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Appearance of the frequency nonreciprocity in a ring chip laser with modulated coupling coefficients of counterpropagating waves

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Abstract. **The effect of harmonic modulation of the coupling coefficients of counterpropagating waves on the type of generation of a solid-state Nd : YAG laser is studied. It is shown that, as the modulation depth is increased, transient generation regimes appear in the laser, which are accompanied by considerable frequency nonreciprocity. The boundaries of the regions of existence of different lasing regimes are found.**

Keywords: *ring solid-state laser, nonlinear radiation dynamics, coupling of counterpropagating waves.*

1. Introduction

New developments and studies in the field of diode-pumped solid-state ring lasers attract considerable recent interest. This is explained by wide applications of these lasers both in fundamental lasers physics (quantum metrology, optical frequency standards, verification of the postulates of the relativity theory, various precision measurements) and in laser techniques (Doppler measurement systems, optical communications, laser gyroscopes, etc.). The study of the nonlinear dynamics of solid-state ring lasers is quite important for searching for new lasing regimes extending the scope of applications of these lasers.

Currently, great attention is being paid to the study of transient (periodic, quasi-periodic, and chaotic) lasing regimes of solid-state ring lasers (see, for example, [1 – 13]). Transient lasing regimes can be excited upon the harmonic modulation of the pump power [1 – 3, 9, 10], intracavity losses [4, 11], and the frequency and amplitude nonreciprocity of the cavity [5 – 7]. The greatest attention has been devoted to the study of the influence of pump modulation on the nonlinear radiation dynamics of solid-state ring lasers, whereas other methods for exciting transient lasing regimes have been investigated to a lesser extent.

In particular, in our opinion, the features of the lasing dynamics of ring lasers upon excitation of transient regimes by modulating the coupling coefficients of counterpropagating waves are not adequately studied. Interest in these regimes is explained by the possibility of their development upon the detection of scattered radiation in optical Doppler location by the self-heterodyning method [14].

In this paper, we present the results of the first theoretical studies of the nonlinear radiation dynamics of a ring chip Nd : YAG laser with the modulated coupling coefficients of counterpropagating waves.

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2. Theoretical model

A two-directional solid-state ring laser is a complex nonlinear oscillating system with the output characteristics depending on many parameters (the pump-power excess of the threshold, the amplitude and frequency nonreciprocity of the cavity, polarisation properties of the cavity, detuning of the laser frequency from the gain line centre, the amplitude and phase of the effective coupling coefficient of counterpropagating waves, and the Q factor of the cavity for counterpropagating waves). We studied the radiation dynamics by using a standard model of a solid-state ring laser, which in the absence of external perturbations is described by the system of differential equations for the complex amplitudes $\tilde{E}_{1,2}$ of counterpropagating waves [15]:

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}$$

$$+\frac{\sigma l}{2T}(1-\mathrm{i}\delta)(N_0\tilde{E}_{1,2} + N_{\mp}\tilde{E}_{2,1}),$$

$$T_1\frac{\mathrm{d}N_0}{\mathrm{d}T} = N_{\mathrm{th}}(1+\eta) - N_0[1 + a(|E_1|^2 + |E_2|^2)] \quad (1)$$

$$-N_{+}aE_1E_2^{*} + N_{-}aE_2E_1^{*},$$

$$T_1\frac{\mathrm{d}N_{+}}{\mathrm{d}T} = -N_{+}[1 + a(|E_1|^2 + |E_2|^2)] - N_0 aE_2E_1^{*},$$

where N_{th} is the threshold inverse population; $\omega/Q_{1,2}$ is the bandwidth of the resonator for counterpropagating waves; L is the perimeter length of the ring resonator; $T = L/c$ is the round-trip transit time of light in the resonator; T_1 is the longitudinal relaxation time; l is the active element length; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; $\sigma = \sigma_0/(1+\delta^2)$ is the laser transition cross section;; $\delta = (\omega - \omega_0)/\Delta\omega_{\mathrm{g}}$ is the relative detuning of the laser frequency from the gain line centre; $\Delta\omega_{\mathrm{g}}$ is the gain line width; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the resonator; ω_1 and ω_2 are the resonator eigenfrequencies for counterpropagating waves; $\tilde{m}_{1,2} = m_{1,2}\exp(\pm\,\mathrm{i}\theta_{1,2})$ are the complex inverse-scattering coupling coefficients of counterpropagating waves; $m_{1,2}$ and $\theta_{1,2}$ are the moduli and phases of coupling coefficients; η is the pump-power excess over the threshold;

$$N_0 = \frac{1}{l}\int_0^l N\mathrm{d}z, \qquad N_\pm = \frac{1}{l}\int_0^l N\mathrm{e}^{\pm \mathrm{i}2kz}\mathrm{d}z$$

are the complex amplitudes of the spatial harmonics of the inverse population N. The modulation of the coupling coefficients was taken into account in the following way. One of the coupling coefficients in the system of equations (1) was represented in the form

$$m_1 = m_1^{(0)}(1 + h\cos\omega_\mathrm{p}t), \tag{2}$$

where $m_1^{(0)}$ is the modulus of the coupling coefficient in the absence of modulation; h is the modulation depth; and ω_p is the modulation frequency. The modulation frequency $\omega_\mathrm{p}/2\pi$ and depth h of the coupling coefficients were used as control parameters.

Theoretical analysis was performed assuming the single-mode generation of linearly polarised waves in each direction. In addition, we assumed for simplicity that $\Omega = 0$ and $\delta = 0$. This, as a rule, is realised in experiments. The system of equations (1) was solved by the Runge–Kutta method.

We determined the intensities $I_1(t)$ and $I_2(t)$ of counterpropagating waves and their spectra $J_1(\omega)$ and $J_2(\omega)$ by numerical simulations. The lasing regimes (especially, the dynamic chaos regime) were identified by analysing the temporal and spectral characteristics of counterpropagating waves and constructing standard phase portraits in the $[I_1(t), I_1(t-\tau)]$ plane. By processing the results obtained, we found the Pearson correlation coefficients for the intensities of counterpropagating waves $K[I_1(t), I_2(t)]$ and their spectra $K[J_1(\omega),\ J_2(\omega)]$, which allowed us to analyse in detail the features of the nonlinear dynamics of a ring chip laser, which appear upon modulation of the coupling coefficients of counterpropagating waves.

3. Results of numerical simulations

We studied the radiation dynamics of a solid-state ring laser with parameters close to those of a monolithic ring chip Nd : YAG laser ($\eta = 20\,\%$, $L = 2.7$ cm, the resonator losses $R_0 = 2.4\,\%$, the self-modulation frequency $\omega_\mathrm{m}/2\pi = 230$ kHz, the relaxation frequency is 55 kHz). The modulus of one of the coupling coefficients of counterpropagating waves was modulated in the frequency range 20 – 60 kHz. The modulation depth was varied from 5 % to 95 %.

By using the numerical simulation of the nonlinear dynamics of the ring laser, we studied the influence of the modulation frequency and depth of the coupling coefficients on the generation of the solid-state ring chip laser and determined the regions of existence of different lasing regimes. We found that if the modulation frequency $\omega_\mathrm{p}/2\pi$ is fixed and the modulation depth of the coupling coefficients of counterpropagating waves increases, the self-modulation generation regime of the first kind, existing in the chip laser in the absence of modulation of the coupling

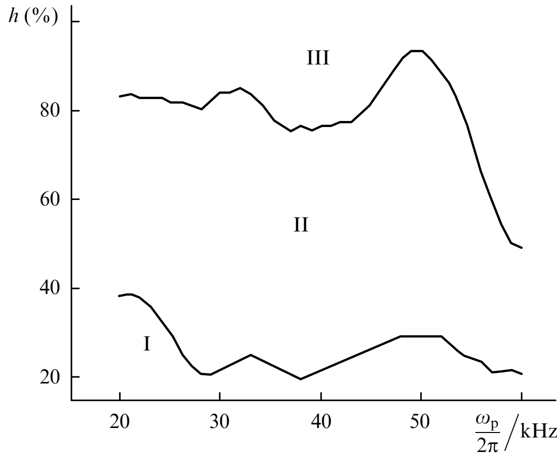

Figure 1. Regions of existence of different lasing regimes depending on the modulation frequency and depth of the coupling coefficients: quasi-sinusoidal (I), pulsed quasi-periodic (II), and chaotic (III).

coefficients, changes first to the quasi-sinusoidal regime, then to the pulsed quasi-periodic regime and, finally, to dynamic chaos. This is illustrated in Fig. 1, where the regions of existence of different lasing regimes are presented in the $(h, \omega_\mathrm{p}/2\pi)$ plane.

Note that the passage from the out-of-phase self-modulation regime of the first kind to the pulsed quasi-periodic regime is accompanied by the appearance of the spectral nonreciprocity of radiation, which is also preserved after passing to dynamic chaos. Figures 2–4 show the typical emission spectra of counterpropagating waves for different lasing regimes.

One can see that while in the self-modulation regime of the first kind the spectra of counterpropagating waves

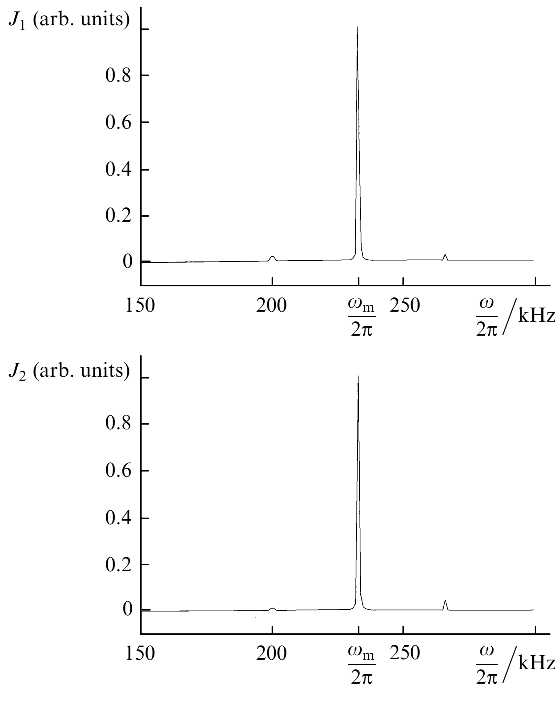

Figure 2. Typical emission spectra of counterpropagating waves in the quasi-sinusoidal lasing regime.

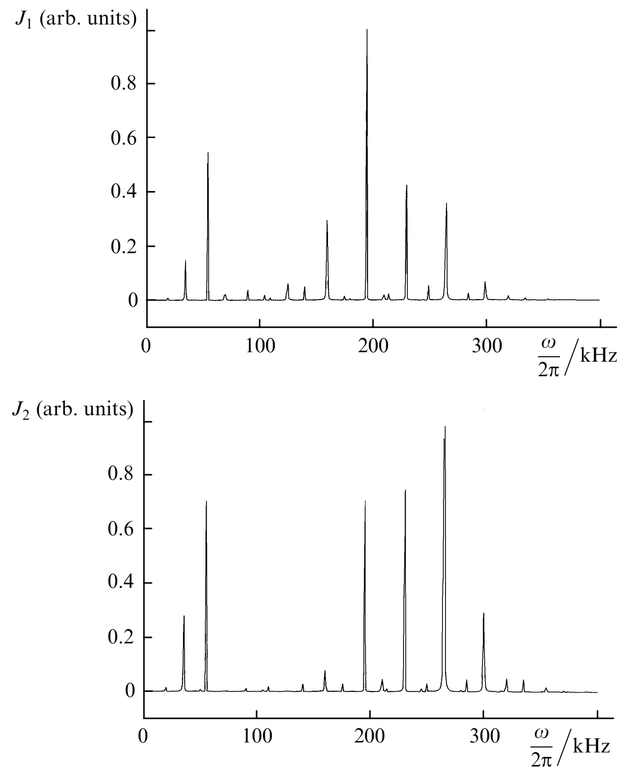

Figure 3. Typical emission spectra of counterpropagating waves in the pulsed quasi-periodic lasing regime.

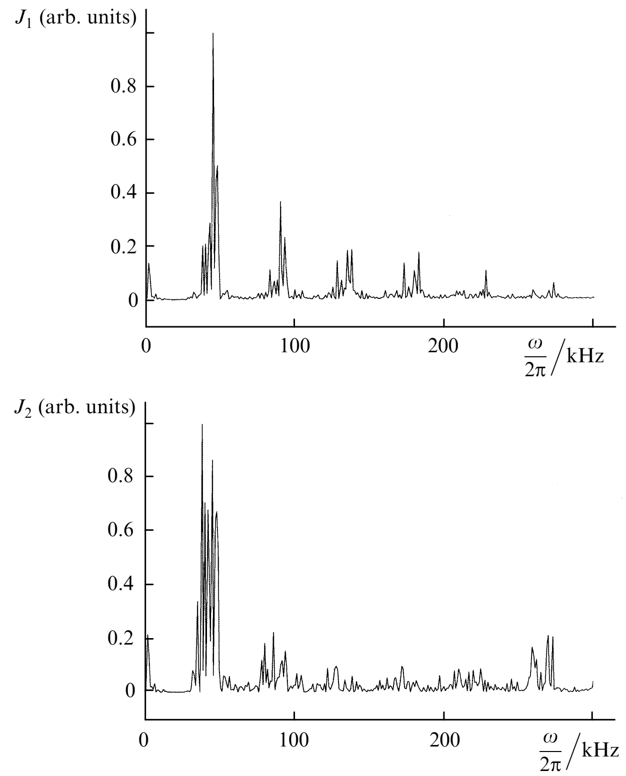

Figure 4. Typical emission spectra of counterpropagating waves in the chaotic lasing regime.

consist of identical spectral components with frequencies coinciding with the self-modulation frequency $\omega_m/2\pi$, a passage to the quasi-sinusoidal regime is accompanied by the appearance of new spectral components at frequencies separated by the frequency $\omega_p/2\pi$ from $\omega_m/2\pi$ (Fig. 2). When the quasi-periodic regime appears, these spectra contain already many components with intensities substantially different for counterpropagating waves (Fig. 3). The frequencies of the spectral components in the quasi-periodic regime are multiples of the relaxation and relaxation frequencies and their combinations. The emission spectra become even more complicated on passing to the dynamic chaos regime, they become quasi-continuous, remaining, however, non-equivalent (Fig. 4).

We determined the Pearson correlation coefficients $K[I_1(t), I_2(t)]$ and $K[J_1(\omega), J_2(\omega)]$ for the lasing regimes studied. The change in the correlation coefficients for the modulation frequency 35 kHz depending on the lasing regime is illustrated in Table 1 (the correlation coefficients were calculated for the region of existence of different lasing regimes). One can see that while in the absence of modulation, the intensity correlation coefficient for counterpropagating waves is -1, indicating the excitation of out-of-phase harmonic oscillations in the laser, this coefficient is close to -1 for relatively small modulation depths and drastically increases with increasing modulation depth.

The frequency nonreciprocity, which is absent in the self-modulation regime of the first kind (the correlation coefficient $K[J_1(\omega), J_2(\omega)] = 1$) increases with increasing modulation depth, which is revealed in the decrease in the correlation coefficient $K[J_1(\omega), J_2(\omega)]$. Note that the correlation coefficients monotonically change within increasing the modulation depth of the coupling coefficients of counterpropagating waves.

Table 1. Correlation coefficients of the intensities and spectra of counterpropagating waves.

Lasing regime	$K[I_1(t), I_2(t)]$	$K[J_1(\omega), J_2(\omega)]$
Self-modulation	-1.000	1.000
Quasi-sinusoidal	-0.997	0.999
Quasi-periodic	-0.633	0.844
Chaotic	0.108	0.746

4. Conclusions

We have studied theoretically the features of the nonlinear radiation dynamics of a ring chip Nd : YAG laser upon harmonic modulation of the coupling coefficients of counterpropagating waves. It is shown that, as the modulation depth was increased, first the quasi-sinusoidal generation regime appeared in the laser, which passed to the pulsed quasi-periodic regime and then to the dynamic chaos regime. The emission spectra of counterpropagating waves in the transient lasing regimes are substantially different. These properties of the radiation dynamics can be useful, for example, for analysis of the operation of solid-state ring lasers in the self-heterodyning regime.

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References

1. Zolotoverkh I.I., Klimenko D.N., Kravtsov N.V., Lariontsev E.G., Firsov V.V. *Kvantovaya Elektron.*, **23**, 938 (1996) [*Quantum Electron.*, **26**, 914 (1996)].
2. Kotomtseva L.A., Kravtsov N.V., Lariontsev E.G., Chekina S.N. *Kvantovaya Elektron.*, **32**, 654 (2002) [*Quantum Electron.*, **32**, 654 (2002)].
3. Klimenko D.N., Kravtsov N.V., Lariontsev E.G., Firsov V.V. *Kvantovaya Elektron.*, **24**, 649 (1997) [*Quantum Electron.*, **27**, 631 (1997)].
4. Zolotoverkh I.I., Klimenko D.N., Kravtsov N.V., Lariontsev E.G. *Kvantovaya Elektron.*, **23**, 625 (1996) [*Quantum Electron.*, **26**, 609 (1996)].
5. Kravtsov N.V., Lariontsev E.G., Sidorov S.S., Firsov V.V., Chekina S.N. *Kvantovaya Elektron.*, **31**, 189 (2001) [*Quantum Electron.*, **31**, 189 (2001)].
6. Kravtsov N.V., Pashinin S.P., Sidorov S.S., Chekina S.N., Firsov V.V. *Kvantovaya Elektron.*, **33**, 321 (2003) [*Quantum Electron.*, **33**, 321 (2003)].
7. Kravtsov N.V., Lariontsev E.G., Shelaev A.N. *Laser Phys.*, **3**, 21 (1993).
8. Uchida A., Sato T., et al. *Phys. Rev. E*, **56**, 7249 (1998).
9. Klische W., Telle H.R., Weiss C.O. *Opt. Lett.*, **9**, 561 (1984).
10. Roy R., Thornburg K.S. *Rhys. Rev. Lett.*, **72**, 2009 (1994).
11. Khandokhin P.A., Khanin Ya.I. *Kvantovaya Elektron.*, **11**, 1483 (1984) [*Sov. J. Quantum Electron.*, **14**, 1004 (1984)].
12. Khandohin P.A., Khanin Ya.I. *J. Opt. Soc. Am. B*, **1**, 226 (1985).
13. Kravtsov N.V., Lariotsev E.G., Pachinin P.P., Sidorov S.S. *Laser Phys.*, **13**, 305 (2003).
14. Kravtsov N.V., Kravtsov N.N., Makarov A.A., Firsov V.V. *Kvantovaya Elektron.*, **23**, 885 (1996) [*Quantum Electron.*, **26**, 862 (1996)].
15. Kravtsov N.V., Lariontsev E.G. *Kvantovaya Elektron.*, **30**, 105 (2000) [*Quantum Electron.*, **30**, 105 (2000)].