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Frequency shift of radiation of an atom near a cluster of two perfectly conducting spherical nanoparticles

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Abstract. **Expressions for the frequency shift of radiation of an atom located near a cluster of two perfectly conducting spherical nanoparticles are obtained within the framework of a classical model. The asymptotic expression is found for the radiation frequency shift of an atom located between spheres approaching each other.**

Keywords*: nanoparticles, radiation frequency shift, nanoclusters.*

1. Introduction

This paper is closely connected with previous paper [1] devoted to the spontaneous emission of an atom located near a cluster consisting of two perfectly conducting nanospheres. In the design of various devices [2 – 6], in which the optical properties of atoms and molecules are used, it is necessary to take into account not only the acceleration or slowing of spontaneous decays of an atom near a nanobody but also the radiation frequency shift, which can be noticeable. In addition, the frequency shift determines the gradient force, which allows one to control the motion of atoms, molecules, and nanoparticles [7, 8].

The aim of this paper is to study analytically the radiation frequency shit of a classical (Lorentz) atom located near a cluster of two arbitrarily positioned perfectly conducting spherical nanoparticles (see Fig. 2 in [1]). The main attention will be devoted to a cluster consisting of two identical nanospheres; however, the method developed in the paper can be also applied to a cluster containing nanospheres of arbitrary radii.

2. Radiation frequency shift for an atom located near a cluster of two perfectly conducting nanospheres

The radiation frequency shift of an atom (molecule) near a nanobody obtained within the framework of the classical (Lorentz) model of an atom [9 – 11] has the form

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3}{4k_0^3|d_0|^2}\mathrm{Re}(\boldsymbol{d}_0^*\boldsymbol{E}^{\mathrm{r}}(\boldsymbol{r}', \boldsymbol{r}'|\omega_0)), \tag{1}$$

where $\boldsymbol{E}^{\mathrm{r}}(\boldsymbol{r}', \boldsymbol{r}'|\omega_0)$ is the reflected part of the field excited by a dipole with the moment $\boldsymbol{d}_0$ at the location point $\boldsymbol{r}'$ of the atom; ω is the radiation frequency of the atom located near a nanobody; ω_0 is the radiation frequency of the atom in free space; γ_0 is the width of the emission line of the atom in vacuum; and $k_0 = \omega/c$. Relation (1) is completely valid only for the classical model of an atom, while the quantum-mechanical model gives different expressions for the frequency shift. The frequency shift in the quantum-mechanical model of a two-level atom can be written within the framework of the perturbation theory [11] as the sum of expression (1), where $\boldsymbol{d}_0$ is the matrix element of the dipole transition moment operator of the atom between the initial and final states, and the additional term [11] described by the integral over imaginary frequencies, which has no resonance properties. Therefore, in the most interesting region of resonance interaction, we can use the frequency shift described by (1), i.e., by the classical model.

The total field can be found by using quasi-static equations (5) from paper [1]. The reflected part of the field excited by an atom can be written in the form

$$E^{\mathrm{r}} = -\nabla\psi^{\mathrm{r}}, \quad \psi^{\mathrm{r}} = (\boldsymbol{d}_0\nabla')\tilde{\psi}^{\mathrm{r}}, \tag{2}$$

where ∇ and ∇' are the gradients over coordinates of the observation point $\boldsymbol{r}$ and the atom position $\boldsymbol{r}'$, respectively. The function $\tilde{\psi}^{\mathrm{r}}$ can be found from the solution of the Poisson equation (7) [1] by subtracting the Green function G_0 of free space from the potential of a unit charge in the presence of two perfectly conducting spheres [see expression (12) in [1]):

$$\tilde{\psi}^{\mathrm{r}} = \tilde{\psi} - G_0. \tag{3}$$

The solution of the Poisson equation for perfectly conducting spheres in bispherical coordinates was obtained in paper [1]. By using expression (9) for potential $\tilde{\psi}$ from [1], we can obtain from (2) the expressions for the field reflected by perfectly conducting spheres at each spatial point for arbitrary positions of the atom and arbitrary directions of its dipole moment.

Expressions for the frequency shift can be obtained by using the scalar product of the reflected field at the atom location and its dipole moment, as follows from (1). In the general case, expressions for the frequency shift are extremely cumbersome, and it is difficult to make any certain conclusions from them about the influence of the cluster on the emission properties of the atom. Because of

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this, we will use general expressions for a cluster of two identical spheres, which is most interesting from the practical point of view.

In expressions obtained below, the bispherical coordinates $-\infty < \eta < \infty$, $0 < \xi \leqslant \pi$ and $0 \leqslant \phi < 2\pi$ are used. The relation between Cartesian and bispherical coordinates is described by expression (8) in [1] (see also [12, 13]). The surface $\eta = \eta_1 < 0$ defines a sphere of radius $R_1 = a/|\sinh \eta_1|$ with a centre at the point $x_1 = y_1 = 0$, $z_1 = a \coth \eta_1$. The second sphere is specified by the relation $\eta = \eta_2 > 0$. The radius of this sphere is $R_2 = a/\sinh \eta_2$ and the centre is located at the point $x_2 = y_2 = 0$, $z_2 = a \coth \eta_2$. Here, a dimensional constant a is half the distance between the poles of the bispherical coordinate system. The distance between the centres of spheres is $R_{12} = z_2 - z_1$. The bispherical coordinates determining the atom position are primed, as in [1].

We will find frequency shifts for the characteristic positions of an atom and directions of its dipole moment ($\phi' = 0$) for the case of two identical spheres with radii $R_1 = R_2 = R_0$ ($-\eta_1 = \eta_2 = \eta_0$).

The dipole moment of the atom is oriented along the x axis ($\boldsymbol{d}_0 = d_0\boldsymbol{e}_x$):

(a) the atom is located on the x axis ($\eta' = 0$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(1-\cos\xi')}{4(k_0a)^3}\left\{\frac{1}{2}\sin^2\xi'\sum_{n=0}^{\infty}\sum_{m=0}^{n}(2-\delta_{m0}) \times \frac{(n-m)!}{(n+m)!}\frac{[P_n^m(\cos\xi')]^2}{\exp[(2n+1)\eta_0]+1} + (1-\cos\xi')\sin\xi' \times \sum_{n=0}^{\infty}\sum_{m=0}^{n}(2-\delta_{m0})\frac{(n-m)!}{(n+m)!}\frac{1}{\exp[(2n+1)\eta_0]+1} \times \frac{\partial[P_n^m(\cos\xi')]^2}{\partial\xi'} + 2(1-\cos\xi')^2\sum_{n=0}^{\infty}\sum_{m=0}^{n}(2-\delta_{m0}) \times \frac{(n-m)!}{(n+m)!}\frac{1}{\exp[(2n+1)\eta_0]+1}\left[\frac{\partial P_n^m(\cos\xi')}{\partial\xi'}\right]^2\right\} + \frac{3(1-\cos\xi')}{C_{11}(k_0a)^3}\left[\frac{1}{2}\sin\xi'\sum_{n=0}^{\infty}\frac{P_n(\cos\xi')}{\exp[(2n+1)\eta_0]+1} + (1-\cos\xi')\sum_{n=0}^{\infty}\frac{1}{\exp[(2n+1)\eta_0]+1}\frac{\partial P_n(\cos\xi')}{\partial\xi'}\right]^2; \quad (4)$$

(b) the atom is located on the z axis between spheres ($\xi' = \pi$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(\cosh\eta'+1)^3}{4(k_0a)^3}\sum_{n=1}^{\infty}n(n+1) \times \left\{\frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1} + \frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\}; \quad (5)$$

(c) the atom is located on the z axis behind one of the spheres ($\xi' \to 0$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(\cosh\eta'-1)^3}{4(k_0a)^3}\sum_{n=1}^{\infty}n(n+1)\times \left\{\frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1} + \frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\}. \quad (6)$$

The dipole moment of the atom is oriented along the y axis ($\boldsymbol{d}_0 = d_0\boldsymbol{e}_y$):

(a) the atom is located on the x axis ($\eta' = 0$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(1-\cos\xi')^3}{(k_0a)^3\sin^2\xi'} \times \sum_{n=1}^{\infty}\sum_{m=1}^{n}m^2\frac{(n-m)!}{(n+m)!}\frac{[P_n^m(\cos\xi')]^2}{\exp[(2n+1)\eta_0]+1}. \quad (7)$$

If the atom is located on the z axis ($\xi' = \pi$ and $\xi' \to 0$), expressions (5) and (6) should be used.

The dipole moment of the atom is oriented along the z axis ($\boldsymbol{d}_0 = d_0\boldsymbol{e}_z$):

(a) the atom is located on the x axis ($\eta' = 0$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(1-\cos\xi')^3}{2(k_0a)^3} \times \sum_{n=0}^{\infty}\sum_{m=0}^{n}(2-\delta_{m0})\frac{(n-m)!}{(n+m)!}\left(n+\frac{1}{2}\right)^2 \times \frac{[P_n^m(\cos\xi')]^2}{\exp[(2n+1)\eta_0]-1} + \frac{3(1-\cos\xi')^3}{(k_0a)^3(2C_{12}+C_{11})} \times \left[\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\frac{P_n(\cos\xi')}{\exp[(2n+1)\eta_0-1}\right]^2; \quad (8)$$

(b) the atom is located on the z axis between spheres ($\xi' = \pi$):

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3(\cosh\eta'+1)}{4(k_0a)^3}\left\{\frac{1}{2}\sinh^2\eta' \times \sum_{n=0}^{\infty}\left\{\frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1} + \frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\} + \sinh\eta'(\cosh\eta'+1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\sinh[(2n+1)\eta'] \times \left\{\frac{1}{\exp[(2n+1)\eta_0]+1} + \frac{1}{\exp[(2n+1)\eta_0]-1}\right\} + 2(\cosh\eta'+1)^2\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)^2\left\{\frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1} + \frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\}\right\} - \frac{3U_1(\cosh\eta'+1)^{1/2}}{2\sqrt{2}d_0k_0^3a} \times \left\{\frac{1}{2}\sinh\eta'\sum_{n=0}^{\infty}(-1)^n\frac{\sinh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]} \times \exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right] - (\cosh\eta'+1)\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right) \times \frac{\cosh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\} -$$

$$-\frac{3U_2(\cosh\eta'+1)^{1/2}}{2\sqrt{2}d_0k_0^3a}\left\{\frac{1}{2}\sinh\eta'\sum_{n=0}^{\infty}(-1)^n\right.$$
$$\times\frac{\sinh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$+(\cosh\eta'+1)\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}, \quad (9)$$

where $U_{1,2}=(\boldsymbol{d}_0\nabla')\tilde{U}_{1,2}$; $\tilde{U}_{1,2}$ is determined by expressions (15) from [1], in which $-\eta_1=\eta_2=\eta_0$, while C_{11} and C_{22} are described by expressions (21) from [1]. Expressions for $\varphi_{1,2}$ in [1] should be replaced by relations

$$\varphi_1=\frac{d_0[2(\cosh\eta'+1)]^{1/2}}{a^2}\left\{\frac{1}{2}\sinh\eta'\sum_{n=0}^{\infty}(-1)^n\right.$$
$$\times\frac{\sinh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$-(\cosh\eta'+1)\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}, \quad (10)$$

$$\varphi_2=\frac{d_0[2(\cosh\eta'+1)]^{1/2}}{a^2}\left\{\frac{1}{2}\sinh\eta'\sum_{n=0}^{\infty}(-1)^n\right.$$
$$\times\frac{\sinh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$+(\cosh\eta'+1)\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}; \quad (11)$$

(c) the atom is located on the z axis behind one of the spheres ($\xi'\to 0$):

$$\frac{\omega-\omega_0}{\gamma_0}=-\frac{3(\cosh\eta'-1)}{4(k_0a)^3}\left\{\frac{1}{2}\sinh^2\eta'\right.$$
$$\times\sum_{n=0}^{\infty}\left\{\frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1}+\frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\}$$
$$+\sinh\eta'(\cosh\eta'-1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\sinh[(2n+1)\eta']$$
$$\times\left\{\frac{1}{\exp[(2n+1)\eta_0]+1}+\frac{1}{\exp[(2n+1)\eta_0]-1}\right\}$$
$$+2(\cosh\eta'-1)^2\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)^2\left\{\frac{\sinh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]+1}\right.$$
$$\left.\left.+\frac{\cosh^2[(n+1/2)\eta']}{\exp[(2n+1)\eta_0]-1}\right\}\right\}+\frac{3U_1(\cosh\eta'-1)^{1/2}}{2\sqrt{2}d_0k_0^3a}\times$$
$$\times\left\{\frac{1}{2}\sinh\eta'\sum_{n=0}^{\infty}\frac{\sinh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\right.$$
$$\times\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]-(\cosh\eta'-1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}$$
$$+\frac{3U_2(\cosh\eta'-1)^{1/2}}{2\sqrt{2}d_0k_0^3a}\left\{\frac{1}{2}\sinh\eta'\right.$$
$$\times\sum_{n=0}^{\infty}\frac{\sinh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$+(\cosh\eta'-1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}, \quad (12)$$

$$\varphi_1=-\frac{d_0[2(\cosh\eta'-1)]^{1/2}}{a^2}\left\{\frac{1}{2}\sinh\eta'\right.$$
$$\times\sum_{n=0}^{\infty}\frac{\sinh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$-(\cosh\eta'-1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta_0-\eta')]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}, \quad (13)$$

$$\varphi_2=-\frac{d_0[2(\cosh\eta'-1)]^{1/2}}{a^2}\left\{\frac{1}{2}\sinh\eta'\right.$$
$$\times\sum_{n=0}^{\infty}\frac{\sinh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]$$
$$+(\cosh\eta'-1)\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)$$
$$\left.\times\frac{\cosh[(n+1/2)(\eta'+\eta_0)]}{\sinh[(2n+1)\eta_0]}\exp\left[-\left(n+\frac{1}{2}\right)\eta_0\right]\right\}. \quad (14)$$

3. Asymptotic expressions for the radiation frequency shift of an atom located near a cluster of two perfectly conducting spheres

The determination of asymptotic expressions in the general case (for different positions of an atom and different directions of its dipole moment) is a difficult and not always solvable problem. In this section, rather simple asymptotic expressions are presented for the radiation frequency shift of an atom located near a cluster of two identical nanospheres ($R_1=R_2=R_0$) with decreasing ($R_{12}\to 2R_0$) or increasing ($R_{12}\to\infty$) the distance between spheres for some particular positions of the atom.

3.1 Asymptotic expressions for the radiation frequency shift of an atom with decreasing distance between spheres

Consider an atom placed exactly midway ($\eta' = 0$ and $\xi' = \pi$) between two identical spheres ($-\eta_1 = \eta_2 = \eta_0$). By using (5) and (9), we find the expressions for the frequency shift ($\phi' = 0$).

The dipole moment of the atom is oriented along the x or *y axis:*

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{6}{(k_0 a)^3} \sum_{n=1}^{\infty} \frac{n(n+1)}{\exp[(2n+1)\eta_0] + 1}. \quad (15)$$

The dipole moment of the atom is oriented along the z axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{12}{(k_0 a)^3} \sum_{n=0}^{\infty} \frac{(n+1/2)^2}{\exp[(n+1)\eta_0] - 1} + \frac{24}{(2C_{12} + C_{11})(k_0 a)^3} \left[\sum_{n=0}^{\infty} (-1)^n \frac{n+1/2}{\exp[(2n+1)\eta_0] - 1} \right]^2. \quad (16)$$

In the case of two spheres approaching each other ($R_{12} \to 2R_0$, $\eta_0 \to 0$), the series in expressions (15) and (16) can be summed by using the Mellin transformation [1, 12] to obtain the following expressions:

The dipole moment of the atom is oriented along the x or y axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{1}{(k_0 R_0)^3} \left\{ \frac{9\zeta(3)}{8\eta_0^6} - \frac{3}{16\eta_0^4}[3\zeta(3) + 4\ln 2] + \frac{1}{320\,\eta_0^2}[51\zeta(3) + 120\ln 2 + 17] - \frac{1}{160}\left[\frac{457}{84}\zeta(3) + 17\ln 2 + \frac{1685}{504}\right] + \ldots \right\}. \quad (17)$$

The dipole moment of the atom is oriented along the z axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{1}{(k_0 R_0)^3} \left\{ \frac{3\zeta(3)}{\eta_0^6} - \frac{1}{4\eta_0^4} \times \left[6\,\zeta(3) + \frac{6}{\gamma_E + \ln(2/\eta_0)} - 1\right] + \frac{1}{480\,\eta_0^2} \times \left\{204\,\zeta(3) + \frac{10}{\gamma_E + \ln(2/\eta_0)}\left[48 + \frac{1}{\gamma_E + \ln(2/\eta_0)}\right] - 67\right\} - \frac{1}{9600}\left\{\frac{18280}{21}\zeta(3) + \frac{1}{\gamma_E + \ln(2/\eta_0)} \times \left\{3140 + \frac{1}{\gamma_E + \ln(2/\eta_0)}\left\{131 + \frac{25}{9[\gamma_E + \ln(2/\eta_0)]}\right\}\right\} - \frac{25520}{63}\right\} + \ldots \right\}, \quad (18)$$

where ζ is the Riemann zeta function and $\gamma_E \approx 0.577216$ is the Euler constant. One can see from (17) and (18) that at a sufficiently small distance between nanospheres, the radiation frequency shift for an atom with the dipole moment oriented along the polar axis occurs $8/3 \approx 2.7$ times faster than that for an atom with the dipole moment oriented perpendicular to this axis.

3.2 Asymptotic expressions for the radiation frequency shift of an atom with increasing distance between spheres

When all the distances are large compared to the nanospheres radius, we can obtain asymptotic expressions corresponding to widely separated spheres ($R_{12} \to \infty$, $\eta_0 \to \infty$). By using (5), (6), and (9), we obtain the following expressions:

The dipole moment of the atom is oriented along the x or y axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{12}{(k_0 R_{12})^3}[\cosh 3\eta' \exp(-3\eta_0) + 3(2\cosh 3\eta' + \cosh 5\eta')\exp(-5\eta_0) - \exp(-6\eta_0) + \ldots]. \quad (19)$$

The dipole moment of the atom is oriented along the z axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{12}{(k_0 R_{12})^3}[4\cosh 3\eta' \exp(-3\eta_0) + 3(7\cosh 3\eta' - 2\cosh 4\eta' + 3\cosh 5\eta') \times \exp(-5\eta_0) + 8\exp(-6\eta_0) + \ldots]. \quad (20)$$

The asymptotic expression for the radiation frequency shift can be obtained by using the model of three interacting dipoles in which spheres are replaced by point dipoles with polarisability equal to that of a sphere in a homogeneous field {expressions (41) and (42) in [1]}. Let $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ be the dipole moments induced on the first and second spheres. We assume that the nanospheres have equal radii ($R_1 = R_2 = R_0$) and will use the z axis as a polar axis assuming that $-z_1 = z_2 = R_{12}/2$. Then, the model of three dipoles [1] for an atom located in the $y' = 0$ plane gives the following expressions for the radiation frequency shift:

The dipole moment of the atom oriented along the x axis:

$$\frac{\omega - \omega_0}{\gamma_0} = -\frac{3}{4k_0^3 d_0}\left\{-\frac{d_{1x}}{R_+^3} - \frac{d_{2x}}{R_-^3} + 3x'^2\left(\frac{d_{1x}}{R_+^5} + \frac{d_{2x}}{R_-^5}\right) + 3x'\left[\frac{d_{1z}(z' + R_{12}/2)}{R_+^5} + \frac{d_{2z}(z' - R_{12}/2)}{R_-^5}\right]\right\}, \quad (21)$$

where the nonzero components of the induced dipole moment have the form

$$\begin{aligned} d_{1x} &= d_0\left(\frac{1}{R_0^6} - \frac{1}{R_{12}^6}\right)^{-1}\left[-\frac{1}{R_0^3 R_+^3} + \frac{1}{R_{12}^3 R_-^3} + 3x'^2\left(\frac{1}{R_0^3 R_+^5} - \frac{1}{R_{12}^3 R_-^5}\right)\right]; \\ d_{2x} &= d_0\left(\frac{1}{R_0^6} - \frac{1}{R_{12}^6}\right)^{-1}\left[-\frac{1}{R_0^3 R_-^3} + \frac{1}{R_{12}^3 R_+^3} + 3x'^2\left(\frac{1}{R_0^3 R_-^5} - \frac{1}{R_{12}^3 R_+^5}\right)\right]; \end{aligned} \quad (22)$$

$$d_{1z}=3d_0\left(\frac{1}{R_0^6}-\frac{4}{R_{12}^6}\right)^{-1}\left(\frac{z'+R_{12}/2}{R_0^3R_+^5}+2\frac{z'-R_{12}/2}{R_{12}^3R_-^5}\right)x';$$

$$d_{2z}=3d_0\left(\frac{1}{R_0^6}-\frac{4}{R_{12}^6}\right)^{-1}\left(\frac{z'-R_{12}/2}{R_0^3R_-^5}+2\frac{z'+R_{12}/2}{R_{12}^3R_+^5}\right)x'. \tag{23}$$

The dipole moment of the atom is oriented along the y axis:

$$\frac{\omega-\omega_0}{\gamma_0}=\frac{3}{4k_0^3d_0}\left(\frac{d_{1y}}{R_+^3}+\frac{d_{2y}}{R_-^3}\right), \tag{24}$$

where

$$d_{1y}=d_0\left(\frac{1}{R_0^6}-\frac{1}{R_{12}^6}\right)^{-1}\left(-\frac{1}{R_0^3R_+^3}+\frac{1}{R_{12}^3R_-^3}\right);$$

$$d_{2y}=d_0\left(\frac{1}{R_0^6}-\frac{1}{R_{12}^6}\right)^{-1}\left(-\frac{1}{R_0^3R_-^3}+\frac{1}{R_{12}^3R_+^3}\right). \tag{25}$$

The dipole moment of the atom is oriented along the z axis:

$$\frac{\omega-\omega_0}{\gamma_0}=-\frac{3}{4k_0^3d_0}\left\{-\frac{d_{1z}}{R_+^3}-\frac{d_{2z}}{R_-^3}+3\left[\frac{d_{1z}(z'+R_{12}/2)^2}{R_+^5}+\frac{d_{2z}(z'-R_{12}/2)^2}{R_-^5}\right]+3x'\left[\frac{d_{1x}(z'+R_{12}/2)}{R_+^5}+\frac{d_{2x}(z'-R_{12}/2)}{R_-^5}\right]\right\}, \tag{26}$$

where

$$d_{1x}=3d_0\left(\frac{1}{R_0^6}-\frac{1}{R_{12}^6}\right)^{-1}\left(\frac{z'+R_{12}/2}{R_0^3R_+^5}-\frac{z'-R_{12}/2}{R_{12}^3R_-^5}\right)x';$$

$$d_{2x}=3d_0\left(\frac{1}{R_0^6}-\frac{1}{R_{12}^6}\right)^{-1}\left(\frac{z'-R_{12}/2}{R_0^3R_-^5}-\frac{z'+R_{12}/2}{R_{12}^3R_+^5}\right)x'; \tag{27}$$

$$d_{1z}=d_0\left(\frac{1}{R_0^6}-\frac{4}{R_{12}^6}\right)^{-1}\left\{-\frac{1}{R_0^3R_+^3}-\frac{2}{R_{12}^3R_-^3}+3\left[\frac{(z'+R_{12}/2)^2}{R_0^3R_+^5}+2\,\frac{(z'-R_{12}/2)^2}{R_{12}^3R_-^5}\right]\right\};$$

$$d_{2z}=d_0\left(\frac{1}{R_0^6}-\frac{4}{R_{12}^6}\right)^{-1}\left\{-\frac{1}{R_0^3R_-^3}-\frac{2}{R_{12}^3R_+^3}+3\left[\frac{(z'-R_{12}/2)^2}{R_0^3R_-^5}+2\,\frac{(z'+R_{12}/2)^2}{R_{12}^3R_+^5}\right]\right\}; \tag{28}$$

$$R_\pm=\left[x'^2+\left(z'\pm\frac{R_{12}}{2}\right)^2\right]^{1/2}.$$

All the components of the induced dipole moment which are absent in (21), (24), and (26) are zero. By using expressions for coordinates (8) from [1] and expanding (21)–(28) into a series for $\eta_0\to\infty$, we find asymptotic expressions, which can be compared with exact formulas (19) and (20). As a result, we see that the three-dipole mode correctly describes the frequency shift at distances larger compared to the sphere radius. The principal terms of expansions, which contain only contribution from dipole radiation, coincide with analogous terms in expressions obtained by expanding into series.

4. Graphic illustrations and discussion of results

As an example we consider a cluster of two identical nanospheres with radius $R_0=50$ nm. Let us assume for definiteness that $k_0R_0=0.1$. Figure 1 presents the radiation frequency shift of an atom located in the $y'=0$ plane as a function of the distance between the atom and cluster. As expected, the radiation frequency at large distances from the cluster tends to the radiation frequency of the atom in free space (in the absence of the cluster). One can see that the frequency shifts for the atom with the dipole moments oriented along the x and y axes (perpendicular to the polar axis) coincide at sufficiently small distances from the cluster centre (Fig. 1a). When the atom moves along the z axis, i.e., the polar axis, the frequency shifts corresponding to the atomic dipole moment oriented perpendicular to the z axis (along the x and y axes) coincide, as shown in Fig. 1. As the atom approaches the sphere surface, the frequency shift becomes an infinitely large negative quantity.

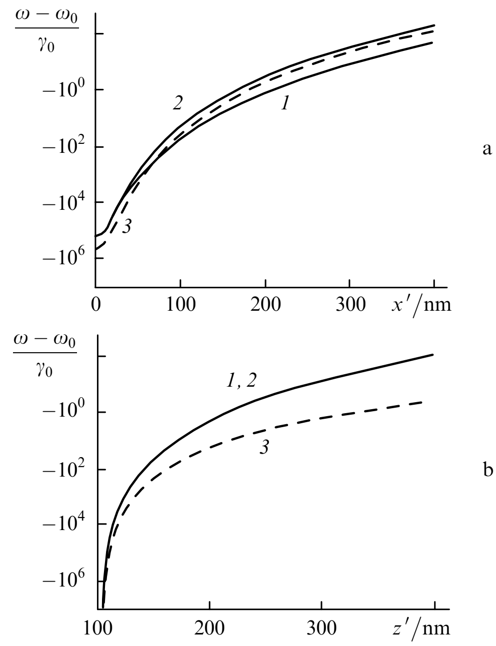

Figure 1. Radiation frequency shift of an atom (molecule) as a function of the distance between the atom and a cluster of two identical nanospheres. The spheres have radii 50 nm and their centres are separated by 101 nm, $k_0R_0=0.1$. The atom moves away from the cluster along the x axis from the exact centre between spheres (a) and along the z axis from the external surface of one of the spheres (b). The dipole moment is oriented along the x (*1*), y (*2*), and z (*3*) axes.

Figure 2 shows the radiation frequency shifts for an atom located exactly midway between nanospheres with the dipole moment directed perpendicular and along the polar axis as a function of the distance between spheres. Also, the corresponding asymptotics (17), (18) and (21), (26) are presented. One can see that asymptotic expressions allow us to describe the entire range of the radiation frequency shift.

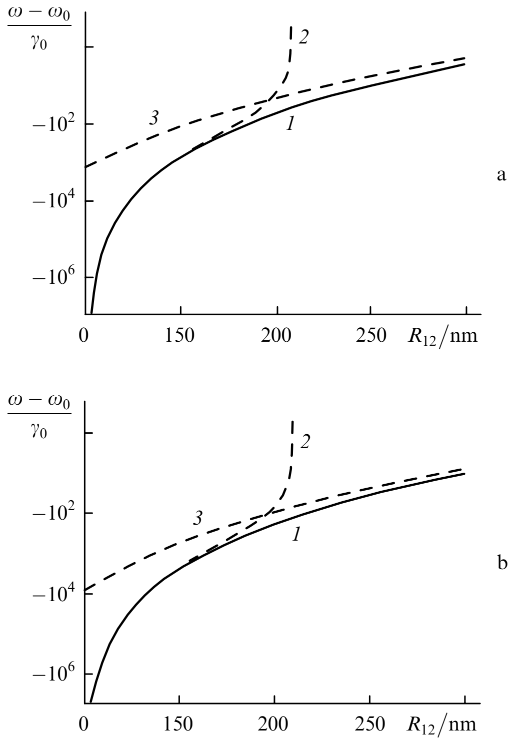

Figure 2. Radiation frequency shift of an atom (molecule) as a function of the distance between spheres (*1*). The spheres have radii 50 nm, $k_0R_0 = 0.1$. The atom is located exactly midway between nanospheres. The dipole moment is oriented along the x (a) and z (b) axes. Curves (*2*) and (*3*) are asymptotics $\eta_0 \to 0$ and $\eta_0 \to \infty$, respectively.

5. Conclusions

We have studied the radiation frequency shift of an atom located near a cluster consisting of two perfectly conducting nanospheres. Rather simple expressions have been obtained for a cluster of two identical spheres and a few characteristic positions of the atom. These expressions describe the radiation frequency shift within the framework of the model used. Simple asymptotic dependences have been found which allow a prompt estimate of the radiation frequency shift for an atom located between spheres. These analytic expressions can be used to estimate the accuracy of numerical calculations in more complicated and realistic cases corresponding to real materials of nanospheres.

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