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Quantum memory and quantum computations in the optical subradiance regime

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***Abstract*.** **The possibilities of creation and manipulation of subradiant states in an extended atomic system by coherent 2π pulses are analysed. It is shown that excitation of the atomic system to collective subradiant states eliminates the superradiant broadening of the resonance line in quantum optical memory devices. The scheme of a nonlinear sign-shift two-qubit gate is proposed, which can be used in optical quantum computers.**

Keywords: *superradiance, subradiance, quantum computer, quantum gate, quantum memory*.

1. Introduction

At present extensive efforts are underway toward the development of optical quantum computers in which qubits are realised by means of one-photon two-mode quantum states of the electromagnetic field. Photons are ideal carriers of quantum information because they can be easily sent over large distances and they lose quantum coherence in the least extent. In addition, one-photon quantum states can be easily manipulated by using only linear optical elements. The main disadvantage of the optical approach is the complexity of realisation of two-qubit gates on which any quantum calculations are based. A natural solution is the use of the interaction of photons in nonlinear media. However, the efficiency of traditional nonlinear optical effects such as second harmonic generation is too low for producing the conditional quantum dynamics of photons [1 – 5]. It was shown in [6] that quantum computations can be performed by using only linear optical elements, additional photons, and one-photon detectors. The concept of linear optical quantum computers attracted great interest both in theoretical and experimental aspects [7 – 11]. However, is should be noted that within the framework of linear optics, only nondeterministic two-qubit gates are obtained, for which the failure probability can be decreased by increasing the number of additional photons (see, for example, [12]).

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One of the possible ways to increase the efficiency of optical quantum computations is the use of quantum memory devices capable of writing and reproducing the quantum states of an electromagnetic field. It was proposed, in particular, to use optically dense impurity crystals as carriers of quantum information and to perform information writing and reading by using electromagnetically induced transparency [13 – 15] or photon echo [16 – 19]. In this case, the conditional quantum dynamics of two photons can be realised during the storage of information on their quantum state in an ensemble of impurity centres [14]. In this paper, we analyse the possibility of realisation of optical quantum computations by means of the subradiant states of a quantum optical memory device.

2. Superradiant and subradiant states of a system of two-level atoms

In his basic paper [20] Dicke considered the two regimes of collective spontaneous radiation of photons: superradiance and subradiance, which result from the constructive and destructive interference of atomic states, respectively. In the first case, a system of inverted atoms undergoes the spontaneous transition to the ground state for the time inversely proportional to the number of atoms, while in the second case the rate of collective spontaneous radiation, on the contrary, decreases compared to the rate of spontaneous radiation of single atoms. In the ideal case of a localised system occupying a region with the characteristic size smaller than the radiation wavelength, the rate of collective spontaneous radiation of photons is zero if atoms are in the antisymmetric collective state. If a system of atoms is extended, the rate of collective spontaneous radiation can be suppressed only for individual radiation modes of the medium. In particular, single-mode subradiance can be observed in samples having a prolate shape characterised by the Fresnel number $N_F \sim 1$. The subradiance process was analysed in detail in papers [21 – 25].

Consider a system consisting of N identical two-level atoms interacting with a single-mode electromagnetic field. Let us denote the ground and excited states of the jth atom by $|0_j\rangle$ and $|1_j\rangle$, respectively. Here, as usual in a single-mode model, is assumed that the atomic states are defined taking the delay into account, i.e., they contain the phase factor $\exp(i\boldsymbol{k}\boldsymbol{r}_j)$, where $\boldsymbol{k}$ is the wave vector of the mode and $\boldsymbol{r}_j$ is the radius vector of the jth atom [26]. During absorption of photons, the atomic system undergoes successive transitions from the initial $|0_N\rangle = |0, ..., 0\rangle$ state to symmetric superradiant Dicke states

$$|1_N\rangle = \frac{1}{\sqrt{N}}\sum_i |0, ..., 1_i, ..., 0\rangle, \quad (1)$$

$$|2_N\rangle = \left[\frac{2}{N(N-1)}\right]^{1/2} \sum_{j>i} |0, ..., 1_i, ..., 1_j, ..., 0\rangle \text{ etc.}, \quad (2)$$

the rate of collective spontaneous emission or absorption of photons being increased during excitation of the medium. Thus, if $1/T_1$ is the rate of spontaneous transition in a single atom, then the spontaneous emission of a photon from the $|1_N\rangle$ state occurs with the rate N/T_1, from the $|2_N\rangle$ state with the rate $2(N-1)/T_1$, etc.

We assume now that atoms have the additional level $|2\rangle$ and the transition frequency ω_{12} differs from ω_{01}, while the spectral width $\Delta\omega_{12}$ of the transition considerably exceeds the spectral width $\Delta\omega_{01}$ (Fig. 1). Then, irradiation by a short coherent 2π pulse at the frequency ω_{12} can almost instantly change the phase of the $|1\rangle$ state, thereby changing the phase of the collective $|1_N\rangle$ state to the opposite one, while the phase of the $|2_N\rangle$ state remains invariable.

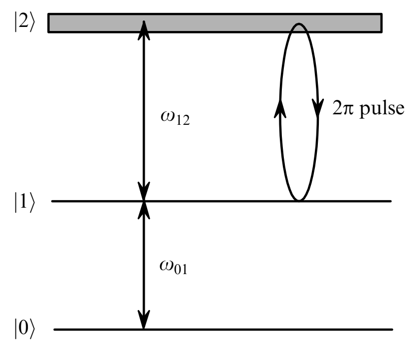

Figure 1. Scheme of the operating atomic levels for preparing subradiant collective states.

Let us divide a sample into two parts, A and B, each containing $N/2$ atoms. It is convenient to represent superradiant states (1) and (2) in the form

$$|1_{A+B}\rangle = \frac{1}{\sqrt{2}}(|1_A, 0_B\rangle + |0_A, 1_B\rangle), \quad (3)$$

$$|2_{A+B}\rangle = \frac{1}{2(N-1)^{1/2}}\{(N-2)^{1/2}[|2_A, 0_B\rangle + |0_A, 2_B\rangle] + \sqrt{2N}|1_A, 1_B\rangle\}. \quad (4)$$

Upon irradiation of atoms in one of the parts, for example B, by a coherent 2π pulse, the atomic system will pass from states (3) and (4) to the antisymmetric single-mode subradiant states

$$|1_{A-B}\rangle = \frac{1}{\sqrt{2}}(|1_A, 0_B\rangle - |0_A, 1_B\rangle), \quad (5)$$

$$|2_{A-B}\rangle = \frac{1}{2(N-1)^{1/2}}\{(N-2)^{1/2}[|2_A, 0_B\rangle + |0_A, 2_B\rangle] - \sqrt{2N}|1_A, 1_B\rangle\}. \quad (6)$$

The rate of photon emission from state (5) is identically zero, while that from state (6) is $(2/T_1)(N-1)^{-1}$, i.e., tends to zero at large N.

Let us assume now that the atomic system in state (5) absorbs a photon and passes to the state

$$|2'_{A-B}\rangle = \frac{1}{\sqrt{2}}(|2_A, 0_B\rangle - |0_A, 2_B\rangle). \quad (7)$$

Note at once that the rate of photon emission in this case is $(N-2)/T_1$, i.e., coincides in fact with the rate of spontaneous decay of the $|1_N\rangle$ state at large N. To convert state (7) to the subradiant state, it is necessary to irradiate again by the 2π pulse half the atoms, whose phase has changed after absorption of the first photon, and another half the atoms, whose phase has not changed. As a result, the atomic system will be divided into four equal parts, which we denote as C, D, E, and F, while the state of the system will take the form

$$|2_{C-D+E-F}\rangle = \frac{1}{\sqrt{2}}(|2_{C-D}, 0_{E+F}\rangle - |0_{C+D}, 2_{E-F}\rangle). \quad (8)$$

Therefore, by irradiating the system of atoms by 2π pulses at the additional transition after each absorption of a photon, we can transfer it to the higher excited subradiant state, the spontaneous transition from this state to the previous subradiant state becoming forbidden. The action of the reverse sequence of 2π pulses will result in the emission of absorbed photons in the reverse order. It is important to note here that the rates of absorption and emission of photons in such a regime almost do not change upon excitation or deexcitation of the medium.

3. Quantum memory in subradiant states

Let us assume for definiteness that the writing and reproduction of quantum states of light is performed in the regime of the external-field-controlled inhomogeneous broadening of the resonance $|0\rangle - |1\rangle$ optical transition of the atomic system [19] (Fig. 2). As qubits, it is convenient to use one-photon wave packets with double-humped tempo-

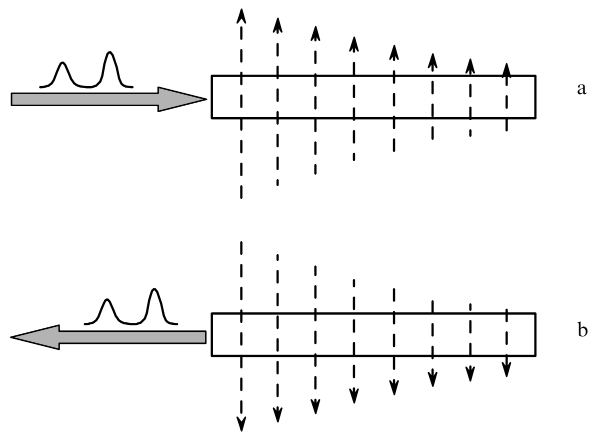

Figure 2. Scheme of the operation of the quantum optical memory proposed in [19]. The dashed arrows are the external inhomogeneous electric field producing the inhomogeneous broadening of the resonance transition. The change in the field direction to opposite leads to the transition from the writing of quantum states of the field (a) to their reproduction (b). The reproduced field is reversed in time and propagates in the opposite direction.

ral shape, which are used in quantum cryptography systems [27]. The minimum duration of the one-photon wave packet, whose state can be transferred to the atomic system, is inversely proportional to the inhomogeneous broadening Γ_i of the resonance line, while the maximum length of the sequence of such packets is inversely proportional to the homogeneous broadening Γ_h. Therefore, the information capacity of a quantum memory device is determined by the ratio Γ_i/Γ_h. Note that despite the inhomogeneous broadening, absorption and emission of photons by the atomic system have a collective nature. This means that during absorption of photons and excitation of the medium, the rate of spontaneous transition, i.e., the homogeneous broadening of the resonance line, increases. As a result, upon the successive writing of one-photon states, the conditions required for the reproduction of their temporal shape are violated. This restriction can be eliminated, if during the writing and reproduction of photons the resonance medium is excited to subradiant states, whose formation, as pointed out above, is accompanied by the narrowing of the resonance line.

We assume that 'proportions' of the atomic system are characterised by the Fresnel number $N_F \sim 1$, so that the collective interaction of atoms with the quantum field takes place only for longitudinal modes, with respect to which the atomic system is an optically dense medium. The interaction of atoms with transverse electromagnetic modes is not collective, so that they can be used to control the phase of atomic states by means of classical coherent 2π pulses (Fig. 3a).

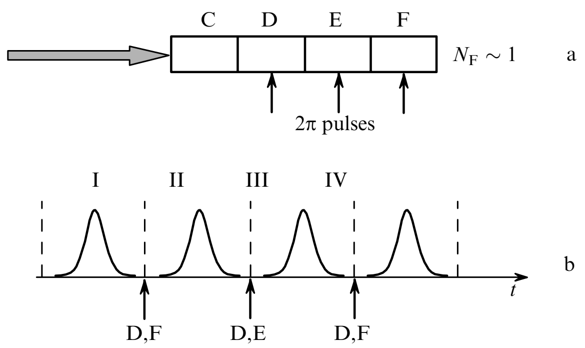

Figure 3. Geometry of the action of 2π pulses preparing the subradiant states of an extended medium characterised by the Fresnel number $N_F \sim 1$ (a), and the temporal order of the action of 2π pulses (b) upon writing the quantum state of two qubits. Upon reading, the 2π pulses act in the reverse order.

The process of writing a two-qubit state can be divided into four time intervals I–IV (Fig. 3b), two states for each qubit. Between the intervals, the atomic system is subjected to 2π pulses, which transfer it to different subradiant states. The first pulse acts on the regions D and F, the second one on the regions D and E, the third one again on the regions D and F. Depending on the initial state of photons, i.e., on the time interval they get into, the evolution of the state of the atomic system during the writing of one-photon states looks as follows

$$\begin{aligned} \text{I, II:}\quad & \frac{1}{\sqrt{2}}[|1_{C+E}0_{D+F}\rangle - |0_{C+E}1_{D+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-E}0_{D+F}\rangle - |0_{C+E}2_{F-D}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-E}0_{D+F}\rangle - |0_{C+E}2_{D-F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-E}0_{D+F}\rangle - |0_{C+E}2_{D-F}\rangle], \end{aligned} \tag{9}$$

$$\begin{aligned} \text{I, III:}\quad & \frac{1}{\sqrt{2}}[|1_{C-D}0_{E+F}\rangle + |0_{C+D}1_{E-F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+D}0_{E+F}\rangle - |0_{C+D}1_{E+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-D}0_{E+F}\rangle - |0_{C+D}2_{E-F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-D}0_{E+F}\rangle - |0_{C+D}2_{E-F}\rangle], \end{aligned} \tag{10}$$

$$\begin{aligned} \text{I, IV:}\quad & \frac{1}{\sqrt{2}}[|1_{C-D}0_{E+F}\rangle + |0_{C+D}1_{E-F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+D}0_{E+F}\rangle - |0_{C+D}1_{E+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+F}0_{D+E}\rangle - |0_{C+F}1_{D+E}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C+F}0_{D+E}\rangle - |0_{C+F}2_{D+E}\rangle], \end{aligned} \tag{11}$$

$$\begin{aligned} \text{II, III:}\quad & \frac{1}{\sqrt{2}}[|0_{C+D}0_{E+F}\rangle + |0_{C+D}0_{E+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+F}0_{D+E}\rangle - |0_{C+F}1_{D+E}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-F}0_{D+E}\rangle - |0_{C+F}2_{D-E}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C-F}0_{D+E}\rangle - |0_{C+F}2_{D-E}\rangle], \end{aligned} \tag{12}$$

$$\begin{aligned} \text{II, IV:}\quad & \frac{1}{\sqrt{2}}[|0_{C+D}0_{E+F}\rangle + |0_{C+D}0_{E+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+F}0_{D+E}\rangle - |0_{C+F}1_{D+E}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+D}0_{E+F}\rangle - |0_{C+D}1_{E+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C+D}0_{E+F}\rangle - |0_{C+D}2_{E+F}\rangle], \end{aligned} \tag{13}$$

$$\begin{aligned} \text{III, IV:}\quad & \frac{1}{\sqrt{2}}[|0_{C+E}0_{D+F}\rangle + |0_{C+E}0_{D+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|0_{C+E}0_{D+F}\rangle + |0_{C+E}0_{D+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|1_{C+E}0_{D+F}\rangle - |0_{C+E}1_{D+F}\rangle] \rightarrow \\ & \rightarrow \frac{1}{\sqrt{2}}[|2_{C+E}0_{D+F}\rangle - |0_{C+E}2_{D+F}\rangle]. \end{aligned} \tag{14}$$

In these expressions, the states are indicated which are obtained at the end of each time interval immediately after the 2π pulse action.

The subsequent reversal of the external electric field producing the inhomogeneously broadened line will result in the emission of photons in the opposite direction and in the reverse temporal order, but only if the atomic system is subjected to the reverse sequence of 2π poulses.

4. Nonlinear sign-shift gate in the optical subradiance regime

Consider the scheme of a nonlinear sign-shift two-qubit gate with the use of subradiant states (Fig. 4). Let us assume that the states of qubits are a linear superposition of two one-photon states corresponding to different propagation directions of signals (the so-called spatial coding). To write and reproduce the states of such qubits, it is necessary to use samples with the Fresnel number $N_F \gg 1$. First the state of one qubit is written (the superposition of the states $|0_1\rangle$ and $|1_1\rangle$). Then, the atomic system is excited to the subradiant state by irradiating half the atomic system by a short 2π pulse. As a result, the homogeneous line broadening before absorption of the second photon will be equal to the initial broadening. Then, the state of the second qubit is written (the superposition of the states $|0_2\rangle$ and $|1_2\rangle$) so that the propagation directions of the first photon in the $|1_1\rangle$ state and the second photon in the $|1_2\rangle$ state would be coincident. If now all the atoms are subjected to the additional 2π pulse, the phase of collective atomic states is conjugated in all cases except the only case when qubits are in the $|1_1\rangle|1_2\rangle$ state. Therefore, a nonlinear sign-shift gate can be realised in the regime of optical subradiance.

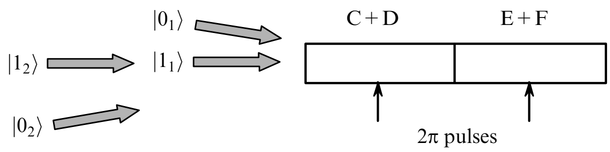

Figure 4. Excitation geometry of a resonance medium for realising a nonlinear sign-shift quantum gate.

5. Conclusions

The problem of eliminating the superradiant broadening of the resonance line in systems of quantum optical memory is important in writing a sequence of one-photon states of the electromagnetic field. Excitation of the atomic system to the collective subradiant state allows one to increase the length of the written sequence, thereby increasing the capacity of the optical quantum memory. The scheme for the preparation of different subradiant states in an extended optically dense medium can be easily generalised to the case of writing and reproduction of the states of many photons. In this case, the action of 2π pulses can be described by using Hadamard matrices. In particular, the example with two 'double-humped' photons considered above corresponds to the fourth-order Hadamard matrix. Note also that the problem of increasing homogeneous broadening during excitation of a medium is also inherent in the classical spectrally selective optical memory based on photon echo [28].

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References

1. Turchette Q.A., Hood C.J., Lange W., Mabuchi H., Kimble H.J. *Phys. Rev. Lett.*, **75**, 4710 (1995).
2. Franson J.D. *Phys. Rev. Lett.*, **78**, 3852 (1997).
3. Kim Y.H., Kulik S.P., Shih Y. *Phys. Rev. Lett.*, **86**, 1370 (2001).
4. Resch K.J., Lundeen J.S., Steinberg A.M. *Phys. Rev. Lett.*, **87**, 123603 (2001).
5. Resch K.J., Lundeen J.S., Steinberg A.M. *Phys. Rev. Lett.*, **89**, 037904 (2002).
6. Knill E., LaFlamme R., Milburn G.J. *Nature*, **409**, 46 (2001).
7. Pittman T.B., Jacobs B.C., Franson J.D. *Phys. Rev. Lett.*, **88**, 257902 (2002).
8. Sanaka K., Kawahara K., Kuga T. *Phys. Rev. A*, **66**, 040301(R) (2002).
9. Pittman T.B., Fitch M.J., Jacobs B.C., Franson J.D. *Phys. Rev. A*, **68**, 032316 (2003).
10. O'Brien J.L., Pryde G.J., White A.G., Ralph T.C., Branning D. *Nature*, **426**, 264 (2003).
11. Zhao Z., Zhang A.-N., Chen Y.-A., et al. *Phys. Rev. Lett.*, **94**, 030501 (2005).
12. Franson J.D., Donegan M.M., Fitch M.J., et al. *Phys. Rev. Lett.*, **89**, 137901 (2004).
13. Fleischhauer M., Yelin S.F., Lukin M.D. *Opt. Commun.*, **179**, 395 (2000).
14. Lukin M.D., Fleischhauer M., Cote R., et al. *Phys. Rev. Lett.*, **87**, 037901 (2001).
15. Lukin M.D. *Rev. Mod. Phys.*, **75**, 457 (2003).
16. Kessel' A.P., Moiseev S.A. *Pis'ma Zh. Eksp. Teor. Fiz.*, **58**, 77 (1993).
17. Moiseev S.A., Kröll S. *Phys. Rev. Lett.*, **87**, 173601 (2001).
18. Ohlsson N., Nilsson M., Kröll S. *Phys. Rev. A*, **68**, 063812 (2003).
19. Kraus B., Tittel W., Gisin N., et al. *arXiv: quant-ph/0502184* (2005).
20. Dicke R.H. *Phys. Rev.*, **93**, 99 (1954).
21. Crubellier A., Liberman S., Pillet P. *Opt. Commun.*, **32**, 143 (1980).
22. Crubellier A., Liberman S., Pavolini D., Pillet P. *J. Phys. B*, **18**, 3811 (1985).
23. Crubellier A., Pavolini D. *J. Phys. B*, **19**, 2109 (1986).
24. Crubellier A. *J. Phys. B*, **20**, 971 (1987).
25. Crubellier A., Pavolini D. *J. Phys. B*, **20**, 1451 (1987).
26. Andreev A.V., Emel'yanov V.I., Il'inskii Yu.A. *Koopertaivnye yavleniya v optike* (Cooperative Phenomena in Optics) (Moscow: Nauka, 1988).
27. Gisin N., Ribordy G., Tittel W., Zbinden H. *Rev. Mod. Phys.*, **74**, 145 (2002).
28. Johnson A.E., Maniloff E.S., Mossberg T.W. *Opt. Lett.*, **24**, 1526 (1999).