

Propagation of a partially coherent laser beam through the inhomogeneous medium of an optical amplifier

A.V. Gulevich, D.V. Evtodiev, O.F. Kukharchuk, A.A. Suvorov

Abstract. The propagation of a partially coherent laser beam through a single-pass optical amplifier is considered in the complex geometric optics approximation. A system of equation is obtained which describes changes in the complex amplitude and complex phase of the coherence function of the laser beam in the inhomogeneous medium of the amplifier. The power and intensity of the amplified radiation are analysed as functions of the coherence radius of the laser beam and the optical strength of a gas lens produced in the amplifier medium due to the spatial inhomogeneity of the energy deposition. It is found that the effect of the gas lens and partial coherence of radiation on the gain depends on the relation between the input beam radius and the amplifier aperture. It is shown that the energy gain increases with increasing the lens strength and improving the beam coherence, whereas the opposite behaviour is inherent in 'narrow' beams.

Keywords: laser–amplifier, gas lens, refraction, partial coherence, coherence radius, complex geometric optics.

1. Introduction

When an optical amplifier operates in the saturated regime, the energy stored in its active medium is most completely converted to the energy of coherent optical radiation. Because the equations describing the interaction of radiation with the atoms of the active medium in the saturated regime are essentially nonlinear, the relation between the amplitude–phase characteristics of the input and output laser beams is rather complicated. This relation is determined by the ratio of the laser beam intensity at the amplifier input to the saturation intensity, the input beam divergence, and the spatial distributions of the refractive index and unsaturated gain of the amplifier medium. As in the cases of nonlinear-optical processes of the thermal self-action of laser radiation [1], generation of harmonics [2] and an increase of the energy of plasma electrons in the

radiation field [3], the gain depends on the degree of coherence of an input laser beam of the amplifier.

Laser systems operating in the master oscillator–amplifier regime and intended for generation of high-quality intense laser beams require typically the pumping of rather large volumes of the active media of the master oscillator and amplifying stages. Therefore, irrespective of the laser type and the method of its pumping (for example, an optically pumped neodymium laser [4] or a nuclear-pumped gas laser [5]), the spatially inhomogeneous distribution of the complex permittivity is formed in the active laser medium. The laser beam generated by the master oscillator is formed in the general case due to the superposition of many statistically independent modes and therefore is partially coherent. The optimisation of operation regimes of such systems, the processing and interpretation of experimental results require the theoretical apparatus for simulating the propagation of a laser beam in the active medium of the amplifier with a sufficient accuracy.

Because the spatial distributions of the refractive index and gain of the active medium of the amplifier are quite complicated in the general case, it is impossible to perform analytic studies. It is necessary to use approximate methods, which make it possible, on the one hand, to consider a variety of factors influencing the propagation of a laser beam through the active medium of the amplifier (diffraction due to partially coherent radiation, refraction from the inhomogeneities of the permittivity, nonlinear gain) and, on the other hand, to simulate rather simply the propagation of a laser beam in an inhomogeneous nonlinear medium. Recently, several such methods were proposed and applied to the problem of the development of an X-ray laser [6] and the problems of radiation propagation in turbulent [7] and dissipative (amplifying) random media [8].

In this paper, we consider the propagation of a partially coherent laser beam in the inhomogeneous medium of an optical amplifier in the geometric optics approximation [9]. This method combines the simplicity of usual geometric optics with the consideration of radiation diffraction in a certain approximation. This circumstance allows us to use the method of complex geometric optics both for a detailed study of the physical features of propagation of a partially coherent beam in the inhomogeneous medium of the amplifier and for a full-scale simulation of the amplifier operation.

We studied in this paper the effect of partially coherent radiation and inhomogeneity of the active medium on the energy parameters of the amplified beam. The calculations were performed for a beam propagating through a nuclear-

A.V. Gulevich, D.V. Evtodiev, O.F. Kukharchuk, A.A. Suvorov State Scientific Center of the Russian Federation–A.I. Leypunsky Institute for Physics and Power Engineering, pl. Bondarenko 1, 249033 Obninsk, Kaluga region, Russia; e-mail: suvorov@ippe.ru

pumped single-pass amplifier. We chose such a laser system for analysis because at present full-scale calculations and experiments are being conducted on the Set B reactor–laser setup at the SSC RF IPPE [10]. One of the important problems is the study of the setup operation in the master oscillator–amplifier regime, when optical radiation produced in the master oscillator is directed to laser elements operating in the amplifier regime. Because the results of studies of nuclear-pumped lasers show that the optical inhomogeneity of the active medium can be accurately described by the quadratic dependence of the refractive index on the distance from the active element axis (see, for example, [5]), the influence of the medium inhomogeneity on the amplification process was analysed in the aberration-free approximation.

2. Formulation of the problem

Consider the propagation of a partially coherent laser beam in a single-pass amplifier. Let us direct the z axis of the Cartesian coordinate system along the amplifier axis. The radius vector in the plane $z = \text{const}$ is denoted by $\boldsymbol{\rho} = \{x, y\}$. We assume that the second-order coherence function of a laser beam entering the amplifier is described by the expression

$$\Gamma_{20}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = I_0 \exp\left(-\frac{\rho_1^2 + \rho_2^2}{2a^2} - \frac{|\rho_1 - \rho_2|^2}{\rho_c^2}\right), \quad (1)$$

where a is the effective radius of the beam; ρ_c is the coherence radius of the beam; I_0 is the peak radiation intensity at the amplifier input. This representation corresponds to the Gaussian model of a partially coherent wave beam. It is known that in the general case the coherence function is the statistical average of the product of the complex amplitudes $U(\rho_1, z)$ and $U^*(\rho_2, z)$ of the wave beam at different spatial points:

$$\Gamma_2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{c}{8\pi} \langle U(\boldsymbol{\rho}_1, z) U^*(\boldsymbol{\rho}_2, z) \rangle. \quad (2)$$

When the values of arguments $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ coincide, this function is equal to the average beam intensity:

$$\Gamma_2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)|_{\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}} = I(\boldsymbol{\rho}, z).$$

The propagation of the laser beam in the active medium of the amplifier is described in the quasi-optical approximation by the equation for the coherence function (2) (see, for example, [2])

$$2ik \frac{\partial}{\partial z} \Gamma_2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) + (A_1 - A_2) \Gamma_2 + k^2 [\Delta \varepsilon(\boldsymbol{\rho}_1, z) - \Delta \varepsilon(\boldsymbol{\rho}_2, z)] \Gamma_2 - ik[\alpha(\boldsymbol{\rho}_1, z) + \alpha(\boldsymbol{\rho}_2, z)] \Gamma_2 = 0 \quad (3)$$

with the initial condition

$$\Gamma_2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)|_{z=0} = \Gamma_{20}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2),$$

where $\Delta \varepsilon(\boldsymbol{\rho}, z)$ is the change of the real part of the permittivity of the medium; $\alpha(\boldsymbol{\rho}_1, z)$ is the gain in the

medium; $k = 2\pi/\lambda$ is the wave number; and $A_{1,2} = \partial^2/\partial x_{1,2}^2 + \partial^2/\partial y_{1,2}^2$.

In the general case, two waves propagate in the amplifier volume: the wave transmitted through the input aperture of the amplifier and the diffracted wave formed upon reflection of the transmitted wave from the side surface of the laser element. We will neglect here the contribution of the diffracted wave in the amplified radiation field assuming that its intensity is much lower than that of the transmitted wave. This case corresponds to the laser element design at which the side surface of the element completely absorbs the incident radiation and does not produce reflected waves. The problem in this approximation is reduced to the study of the laser beam propagation in the gain medium with an infinite cross section. The finite transverse dimensions of the amplifying element are manifested in their influence on the amount of radiation energy entering the amplifier through its input aperture and leaving the amplifier through its output aperture.

3. Method of complex geometric optics

We will solve problem (3) in the complex geometric optics approximation [9]. First, we pass to new variables

$$\mathbf{R}_1 = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2 \quad \text{and} \quad \mathbf{r} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$$

in the equation for the coherence function. This equation in these variables takes the form

$$2ik \frac{\partial}{\partial z} \Gamma_2(\mathbf{R}, \mathbf{r}, z) + 2\vec{\nabla}_R \vec{\nabla}_r \Gamma_2(\mathbf{R}, \mathbf{r}, z) + k^2 \varepsilon_R(\mathbf{R}, \mathbf{r}, z) \Gamma_2(\mathbf{R}, \mathbf{r}, z) - ik[\alpha(\mathbf{R} + \mathbf{r}/2, z) + \alpha(\mathbf{R} - \mathbf{r}/2, z)] \Gamma_2(\mathbf{R}, \mathbf{r}, z) = 0, \quad (4)$$

where $\varepsilon_R(\mathbf{R}, \mathbf{r}, z) = \Delta \varepsilon(\mathbf{R} + \mathbf{r}/2, z) - \Delta \varepsilon(\mathbf{R} - \mathbf{r}/2, z)$. Second, we represent the solution of Eqn (4) in the form

$$\Gamma_2(\mathbf{R}, \mathbf{r}, z) = A_0(\mathbf{R}, \mathbf{r}, z) \exp[ik\Psi(\mathbf{R}, \mathbf{r}, z)]. \quad (5)$$

By substituting (5) into (4), we obtain in the zeroth-order approximation the system of equations for the complex amplitude A_0

$$\frac{\partial}{\partial z} A_0 + \vec{\nabla}_r \Psi \cdot \vec{\nabla}_R A_0 + \vec{\nabla}_R \Psi \cdot \vec{\nabla}_r A_0 + A_0 \vec{\nabla}_R \vec{\nabla}_r \Psi - \frac{1}{2} [\alpha(\mathbf{R} + \mathbf{r}/2, z) + \alpha(\mathbf{R} - \mathbf{r}/2, z)] A_0 = 0; \quad (6)$$

and the complex phase

$$\frac{\partial \Psi}{\partial z} + \vec{\nabla}_r \Psi \cdot \vec{\nabla}_R \Psi - \frac{\varepsilon_R}{2} = 0; \quad (7)$$

The initial conditions for this system of equations are

$$A_0(\mathbf{R}, \mathbf{r}, z)|_{z=0} = I_0, \quad (8)$$

$$\Psi(\mathbf{R}, \mathbf{r}, z)|_{z=0} = i(a_1 R^2 + a_2 r^2), \tag{9}$$

where $a_1 = 1/ka^2$; $a_2 = (1/4a^2 + 1/\rho_c^2)/k$; and $\vec{\nabla}_{r,R}$ is the nabla operator in the coordinates \mathbf{r} and \mathbf{R} .

The representation of the coherence function in form (5) in the zeroth-order approximation of the geometric optics method reduces the solution of second-order equation (4) to the solution of the system of first-order equations (6) and (7) for the amplitude A_0 and phase Ψ of the coherence function, respectively. Unlike conventional geometric optics, in the method of complex geometric optics the initial value of the phase (for $z = 0$) includes the argument of the Gaussian component (1), and the function Ψ becomes complex (the complex phase of the coherence function). Correspondingly, the amplitude A_0 is also a complex function. Such a representation allows one, by preserving the main concepts and advantages of the geometric optics method, to take into account the diffraction change in the radius of an amplified partially coherent laser beam.

The solution of system (6), (7) depends on the spatial distributions of the permittivity and gain. We will study the combined action of a partial coherence of radiation and the optical inhomogeneities of the active medium on the beam gain by assuming that the field distribution

$$\Delta\varepsilon(\rho, z) = -\beta^2 \rho^2 \tag{10}$$

is formed in the medium, where β^2 is the refraction parameter determined by the scale of variations and the value of inhomogeneities of the permittivity. The function ε_R corresponding to such distribution $\Delta\varepsilon$, which, according to (7), determines variations in the complex phase Ψ , has the form

$$\varepsilon_R = -2\beta^2 \mathbf{R}r. \tag{11}$$

Experiments with nuclear-pumped lasers (see, for example, [5, 11]) showed that distribution (10) describes quite accurately the beam focusing by the active medium in the axial region of the amplifier and can be used to simulate refraction properties over the entire cross section of the amplifier. The dependence of the gain on the average radiation intensity I in nuclear-pumped lasers is described by the expression

$$\alpha = \alpha_0 / (1 + I/I_s), \tag{12}$$

where α_0 is the unsaturated gain and I_s is the saturation intensity. We will assume that α_0 and I_s are constant in the amplifier volume and therefore the inhomogeneity α is determined by the intensity distribution of the amplified beam.

We will solve the system of equations (6), (7) by the method of characteristics for solving the Cauchy problem. The trajectories of Eqn (7) with the distribution ε_R (11) are determined by the system of equations

$$\frac{d^2 \mathbf{R}}{dz^2} = -\beta^2 \mathbf{R}, \tag{13}$$

$$\frac{d^2 \mathbf{r}}{dz^2} = -\beta^2 \mathbf{r}$$

with the initial conditions

$$\mathbf{R}|_{z=0} = \mathbf{R}_0, \quad \left. \frac{d\mathbf{R}}{dz} \right|_{z=0} = i2a_2 \mathbf{r}_0, \tag{14}$$

$$\mathbf{r}|_{z=0} = \mathbf{r}_0, \quad \left. \frac{d\mathbf{r}}{dz} \right|_{z=0} = i2a_1 \mathbf{R}_0.$$

The solutions of the characteristic system have the form

$$\mathbf{R}[\xi] = \frac{\mathbf{R}}{g(z)} \left(\cos \beta z \cos \beta \xi + \frac{4a_1 a_2}{\beta^2} \sin \beta z \sin \beta \xi \right) - i \frac{\mathbf{r}}{g(z)} \frac{2a_2}{\beta} \sin[\beta(z - \xi)], \tag{15}$$

$$\mathbf{r}[\xi] = \frac{\mathbf{r}}{g(z)} \left(\cos \beta z \cos \beta \xi + \frac{4a_1 a_2}{\beta^2} \sin \beta z \sin \beta \xi \right) - i \frac{\mathbf{R}}{g(z)} \frac{2a_1}{\beta} \sin[\beta(z - \xi)], \tag{16}$$

where $g(z) = \cos^2 \beta z + (4a_1 a_2 \sin^2 \beta z) / \beta^2$; \mathbf{R}, \mathbf{r} , and z are the coordinates of observation points and $0 \leq \xi \leq z$. By integrating Eqn (7) with initial condition (9) along trajectories (15), (16), we obtain the expression for the complex phase

$$\Psi(\mathbf{R}, \mathbf{r}, z) = - \frac{[\mathbf{R}r(\beta^2 - 4a_1 a_2) \sin 2\beta z] / 2\beta - i(a_1 R^2 + a_2 r^2)}{g(z)}. \tag{17}$$

Equation (6) for the complex amplitude of the coherence function on trajectories (15), (16) has the form

$$\frac{d}{d\xi} A_0 = -A_0 \vec{\nabla}_R \vec{\nabla}_r \Psi + \frac{1}{2} \alpha_0 \times \left[\frac{A_0}{1 + (A_0/I_s) \exp(-k\Psi_{\text{Im}}^+(\xi))} + \frac{A_0}{1 + (A_0/I_s) \exp(-k\Psi_{\text{Im}}^-(\xi))} \right], \tag{18}$$

where

$$\Psi_{\text{Im}}^\pm(\xi) = [\Psi(\mathbf{R}(\xi) \pm \mathbf{r}(\xi) / 2, 0, \xi) - \text{c.c.}] / 2i. \tag{19}$$

By substituting expression (17) for the complex phase Ψ on trajectories (15), (16) into Eqn (18) and assuming that $\mathbf{r} = 0$, we obtain the equation for the function J determining the intensity of the amplified beam:

$$\frac{dJ}{d\xi} = \frac{1}{2} \alpha_0 J \left\{ \frac{1}{1 + [J/g(\xi)] \exp[-k\Psi_0^+(\xi)]} + \frac{1}{1 + [J/g(\xi)] \exp[-k\Psi_0^-(\xi)]} \right\}, \tag{20}$$

where $J = J(\mathbf{R}, \xi) = g(\xi) A_0(\mathbf{R}, \mathbf{r}, \xi) / I_s|_{r=0}$; $\Psi_0^\pm = \Psi_{\text{Im}}^\pm|_{r=0}$. The initial condition for Eqn (19) is $J|_{\xi=0} = I_0 / I_s$.

The function J obtained by solving Eqn (20) and the imaginary part $\text{Im}\Psi$ of the complex phase (17) determine the intensity of the amplified beam at the point $\rho = \mathbf{R}$ of the section $z = \text{const}$ as

$$I(\mathbf{R}, z) = \frac{J(\mathbf{R}, z)}{g(z) I_s} \exp(-k \text{Im}\Psi(\mathbf{R}, 0, z)). \tag{21}$$

In this paper, we solved Eqn (20) numerically for different values of the beam radius a , the coherence radius ρ_c , the ratio of the peak intensity of the input radiation to the saturation intensity I_s , and the refraction parameter β . All calculations were performed for the unsaturated gain of the active medium $\alpha_0 = 10^{-2} \text{ cm}^{-1}$. We used the values of the parameter β typical for a nuclear-pumped He–Ar–Xe (Ar–Xe) laser [11].

4. Discussion of results

The amplifier efficiency is characterised by the radiation intensity distribution over the beam cross section and radiation power. Our calculations showed that the dependences of these characteristics on the parameters of the problem are similar for different relations between the peak intensity I_0 of the input radiation and the saturation intensity I_s . Therefore, we will discuss here the results obtained for a laser beam propagating through the amplifier in the saturation regime for $I_0/I_s = 1$ and different relations between the radii of the beam and amplifier aperture.

Figure 1 shows the peak intensity of the laser beam normalised to the saturation intensity calculated as a function of the distance propagated by the beam in the amplifier. One can see that, as the refraction parameter β increases, the other conditions being the same, the radiation intensity on the beam axis also increases. This occurs because the focusing action of the medium is enhanced with increasing β , resulting in a more efficient extraction of

the energy stored in the amplifier. A comparison of the solid and dashed curves shows that an increase in the peak intensity of the coherent beam is larger than that for a partially coherent beam. This is caused by an increase in the divergence of the partially coherent beam, which weakens the focusing action of the medium. In addition, it follows from a comparison of the solid and dashed curves in Figs 1a and 1b that the influence of the beam divergence on the growth of the radiation intensity at the amplifier axis decreases with increasing the beam radius. One can also see that a linear dependence of the beam intensity on the distance propagated by the beam, which is typical for the saturated amplification regime, is realised only in the case of amplification of a coherent beam in a homogeneous medium [see dashed curves (1)].

Figure 2 shows the calculated relative radial intensity distributions for the output beam of the amplifier. Curves (4) correspond to the radial distribution of the relative intensity of a laser beam entering the amplifier. One can see that because of a greater divergence, a partially coherent beam is broader than the coherent beam. As the optical strength of a gas lens increases (parameter β increases), the beam narrows down. This effect is more pronounced in the case of a relatively narrow beam entering the amplifier (Fig. 2a).

Figure 3 shows the relative radiation power at the amplifier output calculated as a function of the distance propagated by the beam. The beam power was determined by the integral from the intensity

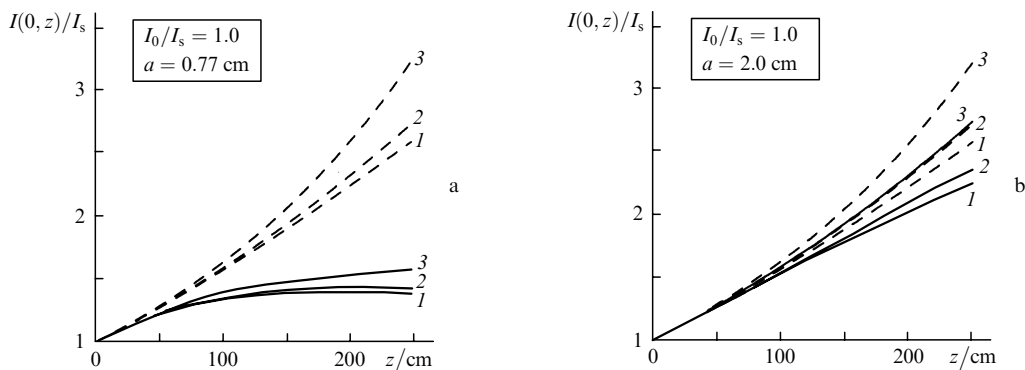


Figure 1. Dependences of the normalised intensity on the beam axis on z for different a and $\beta = 0$ (1), 10^{-3} (2) and $2 \times 10^{-3} \text{ cm}^{-1}$; $\rho_c = 0.2 \text{ mm}$ (solid curves) and $\rho_c = \infty$ (dashed curves).

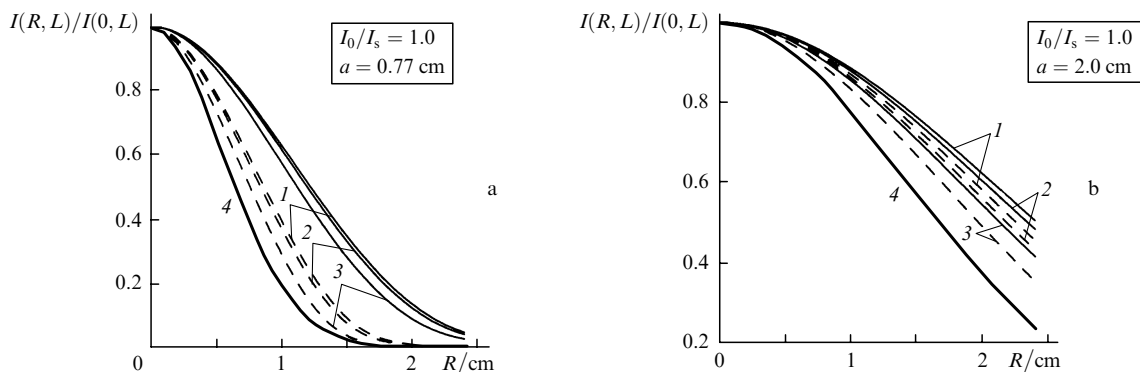


Figure 2. Radial distributions of the normalised beam intensity at the amplifier output for the same parameters as in Fig. 1; curves (4) correspond to the relative intensity distribution of the input beam of the amplifier.

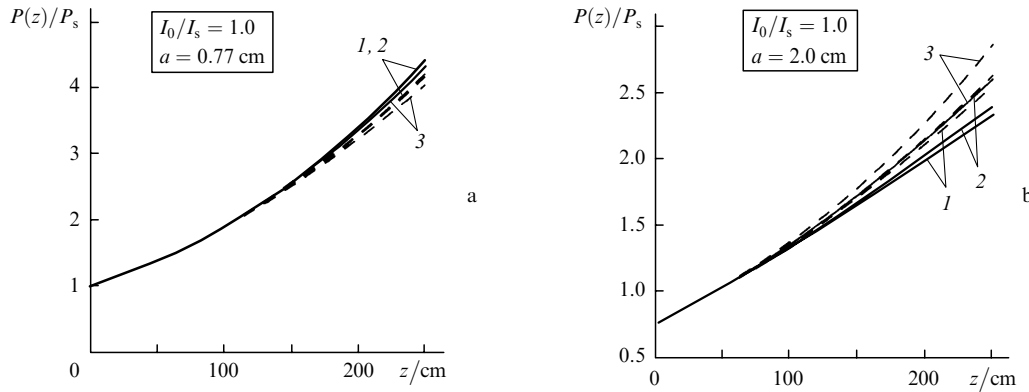


Figure 3. Dependences of the normalised power on the beam axis on z for the same parameters as in Fig. 1.

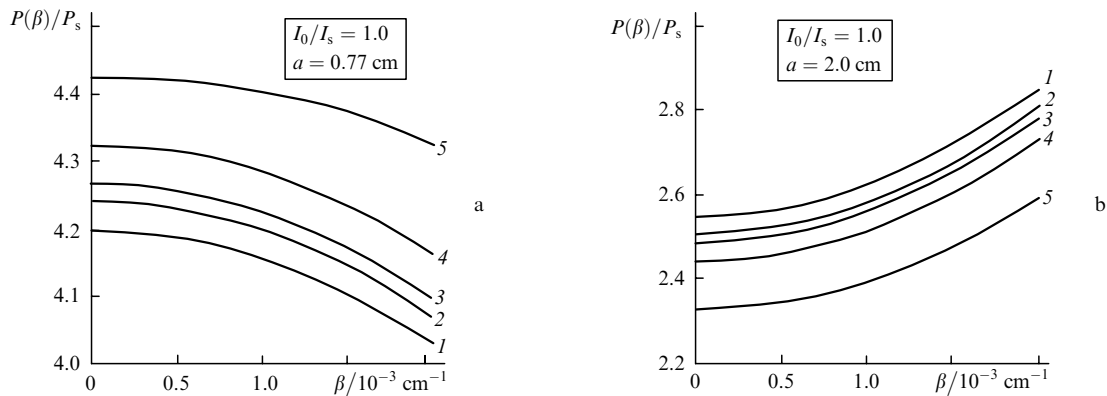


Figure 4. Dependences of the normalised power at the amplifier output on β for $\rho_c = \infty$ (1), 0.5 (2), 0.4 (3), 0.3 (4), and 0.2 mm (5).

$$P(z) = \iint_S d^2R I(R, z), \quad (22)$$

where integration was performed over the amplifier aperture with the radius $R_0 = 2.4$ cm. The saturation power to which normalisation was performed was defined as $P_s = \pi a^2 I_s$. It follows from Fig. 3 that the amplified beam power depends on the degree of coherence and the refraction parameter β much weaker than the intensity (cf. Figs 1 and 3). However, a comparison of Figs 3a and b demonstrates a qualitative difference in the increase in powers of the narrow ($a = 0.77$ cm) and broad ($a = 2.0$ cm) beams. The output power of the amplifier for the narrow beam is higher for a beam with a smaller coherence radius ρ_c propagating in the homogeneous amplifier ($\beta = 0$). For the broad beam, the dependence is opposite. This effect is more clearly illustrated in Fig. 4, where the calculated dependence of the relative power of the output beam on the refraction parameter β is presented for different coherence radii of the input beam.

Let us explain qualitatively these results. The volume occupied by a laser beam in the active region increases with increasing the beam radius. On the one hand, this leads to an increase in the beam energy due to the release of the stored inversion energy. On the other hand, the laser radiation flux incident on the side surface of the amplifier also increases. Because this radiation is completely absorbed by the side surface (in the approximation used in our paper), the energy growth caused by amplification decreases. The total effect (increase of the output power of the amplifier)

depends on the relation between these two competing mechanisms. Thus, if the radius of the input Gaussian beam is 2.0 cm and does not differ strongly from the amplifier aperture radius ($R_0 = 2.4$ cm), then the growth of the laser radiation power due to the reduction of losses on the laser element walls with increasing the focusing action (parameter β) of the medium or improving the beam coherence will be more substantial than a decrease in the power growth due to inversion removal (a decrease in the volume occupied by the beam in the amplifier). Therefore, the amplification of the input radiation power increases (see, for example, Fig. 4b). If the input beam radius is much smaller than the amplifier aperture radius, the situation can be opposite. For example, for $a = 0.77$ cm, the power gain decreases with increasing the refraction parameter β and the coherence radius β_c because in this case the losses on the side surface of the amplifier due to the beam narrowing decrease slower than the rate of the power increase due to the removal of the inversion energy (see, for example, Fig. 4a).

5. Conclusions

We have analysed the propagation of a partially coherent laser beam through a single-pass amplifier. The study has been performed by the method of complex geometric optics allowing the reduction of the second-order partial differential equation for the coherence function to a system of ordinary differential equations taking the beam diffraction into account. Within the framework of this method, we analysed the influence of a partial coherence of the incident

radiation and a regular inhomogeneity of the permittivity of the amplifier active medium on the amplification process.

Our simulations have shown that the power gain in an inhomogeneous medium substantially depends on the relation between the radii of the beam and amplifier aperture. When the input beam radius is much smaller than the amplifier aperture radius, a decrease in the optical inhomogeneity of the medium and divergence of the input beam leads to a more efficient removal of the inversion energy, resulting in the increase in the output power. When the input beam radius is comparable with that of the amplifier aperture, the opposite situation takes place, which is caused by a decrease in the fraction of energy absorbed by the side surface of the amplifier.

Note that in this paper we mainly analysed variations in the energy characteristics of the beam inside the amplifier and at the amplifier output. At the same time, the method developed here can be also used for numerical simulations of the beam propagation at large distances between the amplifier and a detector, when phase variations in the laser beam lead to more considerable variations in its intensity, which should be taken into account in the interpretation of the results of measurements. Note also that the method of complex geometric optics allows one to solve numerically a system of equations for the complex amplitude and phase both in the aberration-free approximation for the medium inhomogeneity and for distributions of the permittivity field in the amplifier corresponding to the real pumping conditions. These circumstances allowed us to use this method not only for studying the influence of numerous factors on the amplification of a partially coherent beam (which is the subject of this paper) but also for simulating amplification experiments performed on the Set B reactor–laser setup. At present, we are preparing for publication the results of our calculations and experimental studies of the operation of this setup in the master oscillator–amplifier regime.

Acknowledgements. The authors thank R.Kh. Almaev for useful discussions of the results of this paper. This work was supported by the Russian Foundation for Basic Research and the government of the Kaluga region (Grant No 04-02-97239).

References

1. Vorob'ev V.V. *Teplovoe samovozdeistvie lazernogo izlucheniya v atmosfere: teoriya i model'nyi eksperiment* (Thermal Self-Action of Laser Radiation in Atmosphere: Theory and Model Experiment) (Moscow: Nauka, 1987).
2. Akhmanov S.A., D'yakov Yu.E., Chirkin A.S. *Vvedenie v statisticheskuyu radiofiziku i optiku* (Introduction to Statistical Radiophysics and Optics) (Moscow: Nauka, 1981).
3. Budnik A.P., Svirkunov P.N. *Trudy Inst. Eksp. Meteor.*, **49**, 31 (1989).
4. Mezenev A.V., Soms L.N., Stepanov A.I. *Termooptika tverdotel'nykh lazerov* (Thermal Optics of Solid-State Lasers) (Leningrad: Mashinostroenie, 1986).
5. Karelin A.V., Sinyanskii A.A., Yakovlenko S.I. *Kvantovaya Elektron.*, **24**, 387 (1997) [*Quantum Electron.*, **27**, 375 (1997)].
6. Gasparyan P.D., Starikov F.A., Starostin A.N. *Usp. Fiz. Nauk*, **168**, 843 (1998).
7. Kandidov V.P. *Usp. Fiz. Nauk*, **166**, 1309 (1996).
8. Dudorov V.V., Kolosov V.V. *Kvantovaya Elektron.*, **28**, 115 (1999) [*Quantum Electron.*, **29**, 672 (1999)].
9. Kravtsov Yu.A. *Izv. Vyssh. Uchebn. Zaved. Ser. Radiofiz.*, **10**, 1283 (1967).
10. D'yachenko P.P., in *Mater. III Mezhdunarodn. konf. 'Problemy lazerov s yadernoi nakachkoi i impul'snye reaktory'* (Proceedings of III International Conference on Problems of Nuclear-Pumped Lasers and Pulsed Reactors) (Snezhinsk; Izd. RFNC–ARRITP, 2003) p. 5.
11. Poletaev E.D., Golovchenko S.A., Dyuzhov Yu.A., et al., in *Trudy regional'nogo konkursa nauchnykh proektov v oblasti estestvennykh nauk* (Proceedings of the Regional Competition of Scientific Projects in the Field of Natural Sciences) (Kaluga: Poligraf-inform, 2004) No 6, p. 173.