

CONTROL OF LASER RADIATION PARAMETERS

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Method for calculating a negative-dispersion resonator-type multilayer mirror

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Abstract. **A method is proposed for the calculation of negative-dispersion mirrors with resonator cavities. The mirror optimisation algorithm combines the capabilities of the gradient method and the random search method. A multilayer mirror structure with a reflectivity $R > 99.9$ % and a group delay dispersion of -60 ± 10 fs^2 in the 930 – 1070 nm wavelength range was calculated. The sensitivity of the obtained structure to random variations of layer thicknesses was analysed.**

Keywords: *femtosecond optics, negative-dispersion multilayer mirrors.*

1. Introduction

Multilayer dielectric mirrors are compact elements employed for efficient dispersion control in femtosecond laser systems. Dielectric mirrors with anomalous (negative) dispersion allow compensating for the positive dispersion of the active medium, which is responsible for laser pulse broadening. The use of multilayer mirrors for the compensation of positive dispersion enables obtaining pulses as short as 5 fs [1].

The dispersion characteristic of an element is described by the dependence of the group delay $G(\omega) = \mathrm{d}\phi(\omega)/\mathrm{d}\omega$ on the radiation frequency ω and/or by the frequency dependence of the group delay dispersion $D(\omega) = \mathrm{d}^2\phi(\omega)/\mathrm{d}\omega^2$, where $\phi(\omega)$ is the phase shift after reflection. The most typical task involves production of a reflective coating (a mirror) with a constant negative group delay dispersion within a specified wavelength range.

At present there are two main approaches to the design of multilayer dispersion mirrors. The first consists in the calculation of multilayer structures wherein the penetration depth of a wave component is frequency-dependent [1 – 3]. These structures are actually Bragg reflectors with a period decreasing towards the substrate. In this case, the wave component experiences reflection at some depth where the local period is equal to a quarter of the corresponding wavelength. Low-frequency components are reflected from the external layers of the multilayer coating and experience practically no delay, while high-frequency components are reflected from the inner layers and traverse a substantial optical path to acquire a long time delay. These structures have come to be known as chirped mirrors. The second approach involves insertion of coupled resonator cavities, similar to the Gire – Tournois interferometer (GTI), inside a multilayer structure [1, 4]. The microresonators 'capture' their corresponding wave components to ensure the time delay. By selecting the strength of coupling between the resonator cavities, it is possible to form the desired profile of group delay as a function of frequency.

GTI mirrors are structurally simpler than chirped mirrors and less sensitive to coating deposition errors. Moreover, at present it is no longer necessary to produce dispersion mirrors with a high reflectivity due to the use of new active media. The resonator-type mirrors consist of two independent parts: a quarter-wave reflector, which determine the reflectivity, and the resonator part, which defines the phase properties. That is why the GTI mirror reflectivity is easy to lower by reducing the number of reflector layers, which in turn permits decreasing the total number of the coating layers. This is impossible in the case of a chirped mirror, because its phase characteristic is determined by its entire structure.

Development of resonator-type dispersion mirrors is a relatively new direction. Although they offer significant advantages over chirped mirrors, they have been given little attention in the literature. Many methods of chirped-mirror calculation have been described. For instance, the Kartner – Keller analytical technique was employed to calculate mirrors with $D = -50 \pm 7$ fs^2 in the wavelength range $\lambda = 550 - 700$ nm [3], and the Fourier technique was used to design chirped structures with $D = -41 \pm 10$ fs^2 in the $\lambda = 720 - 890$ nm range [4]. Several papers were devoted to GTI mirror calculation. In [4] structures with two resonator cavities possessing a negative group delay dispersion $D = -40 \pm 10$ fs^2 for $\lambda = 740 - 920$ nm were considered. A three-resonator structure with $D = -50 \pm 1$ fs^2 for $\lambda = 740 - 860$ nm was presented in Ref. [1]. However, the algorithm of calculation was not described in Refs [1, 4].

In this work, we propose and consider in detail a simple and efficient technique for calculating a dispersion mirror with desired phase characteristics. We show how the dispersion mirror structure can be transformed with the aid of simple scaling for the operation in a different

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wavelength range. Two mirror structures with a negative dispersion $D = -60 \pm 10$ fs^2 for $\lambda = 940 - 1120$ nm and -40 ± 7 fs^2 for $\lambda = 765 - 945$ nm are presented. The immunity of resonator-type mirror structure to random deviations of layer thicknesses during deposition is investigated.

2. Method for the calculation of a dispersion resonator-type mirror

Our method for calculating a resonator-type mirror (a GTI mirror) comprises two stages: definition of the initial (basic) structure and its subsequent numerical optimisation. The structures are calculated for normal light incidence.

In the general case, a GTI can be realised with the aid of the following structure: substrate/(HL)^{10}H 2LH/air, where H and L are quarter-wave ($\lambda_0/4$) layers with high and low refractive indices, respectively, and 2L is a resonator half-wave layer ($\lambda_0/2$). One can see that the structure (HL)^{10} is a conventional quarter-wave reflector. Introducing the resonator layer gives rise to a peak at the resonance frequency λ_0 in the wavelength dependence of the group delay. The peak is Lorentzian in shape, and one of its slopes has a tilt corresponding to negative dispersion. However, the wavelength range for which this structure provides negative group delay dispersion is small and is equal to 40–50 nm. In addition, there occurs undesirable higher-order dispersion.

Employing additional resonator cavities permits broadening the operating wavelength range and cancelling higher-order dispersion (forming the desired quadratic profile of the frequency dependence of the group delay). Based on general physical considerations, we assumed that the basic structure consists of two parts. The first part is a conventional reflector made up of quarter-wave layers $\lambda_0/4$. It determines the mirror reflectivity and does not affect its phase characteristics. Increasing the number of reflector layers increases the total reflectivity. The presence of two resonator cavities enables realising a 100-nm wide operating range.

Our aim was to design a dispersion mirror with an operating range 150–200 nm in width. To comply with this requirement, we used a third resonator. The subsequent optimisation of the basic structure involved minimisation of the so-called multiparametric merit function. This function shows to what extent the phase characteristic of the structure under investigation departs from the desired one. For a merit function, we employed the function $M = [\sum_{i=1}^{N} (G_i^{\text{opt}} - G_i)^2]^{1/2}$, where the wavelength range of interest $[\lambda_0, \lambda_{\text{f}}]$ is represented by N points: $\lambda_i = \lambda_0 + (\lambda_{\text{f}} - \lambda_0)i/N$ $(i = 0, \ldots, N)$, G_i^{opt} and G_i are the desired and current group delays at a point λ_i, respectively. The optimised parameters (objective variables) are layer thicknesses, which make up the parameter vector $\boldsymbol{p}$. The layer thickness is expressed as a fraction of the wavelength λ_0. The refractive indices remain invariable. Also defined are the optimisation increment Δd, the wavelength range, the desired wavelength dependence of the group delay, and the admissible deviation of the phase characteristic from the desired one.

The optimisation procedure comprises the elements of the gradient technique and the random search technique. The merit function possesses many local minima. The gradient search provides a rapid convergence to the nearest minimum, while a random variation of the objective variables permits the search to escape from the neighbourhood of a local minimum, if it is not deep enough.

The structure optimisation procedure consists of several iterations. An iteration involves two stages: random variation of objective variables and optimisation of the resultant structure with the simplified gradient descent method (see below). In this case, upon execution of the first stage the merit function may increase, but if it assumes, after the second stage, a value greater than the initial one, the iteration is considered to be unexecuted and there occurs return to the first stage with the initial parameter values. Therefore, random parameter variation allows escaping from the neighbourhood of a local minimum. However, on transition to the neighbourhood of a minimum with a smaller depth (which corresponds to an increase of the merit function after the second stage), the system reverts to the initial state. Let us consider the iteration stages in greater detail.

Random variation of parameters. Such a variation is realised by simulated annealing optimisation technique [5, 6]. A vector $\boldsymbol{u}$ of random numbers, whose components are uniformly distributed over the interval $[-s, s]$, $-s < u_i < s$, is added to the parameter vector $\boldsymbol{p}$. If M_1 and M_2 are the respective values of the merit function for the initial structure and the structure obtained by parameter variation, the random parameter variation is accepted with a probability $P = 1$ for $M_2 < M_1$, $P = \exp[\beta(M_2 - M_1)/M_1^{g}]$ for $1.5M_1 > M_2 > M_1$, and $P = 0$ for $M_2 \geqslant 1.5M_1$. The constant β determines the percentage of the systems which will possess a higher value of the merit function than the previous one. The quantity M_1^{g} shows how difficult it is for the process to escape from the neighbourhood of a local minimum. The variation of parameters is continued until some variation is accepted. Use was made of the following constant values: $\beta = 0.8$, $g = 0.3$. The variation amplitude s is specified depending on the current value of the merit function M: $s = 0.05$ for $M \geqslant 8$, $s = 0.025$ for $4 \leqslant M < 8$, $s = 0.01$ for $2 \leqslant M < 4$, and $s = 0.005$ for $M < 2$. Therefore, the parameters are randomly varied, and the variation amplitude depends on the current value of the merit function: the smaller its value, the lower the amplitude of parameter variation. Permitted in this case are parameter variations accompanied with an increase in the merit function, but the merit function may increase by no more than a factor of 1.5.

Simplified gradient descent method. Calculations are made of the derivatives of the merit function with respect to every objective variable. Unlike the ordinary method of gradient descent, where the entire parameter vector $\boldsymbol{p}$ is changed in the anti-gradient direction, we vary only one, the most sensitive component p_i. The parameter p_i with the highest value of the derivative is varied in the anti-gradient direction with an increment Δd until this lowers the merit function. Then, the calculation of the derivatives is repeated. This procedure continues while the convergence is efficient, i.e., the merit function lowers by more than 5 % upon variation of the next parameter.

3. Calculation of a three-resonator negative-dispersion GTI mirror for $\lambda = 920 - 1160$ nm

The method described above was employed to calculate the structure of a resonator-type dispersion mirror for the $\lambda =$

920 – 1160 nm range with a linear wavelength dependence of the group delay: G(920 nm) = 15 fs, G(1160 nm) = 40 fs, which corresponds to the group delay dispersion $D \approx -60$ fs^2. The basic structure was defined as follows: substrate/(0.25H 0.25L)^{13} 0.25H (0.25L 0.25H)^4 0.8L 0.25H 0.25L 0.25H 0.65L 0.25H 0.25L 0.25H 0.65L 0.25H 0.25L 0.25H 0.25L/air, where the numerical factors are the optical thicknesses of the corresponding layers in fractions of the central wavelength $\lambda_0 = 1000$ nm. The optimisation step was taken to be $0.001\lambda_0$. This is the highest accuracy which can be realised in the deposition of an optical coating. The refractive indices are $n_H = 2.25$ (NbO_2), $n_L = 1.45$ (SiO_2). Calculating the mirror structure required four iterations, which took about three minutes of computer time when using a desktop computer.

Atfer optimisation of the basic structure, we obtained the following structure: substrate/(0.25H 0.25L)^{13} 0.25H 0.258L 0.249H 0.246L 0.246H 0.25L 0.246H 0.809L 0.256H 0.258L 0.247H 0.75L 0.101H 0.236L 0.247H 0.702L 0.161H 0.14L 0.222H 0.311L/air. Figure 1 shows the wavelength dependence of the group delay dispersion for this structure. The operating range of our calculated mirror is broader than for the analogues considered in Refs [1, 4] and amounts to about 180 nm for an oscillation amplitude of ± 10 fs^2. Throughout the wavelength range the structure possesses a reflectivity $R > 99.9$ %, which can be lowered by decreasing the thickness of the mirror reflector.

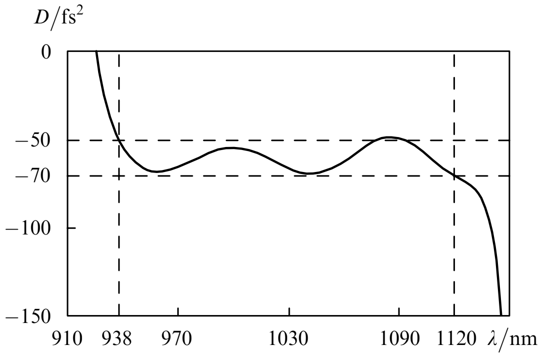

Figure 1. Calculated group delay dispersion D as a function of the wavelength λ for the obtained structure ($\lambda_0 = 1000$ nm).

Note also the fact that by simple scaling it is possible to obtain, proceeding from a structure calculated for one wavelength range, a structure operating in another wavelength range. When the layer thicknesses are expressed in fractions of the central wavelength λ_0, by changing λ_0 we obtain a structure for another wavelength range. The frequency characteristics of the new structure would remain as before, but the modulus of the group delay dispersion D would increase on shifting λ_0 towards longer wavelengths and decrease on shifting λ_0 towards shorter wavelengths. Proceeding from the structure corresponding to Fig. 1, it is possible to calculate another structure operating in the 760 – 920 nm range if we put $\lambda_0 = 800$ nm. Figure 2 shows the wavelength dependence of the group delay dispersion for the scaled structure. One can see that the operating wavelength ranges of the structures corresponding to Figs 1 and 2 are similar. However, the group delay dispersion of the scaled structure (Fig. 2) lowered to -40 fs^2, and the oscillation amplitude also lowered to ± 7 fs^2 in this case.

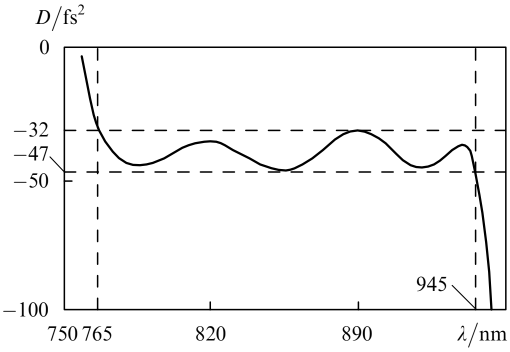

Figure 2. Calculated group delay dispersion D as a function of the wavelength λ for the scaled structure corresponding to Fig. 1 ($\lambda_0 = 800$ nm).

The resonator-type mirror design is relatively stable to random deviations of layer thicknesses emerging during fabrication. We modelled the inaccuracies of fabrication ranging up to 0.5 % and 1 % of the layer thickness, i.e., each layer thickness d_i acquired a random increment $|\delta d_i| < 0.005 d_i$ ($|\delta d_i| < 0.01 d_i$). Figure 3 shows the group delay dispersion D as a function of λ for several structures upon random variation of the layer thicknesses. The oscil-

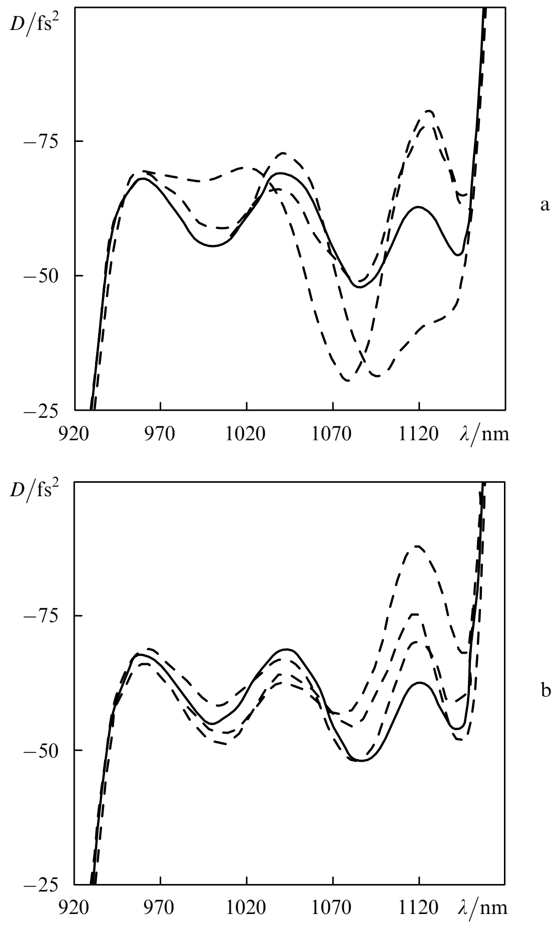

Figure 3. Effect of random layer thickness deviations on the wavelength dependence of the group delay dispersion for a layer thickness variation amplitude of 1 % (a) and 0.5 % (b). The solid curves represent the optimal structure and the dashed curves – the structures with randomised layer thicknesses.

lation amplitudes amounted to only ± 15 and ± 20 fs^2 for $|\delta d_i| < 0.005 d_i$ and $|\delta d_i| < 0.01 d_i$, respectively. One can see that the structure perturbation does not result in critical changes of the phase characteristics of the mirror.

4. Conclusions

We have described a simple and efficient technique for calculating resonator-type dispersion multilayer mirrors with three coupled resonator cavities. The structure design takes place in two stages: definition of the basic structure and its subsequent optimisation. The optimisation procedure includes elements of random search, which broadens the capabilities of the technique as a whole. The optimisation algorithm is considered in detail. The structure calculated by this technique possesses an operating range 180 nm in width (940 – 1120 nm), which exceeds the width of its analogues, and a group delay dispersion $D = -60 \pm 10$ fs^2. We show that by way of simple scaling it is possible to transform the structure of a dispersion mirror for operation in a different wavelength range. The structure described was transformed for the 765 – 945 nm range to exhibit a dispersion $D = -40 \pm 7$ fs^2. The calculated resonator-type dispersion mirror was demonstrated to be immune to random layer thickness deviations occurring in the course of fabrication.

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