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Measurement of wavefront distortions by the method of aperture sounding with spatially separated channels

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***Abstract*.** **Features of the formation of signals in wavefront sensors with the single-frequency light wave phase modulation and spatial separation of control channels are considered. Analysis is performed for sensors in which phase modulation is governed by a controlled element located in the pupil of the optical system of a sensor or in the focal plane of the objective of this system. Peculiarities of the signal formation for a tilted wavefront are considered separately for internal points of the exit pupil in the case of light wave phase modulation in the pupil. It is shown that a signal at the modulation frequency in these wavefront sensors for points located far from the pupil boundaries is determined by the wavefront curvature.**

Keywords: *wavefront sensors, aperture sounding, wavefront curvature, wavefront tilt.*

1. Introduction

Transmitting coherent adaptive optical systems using multichannel phase modulation of the light wave are probably among the first adaptive optical systems demonstrating the potentialities of adaptive optics in the real-time correction of wavefront distortions [1 – 3]. In such systems, a wavefront sensor was a single-element light detector mounted behind the diaphragm whose size was matched with the diffraction spot for the emitting aperture. When the diaphragm was located at the focus of the entrance objective of the wavefront sensor, the signal from the detector was determined by the intensity of the part of the light signal reflected from the target and passing through the focus of the objective.

If the phase modulation of a light wave for individual parts of the emitting aperture corresponds to a certain frequency, a signal associated with a definite region of the emitting aperture can be separated from the output signal of the sensor, which can be used (without intermediate calculations) for controlling the light wave phase in this region (e.g., using a flexible mirror). When the signal at the modulation frequency is zero, this region of the aperture makes the largest contribution to the radiation intensity on the optical axis.

The prevailing opinion formed at present on the basis of the results of numerous theoretical and experimental works is that such a method for measuring wave front distortions has a number of serious drawbacks. The main drawbacks are listed below [3 – 5].

(1) The existence of local extrema ($2\pi N$ problem) in controlling flexible mirrors, which are due to peculiarities in the shape of the flexible mirror during its deformations.

(2) Limited (small) number of control channels.

(3) A low signal-to-noise ratio and, as a consequence, high brightness of the objects required for operation of such a sensor.

For this reason, it is assumed that such wavefront sensors cannot be used (at least, in astronomy) [4]. At the same time, this method seems to be quite attractive due to simplicity of its implementation.

At present, several types of wavefront sensors are being tested and developed, which are analogous to a certain extent to first aperture sounding systems [4, 6]. In these systems, the output signal from the wavefront sensor can be used without any additional processing for controlling the phase of the light wave field in the entrance pupil. For this purpose, an array of single-element detectors is mounted in the exit pupil of the sensor so that each its element is the image of a definite segment of the entrance pupil of the sensor (i.e., spatial separation of control channels is used). The wavefront sensor of this type is called an optical processor [6].

In this paper, we consider the features of formation of signals in wavefront sensors with light wave phase modulation at a single frequency and with spatial separation of channels.

2. Principle of operation of wavefront sensors in aperture sounding systems with spatially separated channels

In Fig. 1, point $\boldsymbol{\rho}$ with coordinates ξ, η of the entrance pupil corresponds to point $\boldsymbol{\rho}'$ with coordinates ξ', η'. We introduce our own coordinate system x, y in the focal plane. We assume that an amplitude – phase filter with a transmission function $T(\boldsymbol{r})$, where $\boldsymbol{r} = \{x, y\}$ is mounted in the focal plane of the optical system of the wavefront sensor.

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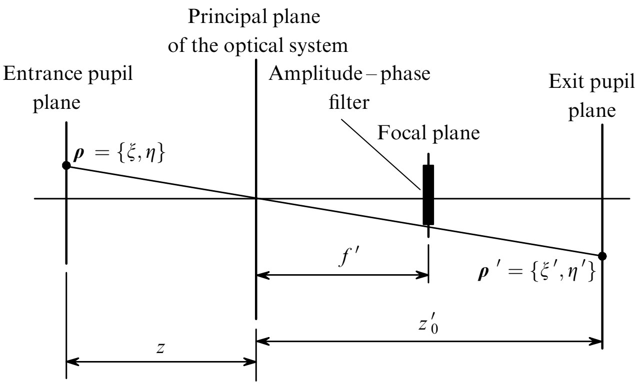

Figure 1. Generalised optical scheme of the wavefront sensor.

The light wave field distribution in the exit pupil for a point light source located at infinity can be calculated to within insignificant phase factors from the expression

$$E_{\text{out}}(\boldsymbol{\rho}') = \frac{\beta}{\lambda^2 f'^2} \int_{\boldsymbol{\rho}} E_{\text{in}}(\boldsymbol{\rho}) H(\boldsymbol{\rho} + \beta\boldsymbol{\rho}') \mathrm{d}^2\boldsymbol{\rho}, \tag{1}$$

where λ is the radiation wavelength; f' is the equivalent focal length of the optical system of the wavefront sensor; $1/\beta$ is the magnification of the optical system of the sensor in the pupils;

$$H(\boldsymbol{\rho} + \beta\boldsymbol{\rho}') = \int_{\boldsymbol{r}} T(\boldsymbol{r}) \exp\left[-\frac{\mathrm{i}k}{f'}\boldsymbol{r}(\boldsymbol{\rho} + \beta\boldsymbol{\rho}')\right] \mathrm{d}^2\boldsymbol{r} \tag{2}$$

is the point-spread function of the optical system of the pupil; and $k = 2\pi/\lambda$ is the wave number.

If $T(\boldsymbol{r}) = 1$, the point-spread function calculated by expression (2) is given by

$$H(\boldsymbol{\rho} + \beta\boldsymbol{\rho}') = (\lambda f')^2 \delta(\boldsymbol{\rho} + \beta\boldsymbol{\rho}'),$$

where $\delta(\boldsymbol{\rho})$ is the delta function. Consequently, for all points lying inside the exit pupil, we have $E_{\text{out}}(\boldsymbol{\rho}') = \beta E_{\text{in}}(\beta\boldsymbol{\rho}')$. It follows from this that the radiation intensity for points lying inside the exit pupil is independent of the light wave phase in the given case.

If, however, the point-spread function differs from the δ function, the relation between the light wave field in the exit and entrance pupils becomes more complicated and the light wave intensity in the exit pupil becomes a function of the phase of the light wave in the entrance pupil. For example, if an amplitude filter with the light wave amplitude transmission function

$$T(\boldsymbol{r}) = \exp\left(-\frac{r^2}{2r_0^2}\right), \tag{3}$$

is mounted in the focal plane, the point-spread function can be calculated by the expression

$$H(\boldsymbol{\rho} + \beta\boldsymbol{\rho}') = 2\pi r_0^2 \exp\left(-\frac{k^2 r_0^2}{2f'^2}|\boldsymbol{\rho} + \beta\boldsymbol{\rho}'|^2\right). \tag{4}$$

It follows from this that contribution to the signal of the sensor whose sensitive area is at point $\boldsymbol{\rho}'$ of the exit pupil is made not by a single point of the entrance pupil as in the above case, but by the whole region for which the transfer function differs significantly from zero. We will refer to this region as the influence zone of the wavefront sensor filter. For a filter with transmission function (3), the order-of-magnitude estimate for the size r_z of the influence zone is given by

$$r_z \approx \frac{f'}{kr_0}. \tag{5}$$

If the light wave field is spatially coherent within the influence zone (e.g., the zone size corresponds to the Fried radius for observations in a turbulent atmosphere), interference of the contributions of the field from various points of the entrance pupil will render the signal from the receiver output at the point of observation a function of the light wave phase in the entrance pupil.

Let us assume that, as in coherent optical systems with multichannel phase modulation, the field of a light wave in the entrance pupil can be written in the form

$$E_{\text{in}}(\boldsymbol{\rho}, t) = \exp\{\mathrm{i}k[\Delta(\boldsymbol{\rho}) + \Delta_{\text{m}}(\boldsymbol{\rho})\sin(\omega t)]\}, \tag{6}$$

where $\Delta(\boldsymbol{\rho})$ is the wave aberration due to the light wave distortions that should be measured; $\Delta_{\text{m}}(\boldsymbol{\rho})\sin(\omega t)$ is a function of wavefront modulation with frequency ω (aperture sounding), which is especially introduced to separate the signal associated with distortions; and $\Delta_{\text{m}}(\boldsymbol{\rho})$ is the modulation amplitude. The main difference from the traditional aperture sounding systems is that the wavefront modulation frequency is the same for all points of the exit pupil.

We can prove that the varying component of the radiation intensity at frequency ω at point of observation $\boldsymbol{\rho}'$ of the exit pupil can be calculated by the expression

$$I_{\text{s}}(\boldsymbol{\rho}') = 2C^2\mathrm{i} \int_{\boldsymbol{\rho}}\int_{\boldsymbol{\rho}} E(\boldsymbol{\rho}_1)E^*(\boldsymbol{\rho}_2) J_1(k[\Delta_{\text{m}}(\boldsymbol{\rho}_1) - \Delta_{\text{m}}(\boldsymbol{\rho}_2)])$$
$$\times H(\boldsymbol{\rho}_1, \boldsymbol{\rho}')H^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}')\mathrm{d}^2\boldsymbol{\rho}_1\mathrm{d}^2\boldsymbol{\rho}_2, \tag{7}$$

where $C = \beta/(\lambda^2 f'^2)$; $J_1(x)$ is a first-order Bessel function of the first kind; and $E \equiv E_{\text{in}}$. It follows from this expression that, if the transfer function of the wavefront sensor filter is complex, a nonzero signal will be observed even in the absence of wavefront distortions at the modulation frequency.

If the transfer function of the filter is real, expression (7) can be reduced to a form that is convenient for numerical calculations (see Appendix 1):

$$I_{\text{s}}(\boldsymbol{\rho}') = 4C^2\Big\{-A_0^{\text{s}}(\boldsymbol{\rho}')A_1^{\text{c}}(\boldsymbol{\rho}') + \sum_{n=1}^{\infty} A_n^{\text{s}}(\boldsymbol{\rho}')$$
$$\times[A_{n-1}^{\text{c}}(\boldsymbol{\rho}') - A_{n+1}^{\text{c}}(\boldsymbol{\rho}')]\Big\}, \tag{8}$$

where

$$A_n^{\text{c}}(\boldsymbol{\rho}') = \int_{\boldsymbol{\rho}} \cos[k\Delta(\boldsymbol{\rho})]H(\boldsymbol{\rho}, \boldsymbol{\rho}')J_n(k\Delta_{\text{m}}(\boldsymbol{\rho}))\mathrm{d}^2\boldsymbol{\rho};$$
$$A_n^{\text{s}}(\boldsymbol{\rho}') = \int_{\boldsymbol{\rho}} \sin[k\Delta(\boldsymbol{\rho})]H(\boldsymbol{\rho}, \boldsymbol{\rho}')J_n(k\Delta_{\text{m}}(\boldsymbol{\rho}))\mathrm{d}^2\boldsymbol{\rho}. \tag{9}$$

It follows from this that, in the absence of distortions $[\Delta(\boldsymbol{\rho})=0]$, the signal at the modulation frequency is equal to zero in this case $[A_n^s(\boldsymbol{\rho}')=0$ for all values of $n]$.

We will carry out subsequent analysis assuming that the field of the light wave in the entrance pupil can be described by the expression

$$E_{\rm in}(\boldsymbol{\rho})=\exp\left[{\rm i}k\Delta(\boldsymbol{\rho})\right],$$

$$\Delta(\boldsymbol{\rho})=\Big\{-\alpha_x(\xi-\xi_0)-\alpha_y(\eta-\eta_0)$$

$$+\Delta_{\max}\left[1-\frac{(\xi-\xi_0)^2+(\eta-\eta_0)^2}{\rho_0^2}\right]\Big\},\ (\xi-\xi_0)^2+ \quad (10)$$

$$+(\eta-\eta_0)^2\leqslant\rho_0^2,$$

$$\Delta(\boldsymbol{\rho})=0,\quad (\xi-\xi_0)^2+(\eta-\eta_0)^2>\rho_0^2,$$

where $\boldsymbol{\alpha}=\{\alpha_x,\alpha_y\}$ is the local tilt of the wavefront and ρ_0 is the spatial scale of wavefront distortions. The modulation function $\Delta_{\rm m}(\boldsymbol{\rho})$ will be described analogously, but with its own spatial scale $\rho_{\rm m}$ and $\boldsymbol{\alpha}=0$.

3. Wavefront sensor with phase modulation defined by a controlled element in the pupil of the optical system of the sensor

If the size of the influence zone of the sensor filter is much smaller than $\min(\rho_0,\rho_{\rm m})$ and the points of the exit pupil, for which the signal is calculated, are far from its edge [in the region where expression (10) is valid], we can neglect the size of the entrance pupil while integrating expression (1). In this case, we can derive the following simple expression for the field intensity ($\boldsymbol{\alpha}=0$):

$$I(\xi',\eta')=\frac{\beta^2}{1+[f'^2q/(kr_0^2)]^2}$$

$$\times\exp\left\{-\left(\frac{f'q}{r_0}\right)^2\frac{(\xi_0+\beta\xi')^2+(\eta_0+\beta\eta')^2}{1+[f'^2q/(kr_0^2)]^2}\right\},\quad (11)$$

where $q(\Delta_{\max},\omega t)=q_0+q_{\rm m}\sin(\omega t)$; $q_0=2\Delta_{\max}/\rho_0^2$ is the local wavefront curvature, which is associated with external distortions; and $q_{\rm m}=2\Delta_{\rm m}/\rho_{\rm m}^2$ is the maximum wavefront curvature associated with phase modulation [see formula (10)].

Let us denote the radius of the sensitive area of the sensor by $\rho'_{\rm fd}$. If the centre of this area element is at a point with coordinates ξ_0,η_0, the expression for the optical signal power incident on the sensitive area of the receiver has the form

$$P(\Delta_{\max},\omega t)=\pi\left(\frac{r_0}{f'q}\right)^2\times$$

$$\times\left\{1-\exp\left[-\left(\frac{f'q\rho'_{\rm fd}}{r_0}\right)2\frac{\beta^2}{1+[f'^2q/(kr_0^2)]^2}\right]\right\}.\qquad (12)$$

Figure 2 shows the signal power $P(\Delta_{\max},\omega t)$ as a function of ωt, plotted in accordance with expression (12). One can see that, for small distortions, the shape of the signal strongly differs from sinusoidal. According to estimates, this shape is successfully reproduced by the first two harmonics (cosine and sine components) of the Fourier expansion of expression (12). The following values of the parameters appearing in expression (11) were used in calculations: $\lambda=0.63$ μm, $f'=330$ mm, $r_0=20$ μm, $\rho_0=10$ mm, $\rho'_{\rm fd}=300$ μm, and $\beta=-12^{\times}$.

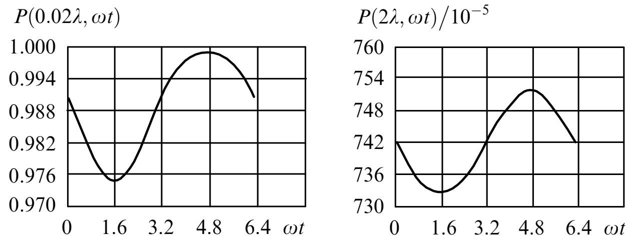

Figure 2. Dependence of the signal power $P(\Delta_{\max},\omega t)$ on ωt for a harmonic modulation of the light wave phase in the pupil of an optical system. An amplitude transparency with a Gaussian transmission function is mounted in the focal plane of the system.

If $k\to\infty$ (geometrical optics approximation), formula (12) gives

$$P(\Delta_{\max},\omega t)=\pi\left(\frac{r_0}{f'q}\right)^2\left\{1-\exp\left[-\left(\frac{f'q\rho'_{\rm fd}\beta}{r_0}\right)^2\right]\right\}.$$

This relation can be derived by the methods of geometrical optics (see Appendix 2).

Figure 3 shows the signal levels $m(\Delta_{\max})$ at the modulation frequency, which were calculated using expression (12) as functions of wavefront distortions $\Delta_{\max}/\lambda$ for two modulation amplitudes $\Delta_{\rm m}$. It follows from Fig. 3 that the maximal signal is independent of the spatial scale of distortions. Distortion $\Delta_{\max}/\lambda$ at which the peak of the signal is achieved is shifted towards smaller values upon a decrease in the spatial scale.

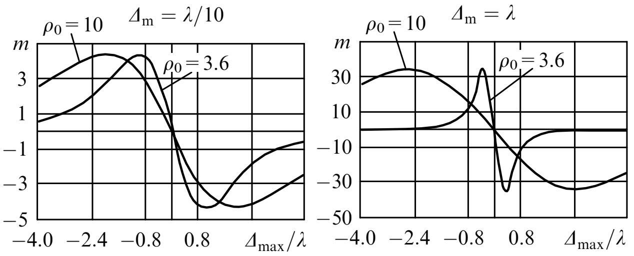

Figure 3. Signal levels $m(\Delta_{\max})$ at the modulation frequency as functions of the wavefront distortions $\Delta_{\max}/\lambda$ for the case when the transverse size ρ_0 of distortions is much larger than the influence zone (for harmonic modulation of the light wave phase in the optical system pupil). An amplitude transparency with a Gaussian transmission function is mounted in the focal plane of the system.

Figure 3 also shows that signal can be very strong for large modulation amplitudes. For this reason, the optical diagram of the wavefront sensor with an intermediate pupil in which a flexible modulator mirror is installed appears promising for increasing the modulation depth. This mirror can be prepared in the form of a single-element mirror with a variable curvature. The modulation depth of the signal at the fundamental frequency may be significant. Such a version of constructing an adaptive system is quite admissible in systems intended for correcting distortions of images in astronomy.

If the size of the influence zone of the wavefront sensor filter is comparable to the spatial scale of distortions, we can derive a dependence analogous to relation (11) for the signal intensity at a point with coordinates $\xi' = -\xi_0/\beta$, $\eta' = -\eta_0/\beta$. The final formula is rather cumbersome and will not be given here; we will confine ourselves only to the results obtained using this expression (Fig. 4). The calculations were made for the above values of the system parameters. One can see from Fig. 4 that additional zero values of the signal emerge upon a decrease in the spatial scale of distortions, which makes the system uncontrollable. This is due to interference between undistorted and distorted parts of the wavefront.

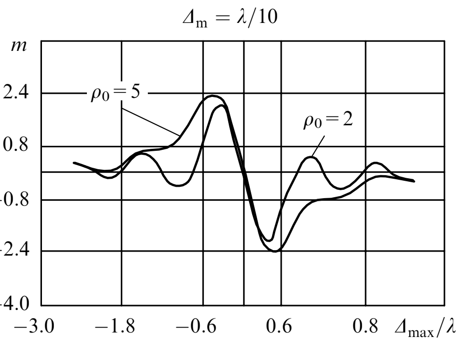

Figure 4. Signal levels $m(\Delta_{max})$ at the modulation frequency as functions of the wavefront distortions Δ_{max}/λ for the case when the transverse size ρ_0 of distortions is comparable with the size of the influence zone (for harmonic modulation of the light wave phase in the optical system pupil). An amplitude transparency with a Gaussian transmission function is mounted in the focal plane of the system.

4. Wavefront sensor with modulation of filter parameters

The second method for obtaining information on phase distortions involves the modulation of parameters of the wavefront sensor filter. In this case, the transfer function of the sensor varies with time. An analysis of this method for constructing wavefront sensors shows that in the case of modulation of parameters of the transfer function of the sensor, the signal at the sensor output without distortions will be equal to zero only if the frequency-modulated point-spread function will have a property analogous to the filtering property of the δ function.

If, for example, a filter with a transmission function

$$T(\boldsymbol{r}, t) = \exp\left[ik q_m(t)\frac{r^2}{2}\right] \tag{13}$$

is installed in the focal plane of the receiving objective of the wavefront sensor, the expression for the point-spread function of the sensor with such a filter will have the form

$$H(\boldsymbol{\rho} + \beta\boldsymbol{\rho}', t) = \frac{2\pi i}{k q_m(t)} \times \exp\left[-i\frac{k}{2}\frac{(\xi+\beta\xi')^2 + (\eta+\beta\eta')^2}{f'^2 q_m(t)}\right]. \tag{14}$$

If $q_m(t) = q_{m0}\sin(\omega t)$, the properties of the point-spread function are determined by increasingly higher values of the spatial frequency as we approach instants $t_n = 2\pi n$ (where $n = \pm 0, 1, 2, ...$) and become analogous to the filtering property of the δ function [7].

A filter with transmission function (13) is prepared in the form of a controllable flexible mirror installed in the focal plane of the receiving objective [4, 6]. In the case of modulation at instants t_n, the plane containing sensitive areas of the photodetectors is the image of the entrance pupil. The order-of-magnitude estimate of the size of the influence zone for a filter with transmission function (13) is given by

$$r_z \approx f'\left(q_{m0}\frac{\lambda}{2}\right)^{1/2}, \tag{15}$$

in order to make this size approximately the same as in the case considered above, we need either larger deformations of the flexible modulator mirror, or a larger focal length of the optical system of the wavefront sensor. We can conditionally assume that the aperture sounding method in which amplitude and phase are modulated at each point of the exit pupil is realised in this case also.

If wavefront distortions are specified in form (10), the field strength in the exit plane of the pupil at points located far from its boundary can be calculated by the expression

$$E_{out}(x) = \frac{\exp(ik\Delta_{max})}{1 + f'^2 q q_m(\omega t)} + \frac{f'^2 q q_m(\omega t)}{1 + f'^2 q q_m(\omega t)} \times \exp\left[-ik\frac{\rho_0^2}{f'^2 q_m(\omega t)}\right]. \tag{16}$$

This expression implies that if $1 + f'^2 q q_m(\omega t) = 0$, we have $E_{out} = 1$. The presence of the second term in expression (16) leads to the emergence of high-frequency components in the output signal of the detector. Figure 5 shows the dependences of the signal power on ωt calculated using expression (16) [curve (*1*)] and on the basis of this expression with allowance for the first term only [curve (*2*)], as well as their representation in the form of a Fourier series [up to the fourth harmonic inclusively, curve (*3*)]. In our calculations,

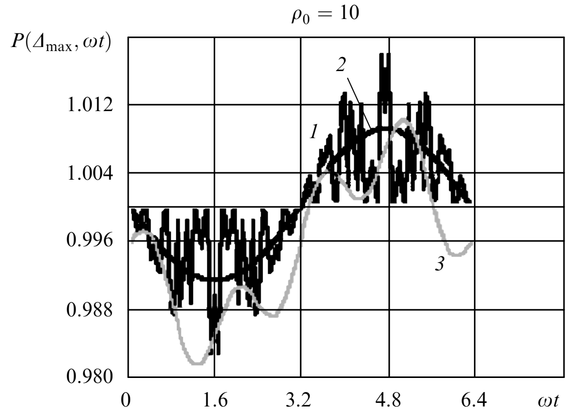

Figure 5. Dependence of the signal power $P(\Delta_{max}, \omega t)$ on ωt for a harmonic modulation of the light wave phase by varying the parameters of the quadratic phase filter mounted in the focal plane of the optical system (see text).

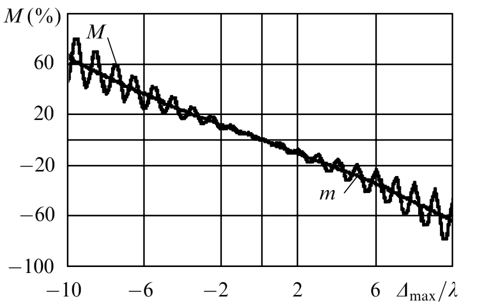

Figure 6. Signal levels $m(\Delta_{max})$ and $M(\Delta_{max})$ at the modulation frequency as functions of the wavefront distortions Δ_{max}/λ for a harmonic modulation of the light wave phase by varying the parameters of the quadratic filter mounted in the focal plane of the optical system.

we assumed that modulation is executed by a flexible mirror with a light diameter of 30 mm and a maximum surface deformation of 20 μm; the focal length of the receiving objective was 1000 mm.

Figure 6 shows the amplitudes of harmonics at the modulation frequency for signals $m(\Delta)$ and $M(\Delta)$ corresponding to curves (*2*) and (*3*) in Fig. 5.

5. Consideration of wavefront tilts

If the wavefront tilt is zero, we can prove that the signal from the inner points of the pupil for each type of wavefront sensors under investigation is determined by the local curvature of the wavefront. In other words, each of these sensors can be treated as a curvature sensor. This is true for a sensor with phase modulation of the light wave in the focal plane, even if the wave front tilt differs from zero. For such a sensor, the radiation intensity is independent of the wavefront tilt at all points of the exit pupil that are situated at large distances from its edge. This is not so obvious for a sensor with modulation in the pupil and an amplitude filter in the focal plane.

Taking into account the tilt of the wavefront of a light wave, we can generalise expression (11) for intensity in the exit pupil and represent it in the form

$$I(\xi',\eta') = \frac{\beta^2}{1+[f'^2q/(kr_0^2)]^2} \times \exp\left\{-\left(\frac{f'}{r_0}\right)^2 \frac{[(\xi_0+\beta\xi')q-\alpha_x]^2+[(\eta_0+\beta\eta')q-\alpha_y]^2}{1+[f'^2q/(kr_0^2)]^2}\right\}, \quad (17)$$

which leads to the expression

$$P(\Delta_{max},\omega t) = \frac{2\pi\beta^2}{1+[f'^2q/(kr_0^2)]^2} \times \exp\left\{-\left(\frac{\alpha f'}{r_0}\right)^2 \frac{1}{1+[f'^2q/(kr_0^2)]^2}\right\} \times \int_0^{\rho'_{fd}} \exp\left\{-\left(\frac{f'\beta q}{r_0}\right)^2 \frac{\rho'^2}{1+[f'^2q/(kr_0^2)]^2}\right\} \times I_0\left\{2\left(\frac{f'}{r_0}\right)^2 \frac{\beta q\alpha\rho'}{1+[f'^2q/(kr_0^2)]^2}\right\}\rho'\mathrm{d}\rho', \quad (18)$$

where $I_0(x)$ is a first-order Bessel function of imaginary argument of the zeroth kind, and $\alpha^2=\alpha_x^2+\alpha_y^2$.

It follows from the above expression that in the absence of wavefront distortion [i.e., for $q=q_m\sin(\omega t)$], the function $P(\Delta_{max},\omega t)$ is an even function of time. Therefore, its expansion into a Fourier series will not contain a harmonic at frequency ω for $\alpha\neq 0$; in other words, the fundamental-frequency signal will be equal to zero if the wavefront curvature in this region of the pupil is zero.

If the wavefront curvature differs from zero, the fundamental-frequency signal will be different for different tilts. Expanding the integrand in expression (18) into a series, we obtain

$$\exp\left\{-\left(\frac{f'\beta q\rho'}{r_0}\right)^2 \frac{1}{1+[f'^2q/(kr_0^2)]^2}\right\} \times I_0\left\{2\left(\frac{f'}{r_0}\right)^2 \frac{\beta q\alpha\rho'}{1+[f'^2q/(kr_0^2)]^2}\right\} \approx 1+\left(\frac{f'}{r_0}\right)^2 \frac{(\beta q\rho')^2}{1+[f'^2q/(kr_0^2)]^2} \times\left\{1-\left(\frac{f'}{r_0}\right)^2\frac{\alpha^2}{1+[f'^2q/(kr_0^2)]^2}\right\}+o(\rho'^4). \quad (19)$$

In the range of small wavefront distortions, the sign of the first harmonic amplitude in the Fourier expansion of the signal is determined by the sign of the coefficient of ρ'^2 in the power expansion of the integrand in expression (18). Expression (19) implies that the sign of the first harmonic amplitude is positive if

$$\alpha < \frac{r_0}{f'}\left\{1+\left[\frac{f'q(\Delta)}{kr_0^2}\right]^2\right\}^{1/2} \approx \frac{r_0}{f'}.$$

If this inequality is violated, the sign of the signal at the modulation frequency will be reversed.

6. Conclusions

The above analysis leads to the following conclusions.

(i) The signal at the modulation frequency for points located far from the pupil boundaries is determined by the wavefront curvature irrespective of the position of the element modulating the phase of the light wave in the wavefront sensor with spatially separated channels (in the entrance pupil plane or in the focal plane). Thus, all such sensors can be treated as curvature sensors of the Roddier type [4].

(ii) The appearance of local extrema upon phase modulation of the light wave in the entrance pupil of the adaptive system may be due to wavefront tilts.

(iii) Local extrema associated with the wavefront tilts are not observed during modulation of parameters of the phase filter installed in the focal plane of the sensor objective.

Appendix 1

It follows from (1) that the intensity of light at points of the exit pupil may be calculated from the expression

$$I(\boldsymbol{\rho}') = C^2 \int_{\boldsymbol{\rho}}\int_{\boldsymbol{\rho}} E(\boldsymbol{\rho}_1)E^*(\boldsymbol{\rho}_2)H(\boldsymbol{\rho}_1,\boldsymbol{\rho}')H^*(\boldsymbol{\rho}_2,\boldsymbol{\rho}')\mathrm{d}^2\boldsymbol{\rho}_1\mathrm{d}^2\boldsymbol{\rho}_2.$$

Taking into account formula (6) and the familiar expression from [8] (p. 987), we arrive at the expression

$$\exp\{\mathrm{i}k[\Delta_\mathrm{m}(\boldsymbol{\rho}_1) - \Delta_\mathrm{m}(\boldsymbol{\rho}_2)]\sin(\omega t)\}$$

$$= J_0(k[\Delta_\mathrm{m}(\boldsymbol{\rho}_1) - \Delta_\mathrm{m}(\boldsymbol{\rho}_2)]) + 2\sum_{n=1}^{\infty} J_{2n}(\Delta_\mathrm{m}(\boldsymbol{\rho}_1) - \Delta_\mathrm{m}(\boldsymbol{\rho}_2))$$

$$\times\cos(2n\omega t) + 2\mathrm{i}\sum_{n=1}^{\infty} J_{2n+1}(\Delta_\mathrm{m}(\boldsymbol{\rho}_1) - \Delta_\mathrm{m}(\boldsymbol{\rho}_2))\sin[(2n+1)\omega t],$$

leading to formula (7). If the transfer function of the wavefront sensor is real-valued, the imaginary part in formula (7) must be equal to zero, which leads to the following expression:

$$I_\mathrm{s}(\boldsymbol{\rho}') = -4C^2 \int_{\boldsymbol{\rho}}\int_{\boldsymbol{\rho}} \sin[k\Delta(\boldsymbol{\rho}_1)]\cos[k\Delta(\boldsymbol{\rho}_2)]J_1(k[\Delta_\mathrm{m}(\boldsymbol{\rho}_1)$$

$$- \Delta_\mathrm{m}(\boldsymbol{\rho}_2)])H(\boldsymbol{\rho}_1)H(\boldsymbol{\rho}_2)\mathrm{d}^2\boldsymbol{\rho}_1\mathrm{d}^2\boldsymbol{\rho}_2.$$

The equality

$$J_n(y+z) = \sum_{m=-\infty}^{\infty} J_m(y)J_{n-m}(z)$$

(see [8], p. 994) shows that

$$J_1(y-z) = -J_0(y)J_1(z) + \sum_{m=1}^{\infty} J_m(y)[J_{m-1}(z) - J_{m+1}(z)],$$

which directly leads to expressions (8) and (9).

Appendix 2

We shall trace a bundle of rays that would form the image at the photodetector in the absence of the screen. In the entrance pupil, this bundle corresponds to a region of size r_fd. In the absence of wavefront distortion, all rays of this bundle intersect at the centre of the amplitude screen where the transmission function is equal to unity, and fall on the photodetector. If the wavefront has a distortion with a local curvature q, the rays of this bundle in the plane of the screen will be distributed in a certain circle of radius r^* (we assume this distribution to be uniform). In this case, the peripheral rays are suppressed by the filter and the power of light falling on the photodiode decreases:

$$P_\mathrm{fd} = \int_{r^*} I(\boldsymbol{r})T^2(\boldsymbol{r})\mathrm{d}^2\boldsymbol{r} = \int_0^{r^*} \frac{\pi r_\mathrm{fd}^2}{\pi r^{*2}}\exp\left(-\frac{r^2}{r_0^2}\right)2\pi r\mathrm{d}r$$

$$= \pi\frac{r_\mathrm{fd}^2 r_0^2}{r^{*2}}\left[1 - \exp\left(-\frac{r^{*2}}{r_0^2}\right)\right].$$

By using the Newton formula, we obtain

$$r^* = r_\mathrm{fd} f q.$$

By substituting this expression in the above formula and taking into account the equality $r_\mathrm{fd} = \beta\rho'_\mathrm{fd}$, the expression for light power on the sensitive area element of the photodetector can be written in the form

$$P_\mathrm{fd} = \pi\left(\frac{r_0}{f'q}\right)^2\left\{1 - \exp\left[-\left(\frac{f'q\beta\rho'_\mathrm{fd}}{r_0}\right)^2\right]\right\}.$$

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