4.2 Angular intensity selection of beams

Cylindrical LC OALs can be used to create an adaptive prism whose angle of deflection depends on the incident-beam intensity. Such a device can be used for angular separation of beams with different intensities.

Love et al. [7] have studied an electrically controlled LC prism as an alternative to traditional deformable mirrors with a piezoelectric drive for AO systems. According to the authors of Ref. [7], the rate of variation of the deflection angle in the case of an LC layer controlled by the nematic transient effect is comparable with the required rate of phase correction.

For a cylindrical LC OAL to work in the adaptive prism mode, it is necessary to ensure a linear distribution of the control voltage along the lens aperture. This is attained by the simultaneous fulfilment of the two conditions: (1) the lens works in the small modal parameter regime, i.e., $|\chi l| \ll 1$; (2) voltages differing in magnitude or phase are

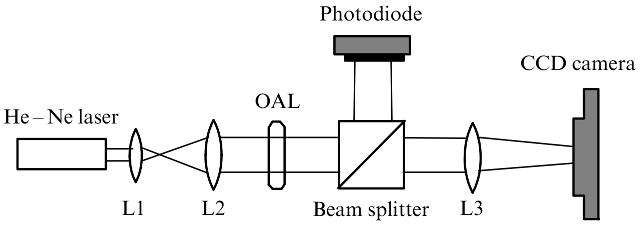

Figure 9. Scheme for the observation of the adaptive deflection of a laser beam using an LC OAL.

applied to the contacts. Figure 9 shows the scheme of the setup used for observing the beam deviation with the help of an LC deflector. A CCD camera is placed in the focal plane of lens L3. The magnitude of the deflection of the laser beam by the LC deflector is determined by the transverse displacement of the focal fringe. Figure 10 shows the change in the position of the focal fringe upon a variation of controlling parameters at the deflector. Note that the parameters of the LC OALs used by us were not optimised for the case considered here. Nevertheless, Fig. 10 shows that the maximum displacement is about half the width of the focal fringe. We intend to study this operational mode of an LC OAL in greater details.

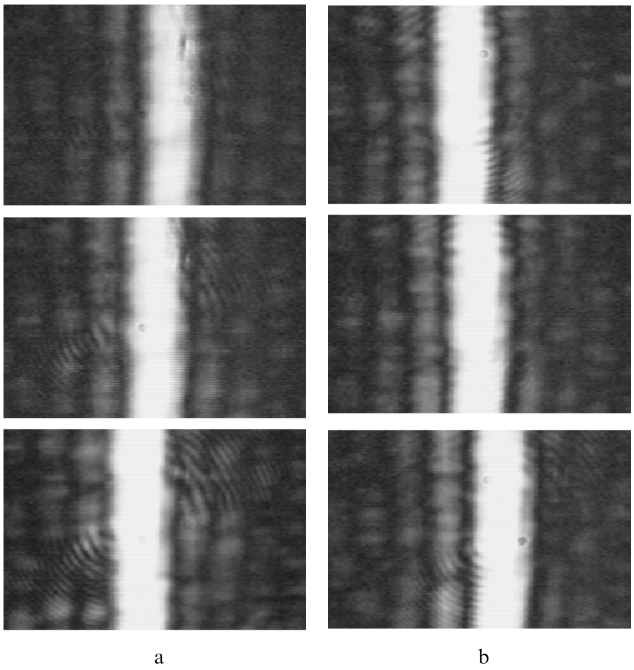

Figure 10. Displacement of the focal fringe for a frequency 40 Hz and intensity (top to bottom) 2.4, 3.5 and 6.8 mW cm^{-2} (a) and for an intensity 4.7 mW cm^{-2} and frequencies 40, 80 and 120 Hz (b). The voltage amplitude in both cases is 9 V.

5. Conclusions

Thus, we have directly measured for the first time the WF shape of spherical LC OALs using a WF sensor, and estimated the magnitude of the lens aberration. These results as well as the experimental investigations of the WF shape formed by cylindrical OALs demonstrate a high sensitivity of the samples. New possibilities of practical applications of LC OALs are illustrated by new AO systems: a defocusing compensator and an adaptive photosensitive LC deflector. Measurements of equivalent electrical parameters of cylindrical and spherical LC OALs have allowed us to determine the contribution of the voltage redistribution over the lens aperture to the lens conductance. The results are in qualitative agreement with the theory of conductance of adaptive modal LC lenses.

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