WAVEGUIDES

PACS numbers: 42.82.Et
DOI: 10.1070/QE2005v035n10ABEH010358

Propagation of light in a cylindrical system of tunnel-coupled waveguides

J.Kh. Nurligareev, K.M. Golant, V.A. Sychugov, B.A. Usievich

Abstract. **Propagation of light in a cylindrical system of tunnel-coupled waveguides is studied. The waveguide system was constructed by layer-by-layer deposition of waveguides on the inner wall of a reference tube by the SPCVD technique developed for synthesising fibre preforms. It is shown that the waveguide light beam in such a system moves along a helical wave-like trajectory and is partially emitted outside at its 'crests'. The angular dependence of the trajectory period and of the number of oscillations along the trajectory is measured.**

Keywords: *dielectric waveguide, tunnel coupling, light beam.*

1. Introduction

The interest towards the problem of propagation, amplification and generation of light in a tunnel-coupled waveguide system has grown considerably during recent years. This is mainly due to the practical need for increasing the power of fibre and semiconductor lasers [1, 2]. However, even passive devices based on a system of channel waveguides require a deep understanding of the peculiarities of light propagation in them [3, 4]. Channel waveguide systems can be divided into two categories: homogeneous and inhomogeneous. The former have been studied quite comprehensively, while the latter kind of waveguides have not been studied so thoroughly as yet, and were reported in only three publications [5 – 7]. The present paper aims at studying the propagation of light in a coupled system of cylindrical waveguides and at demonstrating directly the Bloch oscillations of light emerging in them.

2. Inhomogeneous system of coupled waveguides

A channel waveguide system is called inhomogeneous if the light propagation constant in individual waveguides varies from one channel waveguide to another according to a certain law. In the simplest case that will be considered here, this dependence is linear, which means that the increment $\Delta\beta = \gamma$ of the propagation constant remains unchanged as we pass from one waveguide to another.

It was found in [5, 6] that light coupled into one channel of an inhomogeneous system does not spread to other channels during its propagation over the system (unlike in a homogeneous system of channel waveguides) and remains localised within a few waveguides W ($W \approx 8\chi/\gamma$, where χ is the coupling constant between the waveguides). Moreover, light in such a system is gathered once again into the initially excited waveguide at distances $z_0 = 2\pi/\gamma$, $2z_0$, $3z_0, \ldots$ from the waveguide entrance.

Such a propagation of light is due to the fact that an inhomogeneous system of channel waveguides with c $\Delta\beta = \gamma = \text{const}$ is characterised by an equidistant set of eigenmodes whose interference generates the observed picture.

Two methods of realising an inhomogeneous system of channel waveguides were proposed in [5, 6] and the focusing of light in it was demonstrated experimentally. Here we would like to draw attention to the simplest method of realising such a system of waveguides. In this method, (single-mode) waveguides with identical parameters are arranged equidistantly on concentric circles with a quite large radius (Fig. 1). Assuming that the propagation constants in these waveguides are identical and all waveguides start from one radius of curvature of the system and end at the other, the signal at the system output will experience a phase delay in the case of in-phase excitation during a transition from one waveguide to another. This can be treated as the delay associated with a variation in the light propagation constant in each waveguide relative to the neighbouring waveguide.

Such an approach towards the problem of light propagation in a curvilinear system of channel waveguides makes it possible to obtain a relation establishing the equivalence between an inhomogeneous system of rectilinear waveguides and a system of identical curvilinear waveguides. This relation has the form [7] $\Delta\beta l \approx \beta\Delta l$, where

$$\Delta\beta = k\Delta n^* = kn^*\frac{\Delta R}{R}. \qquad (1)$$

Here, R is the radius of curvature of the waveguide with an effective refractive index n^*; ΔR, and Δn^* are the variations in the radius of curvature of the neighbouring waveguide and its effective refractive index, respectively; $k = 2\pi/\lambda$; where λ is the wavelength of light. Formula (1) leads to an expression for the focusing length of light in a curvilinear system of waveguides excited by a point source:

J.Kh. Nurligareev, V.A. Sychugov, B.A. Usievich Laser Materials and Technology Research Center, A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia; e-mail: borisu@kapella.gpi.ru;
K.M. Golant Fiber Optics Research Center, A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia

Received 16 June 2005
Kvantovaya Elektronika **35** (10) 917 – 920 (2005)
Translated by Ram Wadhwa

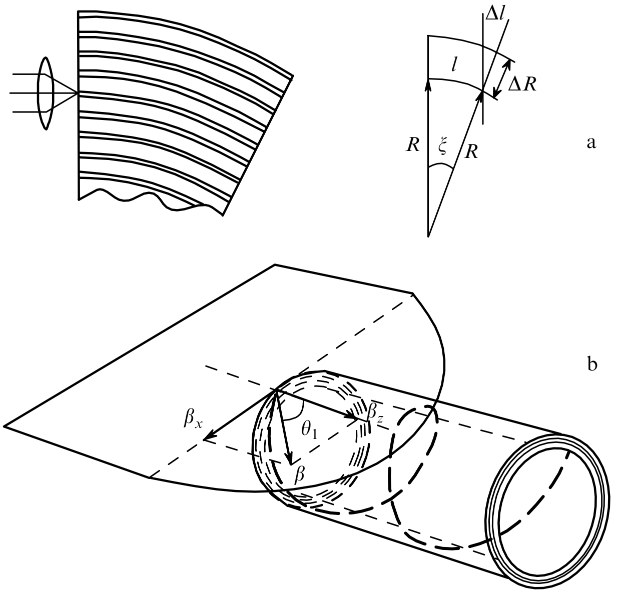

Figure 1. (a) Circular system of coupled waveguides (ξ is the angular size of the waveguide of length l and Δl is the increment in the arc length upon a transition from one waveguide to another); (b) propagation of a light beam along a helical trajectory and the system of coordinates defining this trajectory.

$$z_0 = \frac{2\pi}{\gamma} = \frac{R\lambda}{n^*\Delta R} = \frac{R\lambda}{n^*\Lambda}, \tag{2}$$

where $\Lambda = \Delta R$ is the waveguide period in the structure.

The above expression for z_0 coincides with the one obtained in [8], where the problem of light propagation in a curvilinear system of channel waveguides was solved rigorously.

So far, we have considered an extremely narrow light beam entering the waveguide system and its propagation along it. However, the propagation of broad light beams in an inhomogeneous system of coupled waveguides is also quite interesting for a detailed understanding of the process. Calculations show [9] that the trajectory of propagation of such beams is undulating in nature (Fig. 2).

It should be noted that the light pulsation length $z_0 = 2\pi/\gamma$ is equal to the 'wavelength' of the undulating trajectory of the light beam. Hence, while considering broad light beams below, we shall be speaking of its pulsation length, keeping in mind the 'wavelength' of the undulating trajectory of the light beam.

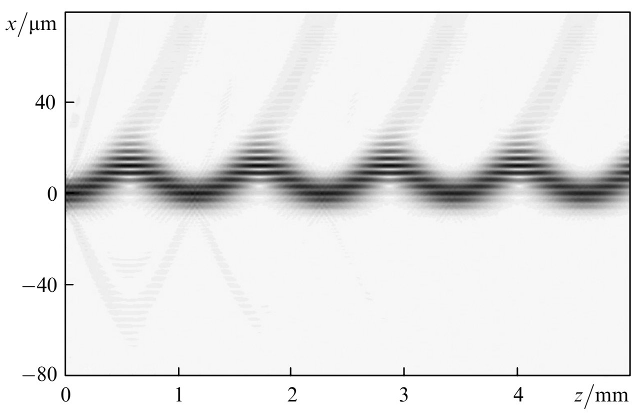

Figure 2. Trajectory of a Guassian beam propagating in a circular system of coupled waveguides.

A homogeneous system of channel waveguides was prepared by depositing 50 pairs of SiO_2 and SiON layers with a refractive index difference $\Delta n = 5 \times 10^{-3}$ on the inner surface of a quartz tube. The waveguide layer thickness was 2 μm, and the separation between the layers was 1 μm. A 50-μm thick layer of SiO_2 was deposited over the last waveguide. The system of waveguides was prepared by the surface plasma chemical vapour deposition (SPCVD) technique developed in [10] for preparing fibre preforms.

Since the method used by us for realising an inhomogeneous system allows us to obtain a system of optically coupled cylindrical waveguides, propagation of light over their cylindrical surface is an interesting subject of investigation. In particular, this pertains to angular dependences of the period and number of light beam pulsations. Fig. 1b shows the helical trajectory of the beam and the coordinate system connected with it.

According to the figure, the longitudinal (i.e., directed along the z axis of the cylinder) component of the light propagation constant β_z is equal to $kn^* \cos\theta_1$ (θ_1 is the angle at which the beam propagates relative to the cylinder axis), while the transverse component β_x is equal to $kn^* \sin\theta_1$. In the adjoining waveguide with a larger radius of curvature, we have

$$\tilde{\beta}_x = kn^* \sin\theta_1 \left(1 + \frac{\Delta R}{R}\right), \quad \tilde{\beta}_z = \beta_z,$$

which leads to the expression

$$\tilde{\beta} = kn^* \left[\cos^2\theta_1 + \sin^2\theta_1 \left(1 + \frac{\Delta R}{R}\right)^2\right]^{1/2}. \tag{3}$$

Considering that $\Delta R/R \ll 1$, we finally obtain after simplification

$$\tilde{\beta} = kn^* \left(1 + \sin^2\theta_1 \frac{\Delta R}{R}\right). \tag{4}$$

The increment in the propagation constant for the adjoining waveguide during propagation of light along a helical trajectory can be presented in the form

$$\Delta\beta = \tilde{\beta} - \beta \approx \beta \frac{\Delta R}{R} \sin^2\theta_1. \tag{5}$$

This increment determines the angular dependence of the pulsation length z_0:

$$z_0(\theta_1) = \frac{\lambda R}{n^*\Delta R \sin^2\theta_1} = \frac{\lambda R}{n^*\Lambda \sin^2\theta_1}. \tag{6}$$

The variation of the pulsation length as a function of the angle of propagation of a light beam over the cylindrical surface leads to a change in the number P of pulsations along the entire length of the trajectory, i.e., in the pattern of emission of a beam at the output. This question was considered by us earlier in [11]. Here, we would like to mention that the change in the pattern of emission of a beam at the output is associated with a change in the phase distribution of the wave across the light beam. It should be recalled that the patterns of beam emission at the output are

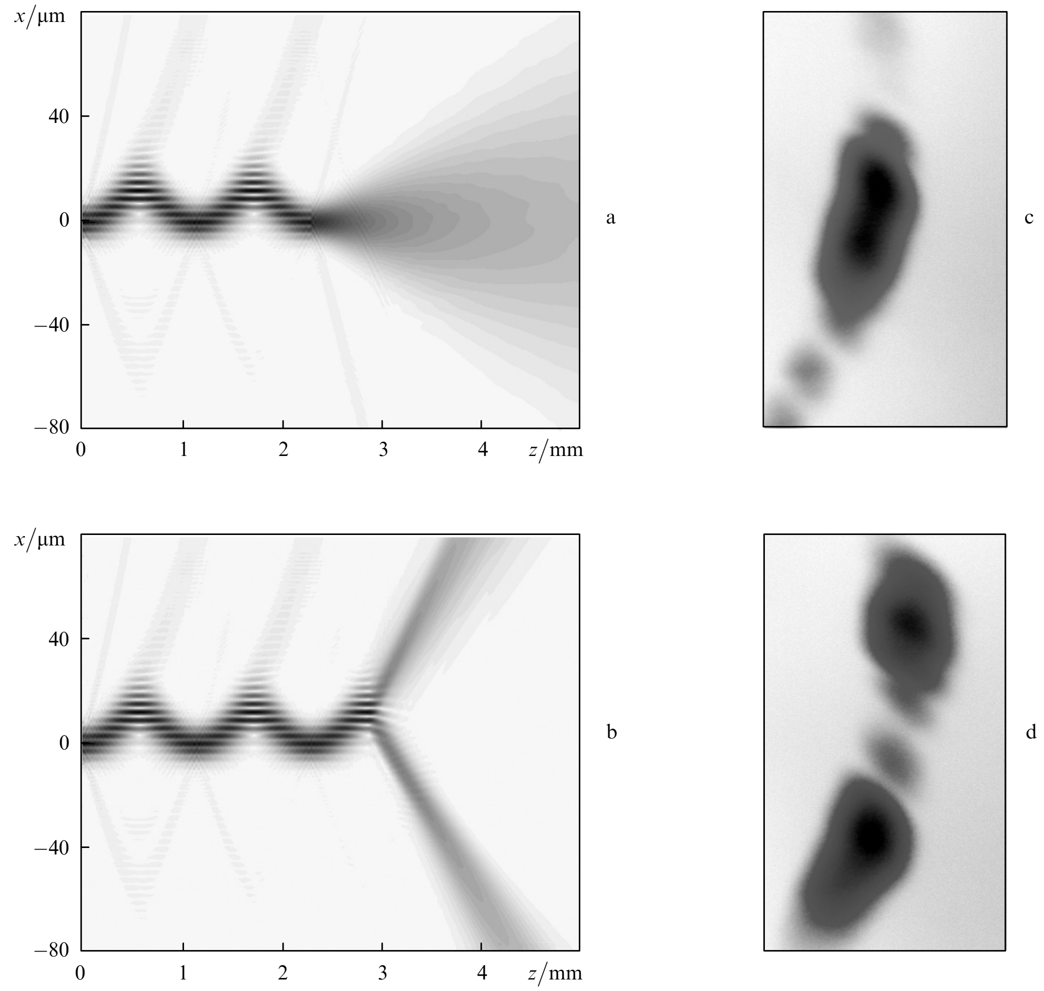

Figure 3. Radiation pattern at the output of a cylindrical system of coupled waveguides for (a) a light beam trajectory containing an integral number of pulsations ($L_1 = Pz_0$) and (b) a trajectory containing a half-integral number of pulsations [$L_2 = (P + \frac{1}{2})z_0$], as well as photographs of radiation patterns observed in far field for an integral (c) and half-integral (d) number of pulsations.

quite different (Fig. 3) for integral and half-integral numbers of pulsations P over the entire beam trajectory. This circumstance enables one to reliably characterise the beam trajectory

3. Experiment

We used two types of samples in our experiments: cylindrical samples of length $L = 30$ mm with a waveguide system at the inner surface of the tube, as well as samples of the same length cut into half along the generatrix of the tube. The inner diameter of the quartz tube in both cases was 16 mm. The end-faces of the samples were polished in order to allow light to enter the waveguide layers. The purpose of the experiment was to reveal the existence of light pulsations inside the waveguide system and to measure the angular dependences of their number and length z_0.

Figure 4a shows the experimental setup with a cylindrical sample. Radiation from a He–Ne laser was focused on the entrance face of the waveguide with the help of an optical system ensuring an input Gaussian beam diameter of 30 μm. The radiation pattern (Bragg's diffraction of light) was observed on the screen behind the exit face of the cylindrical sample. This pattern was an alternating distribution of light or was in the form of two spots having the

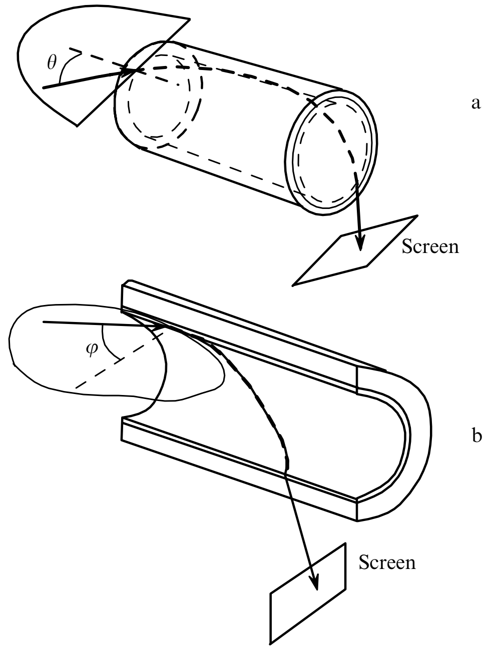

Figure 4. Experimental setup with (a) cylindrical and (b) semicylindrical geometries of samples in the waveguide system.

same intensity (in this case it characterised the waveguide beam trajectory with a half-integral number of pulsations) or a single broader spot characteristic of an integral number of pulsations. Photographs of the radiation patterns observed in the far field and characterising the trajectory of a light beam with integral and half-integral numbers of pulsations are presented in Figs 3c and d.

Transition from one light beam trajectory to another inside a waveguide system occurs as a result of variation of the angle θ at which light enters the waveguide system, this variation being caused by using a G5M goniometer. Upon a variation of the beam entrance angle from 0 to 90°, the angular dependence of the number of pulsations could be measured in the angular interval $0 < \theta_1 < 42°$. The first trajectory observed by us was characterised by the number of pulsations $P = 0.5$ and the last one corresponded to $P = 12$.

Note also that the light beam trajectory in the waveguide system may be visually registered in a dark room and the number of pulsations can be counted on this trajectory. This is just what we did in our measurements.

The number of pulsations obtained in the experiment was compared with the number of pulsations estimated according to the formula

$$P = \frac{L_s}{z_0} = \frac{Ln^* \Lambda}{\lambda R} \tan\theta_1 \sin\theta_1, \qquad (7)$$

where $L_s = L/\cos\theta_1$ is the distance along the beam trajectory.

Figure 5 shows the dependence $P(\theta_1)$ as well as the angular dependence of the pulsation period z_0.

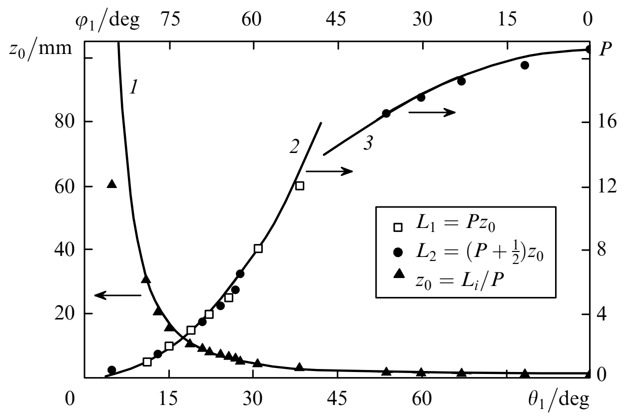

Figure 5. Angular dependences of the pulsation period z_0 [curve (*1*)] and the number of pulsations P [curves (*2*) and (*3*)] obtained by calculations (solid curves) and experiments (symbols).

By using semicylindrical waveguide samples formed by cutting the tubes along their length, we can measure the angular dependence of the number of pulsations P in a slightly different range of angles. This is due to the fact that the angle φ_1 is connected with the angle θ_1 through the equality $\varphi_1 + \theta_1 = \pi/2$ (see Fig. 4b). In this case, the number of pulsations is defined by the relation

$$P = \frac{ln^* \Lambda}{\lambda R} \cos\varphi_1, \qquad (8)$$

where l is the length of the waveguide sample across the semicylindrical tube. The angular dependence of the pulsation length is defined as

$$z_0(\varphi_1) = \frac{\lambda R}{n^* \Lambda \cos^2\varphi_1}. \qquad (9)$$

Figure 5 shows the experimental results of measurement of angles of excitation of light beam trajectory in a waveguide with a half-integral number of pulsations P. The angular dependence of the pulsation period z_0 [curve (*1*)] is determined only by the identity of samples in the waveguide systems used in our experiments (samples with waveguides were cut from the same tube). Hence, this dependence is shown by a single curve [curve (*1*)] in Fig. 5, while the angular dependence of the number of pulsations P is different in different experiments and is therefore shown by two curves [curve (*2*) and curve (*3*)] which do not pass into each other.

However, the experimentally obtained dependences shown by curves (*2*) and (*3*) prove unambiguously that the pulsating nature of a helical trajectory is determined by the motion of a beam of light in the azimuthal direction during its propagation at an angle θ_1 or φ_1 to the generatrix of the cylindrical surface of the waveguide system.

4. Conclusions

Thus, the analysis of the motion of broad light beams along an inhomogeneous system of waveguides with $\Delta\beta = \gamma =$ const clearly reveals their undulating trajectories and allows an experimental measurement of the angular dependence of the period z_0 of light pulsations along these trajectories and their number P.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Project No 03-02-16266-a).

References

1. Beach R.J., Feit M.D., Page R.H., Brasure L.D., Wilcox R., Payne S.A. *J. Opt. Soc. Am. B*, **19** (7), 1521 (2002).
2. Botez D., Napartovich A.P. *IEEE J. Quantum Electron.*, **30**, 975 (1994).
3. Smit M.K. *IEEE J. Selected Topics in Quantum Electron.*, **2**, 236 (1996).
4. Abeeluck A.K., Litchinister N.M., Headley C., Eggleton B.J. *Opt. Express*, **10** (23), 1320 (2002).
5. Pertsch T., Dannberg P., Elflein W., Brauer A. *Phys. Rev. Lett.*, **83**, 4752 (1999).
6. Morandotti R., Perschel U., Aitchinson J.S., Eiesenberg H.S., Silberberg Y. *Phys. Rev. Lett.*, **83**, 4756 (1999).
7. Usievich B.A., Sychugov V.A., Nurligareev J.Kh., Golant K.M., Goncharov A.A., Svidzinskii K.K. *Kvantovaya Elektron.*, **34**, 371 (2004) [*Quantum Electron.*, **34**, 371 (2004)].
8. Lenz G., Talanina I., Martijn de Sterke C. *Phys. Rev. Lett.*, **83**, 963 (1999).
9. Peschel U., Pertsch T., Lederer F. *Opt. Lett.*, **23** (21), 1701 (1998).
10. Pavy D., Moisan M., Saada S., Chollet P., et al. *Proc. XII Europ. Conf. on Opt. Commun.* (Barcelona, 1986) pp 19–22.
11. Usievich B.A., Sychugov V.A., Nurligareev D.Kh., Golant K.M. *Opt. Spektrosk.*, **97** (5), 841 (2004).