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On the renormalisation of the diffusion asymptotics in the problem of reflection of a narrow optical beam from a biological medium

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***Abstract*. An analytic hybrid method is considered for solving the stationary radiation transfer equation in the problem on reflection of a narrow laser beam from biological media such as the 2 % aqueous solution of intralipid and erythrocyte suspension with the volume concentration (hematocrit) $H = 0.41$. The method is based on the reciprocity of the Green function in the radiation transfer theory and on the iteration solution of the integral equation for this function. As a result, the ray intensity is represented as a sum of two terms. The first of them describes the contribution of finite-order scattering to the intensity of a beam diffusely reflected from the medium. The second term contains the explicit analytic expression for a spatially distributed effective source of diffuse radiation emerging from the deep layers of the medium to the surface. This approach substantially improves the diffusion approximation for the problem under study and allows one to obtain the uniform asymptotics of the reflection coefficient at the specified interval of distances between the radiation source and detector on the medium surface with the relative error within ± 6 % for the 2 % intralipid emulsion and erythrocyte suspension ($H = 0.41$).**

Keywords: *radiation transfer theory, scattering medium, monochromatic narrow beam, diffusion approximation, Green function reciprocity.*

1. Introduction

The method of probing biological media by the reflection of a narrow laser beam is widely used in the optics of biological tissues [1 – 11]. In this method, a narrow optical beam is incident on the surface of a medium under study. Radiation propagates in the scattering medium and is detected after diffusion reflection at some distance along the surface from the beam incidence point. In this case, according to experimental papers [3], radiation (photons) incident on the detector propagates in the homogeneous scattering medium to the detector along the paths in the form of a banana with its ends near the beam incidence and emerging points. Figure 1 shows schematically the regions of radiation propagation from the source to two detectors in the medium under study in the case of non-invasive measuring the oxygenation degree of blood by the reflection method [4].

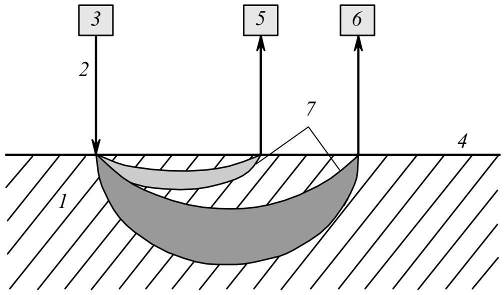

Figure 1. Regions of radiation propagation from a source to a detector: (*1*) tissue region; (*2*) narrow laser beam; (*3*) radiation source; (*4*) medium surface; (*5*,*6*) photodetectors; (*7*) radiation intensity distribution in the radiation propagation regions (bananas).

The distribution of diffusely reflected radiation along the surface and the intensity distribution in the propagation region from the source to detector inside the medium can be calculated by using the conventional diffusion approximation or its modification in the radiation transfer theory (RTT) [5 – 10]. However, this approximation does not always provide a sufficient accuracy and has two main disadvantages. First, it neglects the contribution from small-order scattering to radiation. Second, the diffusion equation uses a point source [10, 11], which reduces the accuracy of this approximation.

The intensity distribution of a diffusely reflected beam along the medium surface was studied in a number of papers [12 – 14] by the Monte-Carlo method in the RTT. However, this method is of the stochastic type and requires time-consuming computer calculations. This stimulated the development of rather economical hybrid models combining the accuracy of the Monte-Carlo method with the diffusion approximation allowing the rapid solution of the problem [15, 16]. Such models improve the accuracy of the diffusion approximation due to the consideration of finite-order scattering at small distances from the source and the construction of a spatially distributed effective source for the diffusion equation. These models are based to a large

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extent on clear heuristic concepts and should be substantiated from the point of view of the radiation transfer equation, which is the object of this paper.

We propose a consistent iteration approach for improving the diffusion approximation at low scattering orders, which uses the reciprocity of the Green function in the RTT and the iteration representation of the integral equation for the Green function in the reciprocal form. Note that the reciprocity of the Green function substantiates the clear intuitive concepts in papers [15, 16]. Such a 'microscopic' hybrid approach from the point of view of the RTT considerably improves the diffusion approximation for the problem on reflection of a narrow collimated beam from a biological medium and gives the uniform asymptotics of the reflection coefficient at the specified interval of distances between the source and detector with the relative error within ±6 % (with respect to the results of standard numerical Monte-Carlo simulations). This error of the improved diffusion approximation is also inherent in media with the strongly elongated indicatrix of the elementary scattering event such as erythrocyte suspensions ($H = 0.41$) [the average cosine of the elementary scattering event (the anisotropy factor) is $g = 0.992$] [17].

Note that the above-mentioned hybrid method is in fact a certain way for renormalising the conventional diffusion asymptotics to improve its accuracy. From the practical point of view, it is interesting to formulate the method for renormalisation of the diffusion asymptotics, which is simpler than the hybrid method, preserving, if possible, the accuracy of the latter. Below, we will present considerations in favour of the existence of such a simplified renormalisation.

2. 'Microscopic' hybrid approach in the RTT

We will use the general radiation transfer equation in a scattering medium (see, for example, [10]) for the ray intensity $I(\boldsymbol{r},\boldsymbol{s})$ at the point $\boldsymbol{r}$ along the unit vector $\boldsymbol{s}$. The solution of this equation can be written, by using the Green function $G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}')$ and the specified radiation source $Q(\boldsymbol{r},\boldsymbol{s})$, in the form

$$I(\boldsymbol{r},\boldsymbol{s}) = \int \mathrm{d}\boldsymbol{r}' \int \mathrm{d}\Omega' G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}')Q(\boldsymbol{r}',\boldsymbol{s}'), \quad (1)$$

or in the symbolic operator form $I = GQ$. In (1), $\mathrm{d}\Omega'$ is the element of the solid angle in the direction of the unit vector $\boldsymbol{s}'$ upon integration over the entire solid angle 4π. It is known that the Green function $G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}')$ satisfies the integral equation (see, for example, [10]), which has the form

$$G(\boldsymbol{r},\boldsymbol{s},\boldsymbol{r}',\boldsymbol{s}') = G_0(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}') + \int \mathrm{d}r'' \int_{4\pi} \mathrm{d}\Omega''$$

$$\times \int_{4\pi} \mathrm{d}\Omega''' G_0(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}'',\boldsymbol{s}'')\Sigma(\boldsymbol{s}'',\boldsymbol{s}''')G(\boldsymbol{r}'',\boldsymbol{s}''';\boldsymbol{r}',\boldsymbol{s}'). \quad (2)$$

Here, G_0 is the Green function for a direct attenuated radiation source, which can be written in the form

$$G_0(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}') = \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|^2} \times$$

$$\times \exp(-\mu_{\mathrm{t}}|\boldsymbol{r}-\boldsymbol{r}'|)\delta(\boldsymbol{s}-\boldsymbol{s}_{\boldsymbol{rr}'})\delta(\boldsymbol{s}-\boldsymbol{s}'), \quad (3)$$

where $\mu_{\mathrm{t}} = \mu_{\mathrm{s}} + \mu_{\mathrm{a}}$ is the extinction coefficient; μ_{s} and μ_{a} are the scattering and absorption coefficients; and $\boldsymbol{s}_{\boldsymbol{rr}'}$ is the unit vector from the point $\boldsymbol{r}'$ to the point $\boldsymbol{r}$. The differential scattering cross section $\Sigma(\boldsymbol{s},\boldsymbol{s}')$ for the elementary volume of the medium in Eqn (2) is determined by the equality

$$\Sigma(\boldsymbol{s},\,\boldsymbol{s}') = \frac{\mu_{\mathrm{t}}}{4\pi}\,p(\boldsymbol{s},\boldsymbol{s}')$$

with the phase function $p(\boldsymbol{s},\boldsymbol{s}')$ normalised to the albedo

$$a = \frac{\mu_{\mathrm{s}}}{\mu_{\mathrm{t}}} = \int \frac{\mathrm{d}\Omega}{4\pi}\,p(\boldsymbol{s},\boldsymbol{s}').$$

The Green function for radiation transfer satisfies the reciprocity relation [18], which has a clear physical meaning and is written in the form

$$G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}') = G(\boldsymbol{r}',-\boldsymbol{s}';\boldsymbol{r},-\boldsymbol{s}). \quad (4)$$

In our approach, this reciprocity property plays a decisive role, allowing one to rewrite integral equation (2) in the reciprocal form

$$G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}') = G_0(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}',\boldsymbol{s}') + \int \mathrm{d}r'' \int_{4\pi} \mathrm{d}\Omega''$$

$$\times \int_{4\pi} \mathrm{d}\Omega''' G(\boldsymbol{r},\boldsymbol{s};\boldsymbol{r}'',\boldsymbol{s}'')\Sigma(\boldsymbol{s}'',\boldsymbol{s}''')G_0(\boldsymbol{r}'',\boldsymbol{s}''';\boldsymbol{r}',\boldsymbol{s}'). \quad (5)$$

Equation (5) was obtained from (2) after elementary transformations taking into account the reciprocity relation $p(\boldsymbol{s},\boldsymbol{s}') = p(-\boldsymbol{s}',-\boldsymbol{s})$ for the phase function and can be written in the symbolic operator form as $G = G_0 + G\Sigma G_0$. The first iteration of this equation gives the relation $G = G_0 + G_0\Sigma G_0 + G\Sigma G_0\Sigma G_0$. Already from this first iteration, after its convolution with the specified radiation source Q, the origin of the term $I^{(0\ldots1)} = G_0Q + G_0\Sigma G_0 Q$ becomes clear. This term describes contributions of the zero- and first-order scattering to the ray radiation intensity. The contribution of double scattering to the effective source $Q^{(2)} = \Sigma G_0 \Sigma G_0 Q$ characterises radiation from the volume of the scattering medium. After multiple iterations of Eqn (5) and its further convolution with the specified radiation source Q, the ray intensity $I(\boldsymbol{r},\boldsymbol{s})$ proves to be equal to the sum of two terms

$$I = I^{(0\ldots N)} + GQ^{(N+1)}. \quad (6)$$

The first term $I^{(0\ldots N)}$ takes into account the contributions $I^{(n)} = G_0(\Sigma G_0)^n Q$ (where $n = 0,\ldots,N$) of finite-order scattering to the ray intensity and is represented by the sum

$$I^{(0\ldots N)} = I^{(0)} + \sum_{n=1}^{N} I^{(n)}. \quad (7)$$

The second term in (6) describes the radiation of the effective source $Q^{(N+1)} = \Sigma I^{(N)}$, which is formed by scattering with the finite order $N+1$. The contribution $I^{(n)}$ of scattering of order n into the ray intensity can be written as the multiple integral along the linear intervals of scattering paths $\boldsymbol{ls}$, $l_1\boldsymbol{s}_1, \ldots, l_n\boldsymbol{s}_n$:

$$I^{(n)}(\boldsymbol{r},\boldsymbol{s}) = \int_0^\infty \mathrm{d}l \exp(-\mu_t l) \int_0^\infty \mathrm{d}l_1 \exp(-\mu_t l_1)$$

$$\ldots \int_0^\infty \mathrm{d}l_n \exp(-\mu_t l_n) \int \Sigma(\boldsymbol{s},\boldsymbol{s}_1)\mathrm{d}\Omega_1 \int \Sigma(\boldsymbol{s}_1,\boldsymbol{s}_2)\mathrm{d}\Omega_2$$

$$\ldots \int \Sigma(\boldsymbol{s}_{n-1},\boldsymbol{s}_n)\mathrm{d}\Omega_n Q(\boldsymbol{r} - l\boldsymbol{s} - l_1\boldsymbol{s}_1 - \ldots - l_n\boldsymbol{s}_n, \boldsymbol{s}_n). \quad (8)$$

After the calculation of this multiple integral along the scattering paths, the density of the effective source takes the form

$$Q^{(N+1)}(\boldsymbol{r},\boldsymbol{s}) = \int \Sigma(\boldsymbol{s},\boldsymbol{s}')I^{(N)}(\boldsymbol{r},\boldsymbol{s}')\mathrm{d}\boldsymbol{s}'. \quad (9)$$

Expressions (8) and (9) form the basis of the 'microscopic' hybrid approach to the calculation of diffusion reflection of a narrow collimated beam from a semibounded scattering medium. They substantiate clear heuristic considerations of the hybrid method developed in [15, 16]. In this case, the radiation source $Q(\boldsymbol{r},\boldsymbol{s})$ is located at the coordinate origin on the medium surface and is specified by the expression $Q(\boldsymbol{r},\boldsymbol{s}) = Q_0\delta(\boldsymbol{r})\delta(\boldsymbol{s}-\boldsymbol{s}_0)$, where $\boldsymbol{s}_0$ is the unit vector directed along the incident beam.

So far we used exact relations within the framework of the RTT. In the 'microscopic' hybrid approach, we replace the exact Green function G in the second term of the right-hand side of Eqn (6) by its conventional diffusion asymptotics [15, 19] determined by the equality

$$G_{\mathrm{diff}}(r-r',\theta-\theta',z') = \frac{1}{4\pi}\left[z'\left(\mu_{\mathrm{eff}} + \frac{1}{d_1}\right)\frac{\exp(-\mu_{\mathrm{eff}}d_1)}{d_1^2}\right.$$

$$\left. + (z'+2z_b)\left(\mu_{\mathrm{eff}} + \frac{1}{d_2}\right)\frac{\exp(-\mu_{\mathrm{eff}}d_2)}{d_2^2}\right]. \quad (10)$$

Here, d_1 is the distance between the point source with coordinates r', θ', z' in the cylindrical coordinate system with the origin located at the point $\boldsymbol{r}=0$ on the medium surface and the z axis directed downward perpendicular to this surface, and the observation point with coordinates r, θ, $z=0$; d_2 is the distance between the observation point and an imaginary source, which is obtained from the point source by mirror reflection with respect to the effective boundary plane $z=-z_b$; and $\mu_{\mathrm{eff}} = \{3\mu_a[\mu_a + \mu_s(1-g)]\}^{1/2}$ is the effective attenuation coefficient of radiation in the diffusion approximation. In the case of the absence of beam refraction at the medium boundary, it is assumed that $z_b = 2\tilde{D}$, where $\tilde{D} = 1/\{3[\mu_a + \mu_s(1-g)]\}$ multiplied by the speed of light in vacuum gives the diffusion constant of radiation in a scattering medium. Such a choice of the position of the mirror reflection plane provides the boundary condition of the absence of the scattered radiation flux entering the medium in the diffusion approximation (see details in [19]).

Note that, in our opinion, Eqn (6) gives the uniform asymptotics of the reflection coefficient for all distances if the number of iterations exceeds some critical value.

We consider a semibounded scattering medium on which surface $z=0$ a collimated infinitely narrow beam is incident from the free half-space $z<0$ along the z axis. Our problem is the calculation of the distribution function for diffusely reflected radiation along the medium surface (the reflection coefficient), which is determined by the equality $R(\boldsymbol{r}) = \int_{\boldsymbol{sn}>0} I(\boldsymbol{r},\boldsymbol{s})\boldsymbol{sn}\mathrm{d}\Omega$, where $\boldsymbol{n}$ is the external normal to the medium surface at the point $\boldsymbol{r}$ in the free half-space direction. This function is calculated from expression (6) for the ray intensity. The calculation of the first term in (6), which takes into account the contributions of finite-order scattering to the reflection coefficient $R(r)$, and of the effective source in the second term is reduced to the calculation of multiple integrals (8) along the linear intervals of scattering paths. We used in these calculations the Monte-Carlo method based on papers [14–16], where the phase function was approximated by the postulated Henyey–Greenstein function [1, 2].

Consider a semibounded medium with the scattering coefficient $\mu_s = 54\ \mathrm{cm}^{-1}$, the anisotropy factor $g = 0.7$, and the absorption coefficient $\mu_a = 0.02\ \mathrm{cm}^{-1}$, which correspond to the optical parameters of the model of a biological scattering medium in the form of the 2 % intralipid emulsion [20]. Figures 2 and 3 show the reflection coefficients calculated from Eqn (6) for different scattering orders N. We performed calculations for the distances along the medium surface in the range from 0 to 2 cm, which corresponds to 108 free paths. For comparison, we also present in these figures the reflection coefficients calculated by solving the radiation transfer equation by the Monte-Carlo method [13]. The case $N=0$ corresponds to the conventional diffusion approximation in the RTT [5, 6].

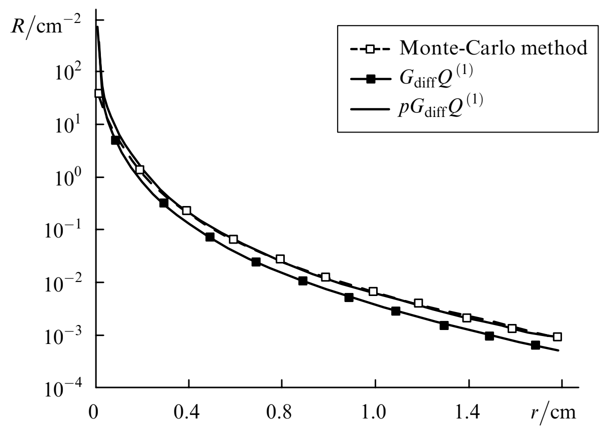

Figure 2. Dependences of the reflection coefficient on the distance between the radiation source and detector obtained by using the conventional diffusion approximation [$N=0$, $G_{\mathrm{diff}}Q^{(1)}$], its renormalisation [$pG_{\mathrm{diff}}Q^{(1)}$], and numerical Monte-Carlo simulation (10^7 photons 'injected' into the medium) for the 2 % intralipid emulsion (renormalisation is described in the text).

One can see from Fig. 2 that the conventional diffusion approximation gives the overstated reflection coefficient at small distances and noticeably understated values of this coefficient at large distances from the incidence point of the beam. In this approximation, small-order scattering described by the first term in (6) was neglected and the effective source $Q^{(1)}$ was mainly concentrated on the interval $0 < z < 1/\mu_t$ along the incident beam axis. As the scattering order N increases, the accuracy of the analytic hybrid method improves. This method involves the calculation of the distribution function $R(r)$ of radiation emerging

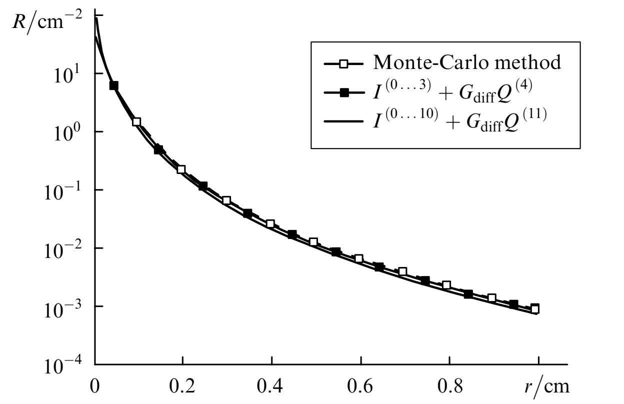

Figure 3. Dependences of the reflection coefficient on the distance between the radiation source and detector obtained by the analytic hybrid method $[I^{(0\ldots N)} + G_{\text{diff}}\,Q^{(N+1)}]$ for $N = 3$ and 10 and the Monte-Carlo method (10^7 photons 'injected' into the medium) for the 2 % intralipid emulsion.

from the medium from Eqn (6). In other words, the hybrid method gives the uniform approximation of the distribution function $R(r)$ both for small and large distances.

Figure 3 shows the profiles of the distribution function $R(r)$ obtained by the standard Monte-Carlo method for solving the formulated problem as a whole and by the analytic hybrid method $[I^{(0\ldots N)} + G_{\text{diff}}Q^{(N+1)}]$ taking into account the scattering order up to $N = 3$ and 10 inclusive. The scattering order N can be treated as a parameter for estimating the accuracy of the analytic hybrid method. The critical scattering order, beginning from which the results obtained by the analytic hybrid method and Monte-Carlo method asymptotically approach each other, is estimated from the expression $N_{\text{cr}} \sim (1-g)^{-1} = 3$. However, the results of calculations of the intensity distribution of radiation leaving the medium presented in Fig. 3 show that this expression is rather rough and the real value of N_{cr} is several times larger.

Figure 4 presents the dependence of the relative error of the analytic hybrid method on r (for the 2 % intralipid emulsion) for $N = 10$. One can see that this error varies from -6 % to $+6$ % for the distances considered, which allows one to set $N_{\text{cr}} = 10$.

3. Approximation of the analytic hybrid method

We described in the previous section the method for constructing an effective distributed source, which improves the accuracy of the diffusion approximation within the framework of the analytic hybrid method. However, it is desirable to use a simplified approximation of this method for practical applications. Therefore, the question arises of the possibility of a rough, but simple simulation of a distributed effective source. To answer this question, consider Fig. 2 where the standard diffusion approximation for $N = 0$ is compared with the result of Monte-Carlo simulations. Note that, beginning from some distance, the curves obtained in the diffusion approximation and by the Monte-Carlo method are equidistant. This allows one to select a renormalisation coefficient p of the diffusion approximation to bring it together with the result of numerical Monte-Carlo simulation at a certain interval [curve $(pG_{\text{diff}}Q^{(1)})$ in Fig. 2]. Such a procedure can be considered as the approximation of the reflection coefficient obtained by the analytic hybrid method. For the 2 % intralipid emulsion, the coefficient $p = 1.7$ and the relative error of the renormalised diffusion approximation lies within ± 6 % (Fig. 4). The practical advantage of this approximation is that it allows the use of the diffusion asymptotics at the distances under study for measuring the relative reflected intensity, which was in fact used in the optical method of measuring the common oxygenation of the venous and arterial blood [4].

Figure 5 presents the results of numerical simulation of the reflection coefficient by the Monte-Carlo method, the analytic hybrid method taking into account scattering orders up to $N = 600$, and by the method of diffusion asymptotic renormalisation with the coefficient $p = 1.6$ for an erythrocyte suspension ($H = 0.41$) with the optical parameters $\mu_s = 668\ \text{cm}^{-1}$, $g = 0.992$, and $\mu_a = 1.68\ \text{cm}^{-1}$ [17]. In this case, the relative error within ± 6 % is provided for the renormalised diffusion asymptotics at the interval 0.5 – 1.0 cm, corresponding to the 335 – 670 free paths.

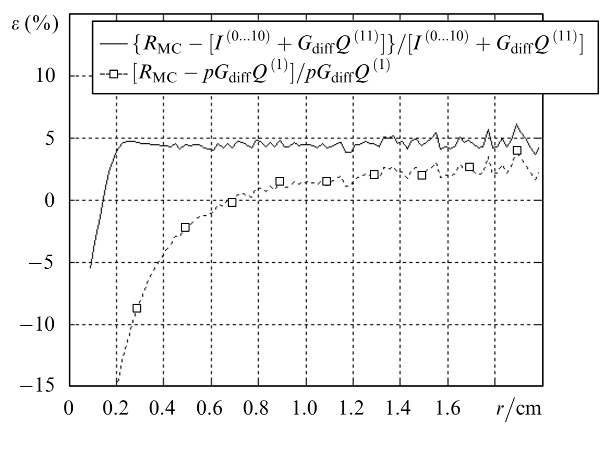

Figure 4. Relative errors ε of the analytic hybrid method ($N = 10$) and renormalised diffusion approximation ($p = 1.7$, $N = 10$) for the 2 % intralipid emulsion; R_{MC} is the reflection coefficient obtained by the Monte-Carlo method.

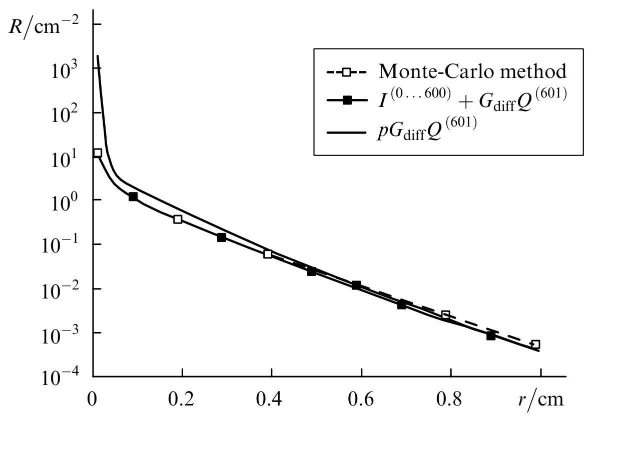

Figure 5. Dependences of the reflection coefficient on the distance between the radiation source and detector obtained by the analytic hybrid method ($N = 600$), the renormalised diffusion approximation ($p = 1.6$), and by the numerical Monte-Carlo simulation (10^7 photons 'injected' into the medium) for an erythrocyte suspension ($H = 0.41$).

4. Nonmonotonic behaviour of the reflection coefficient at small distances and the anisotropic features of the phase function for large scattering angles

In the previous sections, we calculated the first and second terms in Eqn (6) by using the Henyey–Greenstein function [1, 2, 7–11, 13, 15, 20–23]. Strictly speaking, the first term should be calculated by selecting the phase function taking into account more accurately its possible anisotropy at large scattering angles (within $180° - 90°$) because these angles may be responsible for the nonmonotonic dependence of the reflection coefficient on small distances between the observation point and the beam incidence point [23, 24]. In this case, small distances are of the order of the transport free paths $1/\mu_{tr}$, where $\mu_{tr} \sim (1-g)\mu_s$. In addition, it is necessary to take into account a finite width of the directivity diagram of the incident beam and a detector. In this section, we will use the phase function of the elementary scattering act [24]

$$p(\cos\theta) = \frac{\alpha}{4\pi}\frac{M+1}{2^M}(1+\cos\theta)^M + (1-\alpha)\frac{3}{4\pi}\cos^2\theta, \quad (11)$$

where the weight coefficient $\alpha \in [0, 1]$ providing the normalisation of the phase function and the parameter $M = 0, 1, 2, 3, ..$ determine, in particular, the position of a local minimum of the phase function for large scattering angles (within $90° - 180°$).

Figure 6 shows the dependence of the reflection coefficient on small distances measured in free paths. This dependence was calculated by (11) for $M = 21$ and $\alpha = 0.9867$ by using only the first term in Eqn (6) for $N = 60$. This figure also presents the result of calculations of the reflection coefficient by the standard Monte-Carlo method for the indicated phase function [24], which illustrate the nonmonotonic behaviour of the reflection coefficient at small distances ($\mu_t r \sim 3$). One can see from Fig. 6 that the first term in Eqn (6) of the analytic hybrid method describes the reflection coefficient at small distances, where finite-order scattering dominates and the nonmonotonic behaviour of the reflection coefficient is possible.

To interpret the nonmonotonic behaviour of the reflection coefficient, we present in Fig. 7 the phase function (11) with a distinct minimum at the scattering angle $\theta = 90°$. In our opinion, the local nonmonotonic behaviour of the reflection coefficient in the vicinity $\mu_t r \sim 3$ is directly related to the minimum of function (11), which is confirmed by the estimate of $\mu_t r$ in the single-scattering approximation (see the insert in Fig. 7).

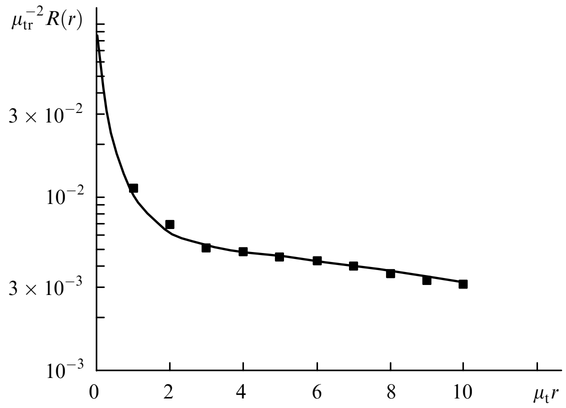

Figure 6. Contribution of the first term $I^{(0...60)}$ in Eqn (6) of the analytic hybrid method to the reflection coefficient (solid curve) at small distances between the radiation source and detector. The squares are data from [24].

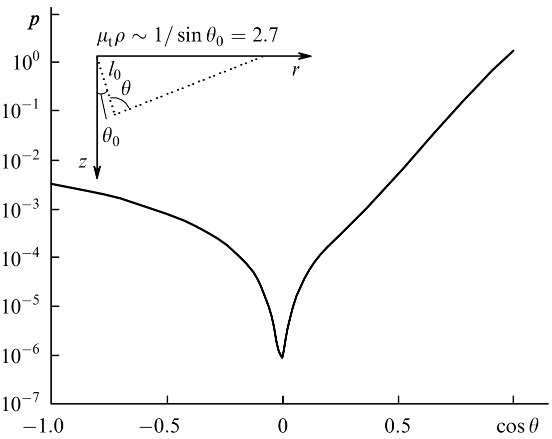

Figure 7. Illustration of the relation of the local nonmonotonic behaviour of the reflection coefficient at small distances between the radiation source and detector with the anisotropic phase function for large scattering angles (within $180° - 90°$); $\theta_0 = 21.7°$ is the half-width of the directivity diagram of the incident beam in our calculation and paper [24]; $l_0 = 1/\mu_t$ is the free path of radiation until the first scattering event; and ρ is the distance between the beam incidence point and the emergence point of singly scattered radiation.

5. Discussion of results

In this paper, we have considered the analytic iteration method for improving the diffusion asymptotics in the RTT, which is based on the Green function reciprocity and is used in the problem of probing biological media such as the 2 % intralipid emulsion and erythrocyte suspension ($H = 0.41$) by the reflection of a narrow laser beam. The diffusion approximation was improved by taking into account the contribution of finite-order scattering into the radiation intensity in the vicinity of a source on the medium surface and by constructing the effective distributed source of radiation emerging from the deep layers of the medium. In this case, finite-order scattering, which dominates near the incident beam, is responsible for the nonmonotonic dependence of the reflection coefficient on the distance between the beam incident point and the radiation detection point. We have shown that this local nonmonotonic behaviour of the reflection coefficient is related to the anisotropy of the phase function of the elementary event of scattering by large angles (in the interval $180° - 90°$) and can change depending on the anisotropy type. In addition, it is sensitive to the width of the directivity diagram of the incident beam and a detector.

The iteration parameter of the analytic hybrid method is the radiation scattering order. As the scattering order increases, the reflection-coefficient curves obtained by the method proposed in the paper and the Monte-Carlo method uniformly approach each other both for small and large distances between the radiation source and detector for a large angular aperture of the detector (within the solid angle

2π). The proposed method can be also applied to media with a strongly elongated indicatrix of the elementary scattering event, although in this case the critical scattering order substantially increases. It is shown that this method can be simplified for using for certain distances between the source and detector, which justifies the practical application of the conventional diffusion asymptotics for estimating the relative intensities of radiation emerging from the medium at the specified distances from the beam incidence point.

Therefore, we have formulated and solved, by using the Green function reciprocity in the RTT and the analytic approach, the problem of obtaining the uniform approximation of the coefficient of diffusion reflection of a laser beam from a homogeneous semibounded medium with the anisotropic phase function of the elementary scattering event both for small and large distances between the radiation source and detector in the case of a broad directivity diagram of the detector.

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