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Phase control of a focused laser beam: the comparison of the efficiency of methods

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Abstract. **The adaptive phase correction of distortions of spatially limited laser beams and a plane wave transmitted through a turbulent atmosphere layer is considered. The requirements to the size of the mirror element and the bandwidth of the correction system are determined, which proved to be the same as in the case of weak intensity fluctuations.**

Keywords: *turbulence, laser beams, phase correction, amplitude.*

1. Introduction

The characteristics of the phase correction of distortions of optical waves in the case of a large variance of phase fluctuations have been considered in a number of papers [1 – 3]. It was assumed that intensity fluctuations were absent. In this paper, on the contrary we analyse another limiting case of strong intensity fluctuations.

The terms 'phase correction' and 'wave-front correction' have long been considered interchangeable, while 'a phase corrector' and 'a wave-front corrector' have been synonyms. Adaptive correction was often treated as the wave-front straightening if the reception of a distorted wave was considered. In the case of adaptive beam focusing, correction is treated as the wave-front predistortion. Within the framework of wave optics, the focusing of a beam or an image can be considered as the summation of partial waves taking their phases into account [1].

Under conditions when the wave front is a sufficiently smooth surface, both these approaches are indeed virtually equivalent. However, the situation changes when the wave-front smoothness is distorted. This occurs, for example, in the turbulent atmosphere when the intensity fluctuations caused by turbulent fluctuations of the refractive index become strong enough.

The aim of this paper is to study the phase-correction efficiency in the case of strong intensity fluctuations.

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2. Phase correction: the potential and dislocation phases

It is known that phase distortions appearing during the propagation of a wave through an optically inhomogeneous medium are gradually transformed to the modulation of the spatial intensity distribution. In the case of a sufficiently deep modulation, the zero-intensity points can appear. If the wave is described in terms of the complex amplitude U, such points will be formed in the intersections (or contacts) of the lines, where the real and imaginary parts of the amplitude vanish. When $\mathrm{Re}\,U$ and $\mathrm{Im}\,U$ change their sign from positive to negative after passing through these lines, such points of intersection are the dislocation points of the wave front. From the point of view of adaptive phase correction, it is important that in the presence of dislocation the continuity of the two-dimensional phase distribution is violated and phase interruptions appear.

Note that, when the phase interruptions appear, the error of the wave-front approximation by a deformable adaptive mirror will significantly increase. The use of special correctors (in which the possibilities of composite and deformable mirrors are combined) is not efficient in the general case because upon correction of turbulent distortions the dislocations appear at random points of the aperture. The aberration map of the reference wave constructed with the help of algorithms used at present in most wave-front sensors represents a continuous function of transverse coordinates. In this case, the vortex part of the measurements is filtered [2, 4].

2.1 Efficiency of the phase conjugation algorithm

Consider the results [2, 5] of numerical experiments demonstrating the influence of intensity fluctuations and

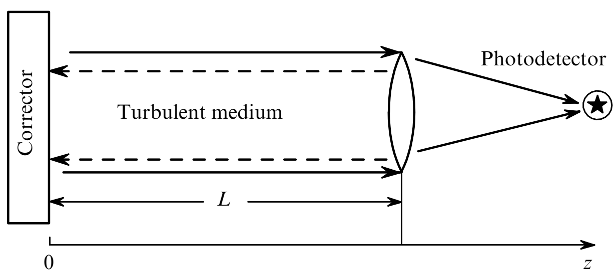

Figure 1. Optical scheme of the numerical experiment: the scheme of phase conjugation.

phase dislocations on the correction of turbulent distortions. Numerical experiments were performed for the correction scheme shown in Fig. 1, which performs the adaptive formation of a coherent radiation beam based on the predistortions introduced into the optical wave in the radiation plane ($z = 0$). The radiation being corrected (main radiation) propagates from the corrector plane ($z = 0$) to a removed object. The reference radiation propagates toward the main radiation. Thus, the scheme of adaptation is realised with the help of phase-conjugation (PC) algorithm using measurements in the reference wave. The lens of the detecting system is located in the $z = L$ plane, and the adaptive system in the $z + 0$ plane. The plane reference wave propagates from the $z = L$ plane to the $z = 0$ plane. In the $z = 0$ plane, the adaptive system measures the distortions of the reference radiation and performs the wave correction in accordance with the PC algorithm.

Of practical and scientific interest are the two aspects. The first one is how much the loss of the amplitude information affects the phase-correction efficiency. The second one is how much the loss of information contained in the vortex part of phase measurements reduces the adaptation efficiency.

Two variants of the phase-measuring algorithm were considered. In the first variant, the ideal adaptive system instantly and exactly reproduces the reference-wave phase over the entire cross-sectional plane, including the wave-front singularities (dislocations). In the second variant, only the component corresponding to the potential part of the vector field of the local tilts of the wave front is corrected. Below, we will call this correction the correction of the 'potential' (or 'vortex-free') phase. This scheme can be used to realise four algorithms of the numerical experiment: (i) the ideal compensation system (ii) the compensation system for the 'potential' part of aberrations; (iii) the ideal system based on the PC algorithm; and (iv) the system based on the PC algorithm in which only the 'potential' part of aberrations is used.

This scheme of the optical experiment can be described by four numerical parameters of the problem: the propagation path length L, the lens diameter D, the wavelength λ, and the turbulence intensity C_n^2. According to the Gurvich similarity theory [3], the propagation of a plane wave in the turbulent atmosphere is characterised only by two scales: the transverse one, which can be represented by the coherence radius r_0 of the optical wave, and the longitudinal one, represented by the diffraction length $L_d = kr_0^2$ over the coherence radius. Then, the problem will be characterised by the normalised path length L/L_d and normalised aperture diameter D/r_0. The scintillation index β_0^2 of a plane wave for the power turbulence spectrum is uniquely related to the ratio L/L_d as $\beta_0^2 = 2.9(L/L_d)^{5/6}$. It can be used as a parameter instead of the ratio L/L_d.

The propagation of a spatially limited coherent beam in a medium with the Kolmogorov turbulence was considered in [3]. Computer-aided numerical calculations [2] showed (see Fig. 2) that in the case of ideal compensation for only the 'vortex-free' part of the phase (the lower group of curves), the correction efficiency significantly decreases with increasing scintillation index. The Strehl parameter Sr decreases by half already for $\beta_0^2 \sim 1 - 1.5$. As the intensity fluctuations further increase, the radiation intensity at the lens focus tends to an uncorrected value (Fig. 2). This means that the phase correction becomes inefficient when the path

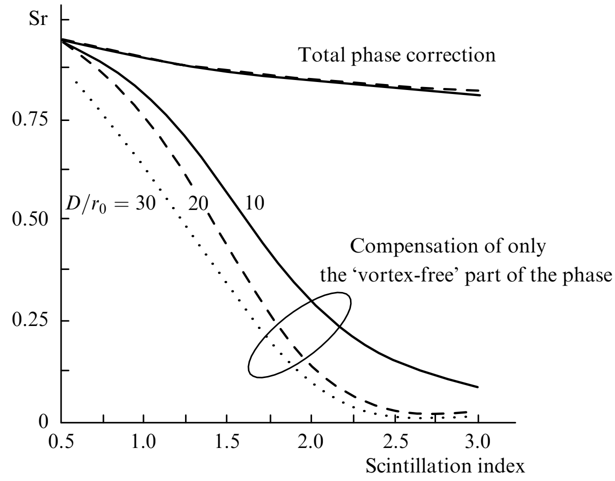

Figure 2. Dependences of the Strehl parameter Sr on the scintillation index in the phase conjugation scheme.

length reaches the diffraction length over the coherence radius. In addition, it was found that the correction efficiency in the scheme of ideal phase conjugation becomes to depend on the magnitude of intensity fluctuations. However, this dependence is not such strong as would be expected. For $\beta_0^2 = 3$, the parameter Sr decreases down to 0.8 (see the upper group of curves) and is virtually independent of the aperture diameter (or the ratio D/r_0).

2.2 Experiment at the Lincoln Laboratory: Comparison with calculations

The results of numerical calculations presented above can be compared with the experimental data [6] obtained at the Lincoln Laboratory for the 5.5-km path. The adaptive system contained a Hartmann wave-front sensor and a deformable mirror, and the PC algorithm for a focused beam was used. The wavelengths of the reference and corrected beams were 633 and 514 nm, respectively.

Figure 3 shows the results of this experiment. On the horizontal axis the variance of fluctuations of the amplitude logarithm χ for a spherical wave is plotted. It is difficult to judge from the data obtained at the Lincoln Laboratory to which extent the correction efficiency is reduced due to the

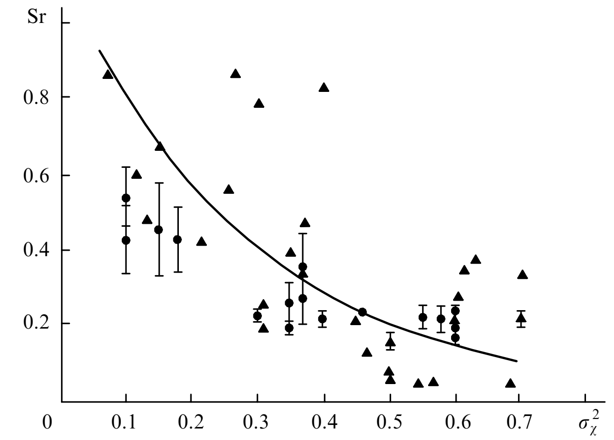

Figure 3. Dependences of the Strehl parameter Sr ($\lambda = 514$ nm) on the variance of fluctuations of the spherical wave amplitude: the solid curve is calculated by the smooth perturbation method; points are the experimental data [6].

phase interruption and filtration of phase dislocations and to which extent it occurs due to an increase in the ratio of the aperture diameter to the coherence radius, the errors of the wave-front-distortion sensor and other errors of the adaptive system.

The agreement of the experimental results with our calculations shows that it is the use of the algorithm for reconstructing the wave front of the reference beam, which filters the 'vortex' phase, is a decisive factor reducing the correction efficiency [1].

Table 1 presents the ratios of the corrected Strehl parameter $\mathrm{Sr_{cor}}$ to the uncorrected parameter $\mathrm{Sr_{n\,cor}}$ for the compensation scheme for the 'vortex-free' phase at $\beta_0^2 = 3$. One can see that for all values of D/r_0 the corrected Strehl parameter is larger than the uncorrected one by approximately a factor of four. Thus, we can conclude that at large amplitude fluctuations (when $\beta_0^2 = 3$) a substantial advantage compared to the value of the Strehl parameter in the absence of correction takes place as well.

Table 1.

D/r_0	$\mathrm{Sr_{n\,cor}}$	$\mathrm{Sr_{cor}}$	$\mathrm{Sr_{n\,cor}/Sr_{cor}}$
10	0.0324	0.129	3.98
20	0.0106	0.038	3.58
30	0.0051	0.025	4.90

3. Comparison of the efficiencies of composite and flexible mirrors

It is known that the wave-front dislocations, whose positions coincide with the points where the instant intensity value is zero, appear when the propagation distance of an optical wave in a random medium is approximately equal to the diffraction length $L_\mathrm{d} = kr_0^2$ where k is the wave number of radiation. In the presence of such points in the wave front of the reference wave, the efficiency of adaptive systems with flexible mirrors decreases. At the same time, the numerical experiment [2] with the model of a composite phase corrector showed that the efficiency of this corrector virtually does not change in passing to the region of strong intensity fluctuations.

3.1 Plane wave

Before proceeding to the results obtained for a focused wave beam in the adaptive system operating for the 'transmission', i.e., formation of a focused beam, we recall the most important data obtained in [2, 7] for a plane wave in the 'reception' regime. We found that for a plane wave the efficiency of the adaptive system with a composite corrector did not change in passing from the region of weak intensity fluctuations to the region of strong fluctuations. At first, this conclusion appeared to us even somewhat paradoxical because we expected that the presence of phase singularities and wave-front interruptions will require the use of an adaptive corrector with many elements. However, this did not happen.

At the same time, if the adaptive system is treated as a system for phasing partial waves, this should be the case. Indeed, the transformation of phase distortions to the amplitude ones in passing from a short path to the equivalent (from the point of view of turbulence) long path is by no means favourable for reducing the coherence area but, on the contrary, leads to its increase. Therefore, having a composite mirror with the size of elements equal to the coherence radius, we can perform the mutual phasing of these areas, thereby providing the coherent summation of the waves in the telescope focus.

From this point of view, another result obtained for a plane wave [2, 7] can be readily explained, namely, the fact that the dependence of the adaptive system efficiency on the delay in the correction scheme virtually does not change in passing to the region of strong fluctuations.

3.2 Spatially limited Gaussian beam and an 'ideal' corrector

In this section, we studied a focused Gaussian beam to test whether the quality of its correction can be as high as that for a plane wave. In addition, in this case, the adaptive system operates for 'transmission', which can also lead to different results.

It is known that the 'ideal' adaptive system in this case provides a high-quality correction of the focused beam [7, 8]. The 'ideal' means that the size of the element of the phase corrector is infinitely small and the boundary conditions describing the field on the emitting aperture of the adaptive system have the form

$$U(\boldsymbol{\rho}) = A_0(\boldsymbol{\rho})\exp[-\arg u(\boldsymbol{\rho})], \tag{1}$$

where $\boldsymbol{\rho}$ is the coordinate of the observation plane and A_0 is the initial amplitude of the focused beam field.

Consider the results for the 'ideal' sensor and corrector. Here, it is interesting first of all to compare the efficiencies of adaptive correction for the initial plane wave and focused beam. The result of calculations is shown in Fig. 4. In both cases, the optical system operates in the 'transmission' regime, i.e., the emitted wave is first modulated by the adaptive phase corrector and then propagates through the inhomogeneities of the refractive index. Therefore, the adaptive correction is introduced into the optical wave as the predistortion of the initial radiation. The correction quality of a focused Gaussian beam is characterised by the average radiation intensity at the focus, while that of a plane wave – by the far-field intensity of the wave, i.e., the

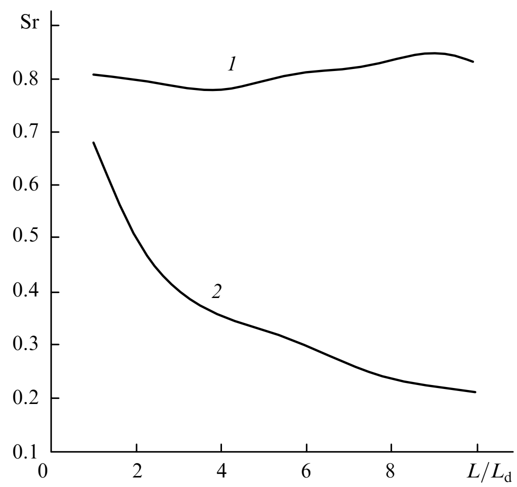

Figure 4. Dependences of the Strehl parameter Sr on the normalised path length for $D/r_0 = 10$ for a plane wave (*1*) and a focused Gaussian beam (*2*).

intensity measured at the focus of a lens located on the other end of the path in the $z = L$ plane. This case corresponds approximately to a broad collimated beam or a broad beam focused far behind a layer of a random medium.

It follows from Fig. 4 that the results obtained for a focused Gaussian beam and plane wave are substantially different. In the numerical experiment we varied the path length L from $10^{-1}L_\mathrm{d}$ to $10L_\mathrm{d}$ and did not find any substantial decrease in the correction efficiency for a plane wave. A different result was obtained for a Gaussian beam. Already for $L = 2L_\mathrm{d}$, the focal intensity decreased by half; for $L = 5L_\mathrm{d}$, it decreased by a factor of three; and for $L = 7L_\mathrm{d}$, it decreased by a factor of four compared to the diffraction-limited value. This means that there exists a principal restriction on a purely phase correction of the turbulent broadening of the focused beam. Whatever the adaptive system is used, it is impossible to compensate completely turbulent effects at long paths. Table 2 demonstrates the meaning of the term 'long path' in this case. Here, the values of L and the achievable values of the Strehl parameter Sr are presented for $r_0 = 10$ cm and a wavelength of 0.5 μm. The calculation was performed for the aperture $D = 10r_0$. For $D > 10r_0$, the dependence of the parameter Sr on L/L_d should be approximately the same, at least for $0.1 < \mathrm{Sr} < 1$.

Table 2.

Sr	L/km	L/L_d
0.68	125	1
0.48	250	2
0.33	625	5
0.25	825	7

3.3 Correction of only the 'vortex-free' part of the phase

Consider another variant – the correction of only the 'vortex-free' part of phase distortions. This variant corresponds to the conventional adaptive system with a flexible mirror and a detector whose data are processed by using a standard algorithm for phase reconstruction from its differences. In papers [2, 7], we presented the results obtained for a plane wave. Let us compare them with calculations performed for the focused beam (Fig. 5). At first glance it is somewhat unexpected that the efficiency of adaptive phase correction for the plane wave is lower than that for the focused beam. On the contrary, the correction efficiency in the focused beam presented in Fig. 4 decreases faster. This can be easily explained taking into account that the reference radiation in the system with a focused beam is a diverging wave, in which the intensity fluctuations are developing slower than in a plane wave. This is clearly seen from a comparison of the scintillation indices [3] for a plane wave,

$$\beta_0^2 = 1.24C_n^2k^{7/6}L^{11/6}, \tag{2}$$

and a diverging spherical wave,

$$\beta_0^2 = 0.42C_n^2k^{7/6}L^{11/6}. \tag{3}$$

It follows from (2) and (3) that for the same values of $C_n^2k^{7/6}$, the value of the scintillation index for a diverging wave will be equal to that for a plane wave when the propagation path of the former wave is almost twice as large as that for the latter. Therefore, for the same path lengths L, the number of phase dislocations in a diverging reference wave will be smaller than in a plane wave, i.e., the correction efficiency will be higher.

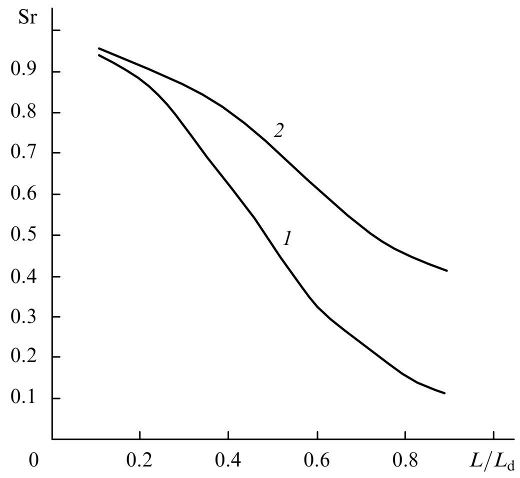

Figure 5. Dependences of the Strehl parameter Sr on the normalised path length with correction of the 'vortex-free' part of the phase for a plane wave (*1*) and a focused Gaussian beam (*2*) for $d \ll r_0$ (d is the size of an element of the active mirror).

Thus, we compared the efficiencies of adaptive correction for a plane wave and a focused beam and found out that in the case of correction for all phase aberrations (including phase dislocations), the focal efficiency for the focused beam decreases faster with increasing path length, whereas upon correction for only smoothed ('vortex-free') part of phase aberrations, the situation is opposite. In this case, the scales of path lengths differ almost by an order of magnitude (cf. Figs 4 and 5). Note that in both cases the spatial resolution of the adaptive system was assumed infinite, i.e., we assumed that the dimensions d of the corrector and detector elements were much smaller than the coherence Fried radius r_0.

4. Effect of the size of a wave-front sensor element

Consider the efficiency of an adaptive system with a finite element of size $d = r_0$. It is shown [2, 7] that the spatial resolution ($d = r_0$) is sufficient for the adaptive correction of distortions in a plane wave in the regions of weak and strong intensity fluctuations. Let us verify whether this is valid for a focused beam. Recall that we will determine the phase [2, 3] on the subaperture element of size d in terms of the complex amplitude as

$$\varphi = \arg(\bar{U}), \quad \bar{U} = \frac{1}{d^2}\iint_d U(x, y)\mathrm{d}x\mathrm{d}y. \tag{4}$$

In fact, we interchange the operations of averaging over the area and calculation of arctangent (more exactly, the principal value of arctangent), thereby eliminating difficulties encountered in the determination of a continuous phase over the entire aperture in the presence of phase dislocations [9, 10].

Consider the results of numerical simulations presented in Fig. 6. Curve (*1*) in this figure corresponds to the infinite

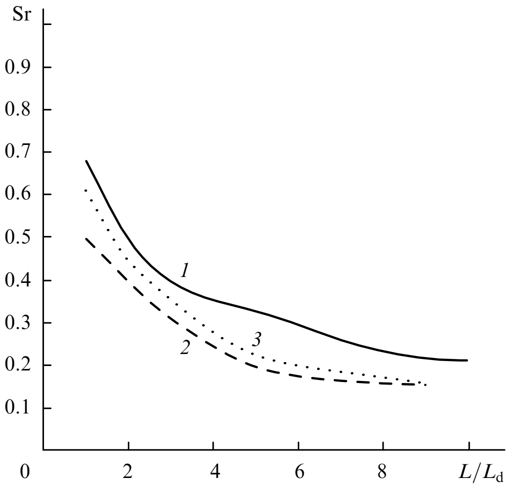

Figure 6. Dependences of the Strehl parameter Sr on the normalised path length obtained with correction with a finite size of elements of the adaptive system for $d \ll r_0$ (*1*). The correction of the average phase and the wave-front tilt (*2*) and correction of the average phase only (*3*).

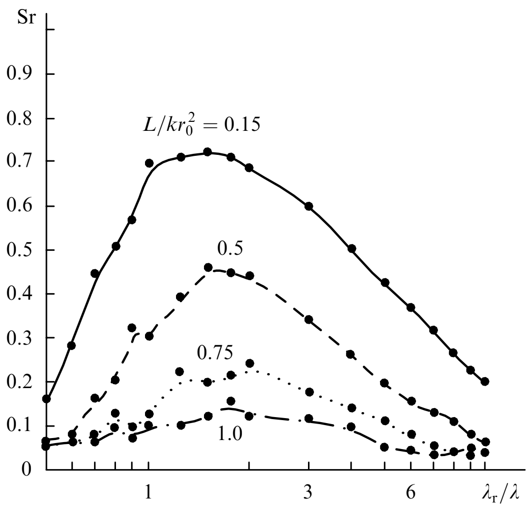

Figure 7. Dependences of the normalised intensity (Sr) on the reference radiation wavelength for different normalised path lengths L/kr_0^2 (normalisation at the wavelength of corrected radiation).

spatial resolution (the area size is $d = 0$), while curve (*2*) corresponds to the correction of the average phase and the local tilt of the wave front, and curve (*3*) corresponds to the correction of the average phase only; the area size for curves (*2*) and (*3*) being $d = r_0$. One can see that the difference between these three curves is not substantial. And although the efficiency of the adaptive system with the infinite spatial resolution is higher, the dependence on the path length is more important.

5. Two-colour adaptive system

Amplitude fluctuations and related phase dislocations in the case of two-colour adaptive correction are manifested quite differently. The main problem appears due to the necessity of scaling the phase aberrations measured at the reference radiation wavelength λ_r to the wavelength λ of the corrected radiation [11]. Therefore, the statement of the problem itself of using reference radiation at a different wavelength proves to be related to the algorithm on which the operation of the wave-front sensor is based. In principle, it is more reasonable to measure in a two-colour adaptive system the wave-front aberrations characterising the fluctuations of the difference between optical paths rather than the fluctuations of the phase difference. Because the problem of two-colour correction is difficult and depends on many factors, we will not complicate it by introducing the additional spatial scale – the beam size. Consider some results of the numerical experiment with a plane wave.

The results of the numerical simulation of this problem [12, 13] are presented in Fig. 7. All the calculations were performed for a plane wave for the relations $D/r_0 = 10$ and $d = r_0$ between the diameter D of a focusing lens, the coherence radius r_0, and the subaperture diameter d. A random medium was simulated by ten random screens, and the radiation intensity in the focal plane of the lens was averaged over ten random realisations.

We varied the wavelength of reference radiation. Figure 7 shows the dependences of the normalised intensity in the lens focus (in fact, the Strehl parameter) for four paths of different lengths. One can see that even for equal wavelengths, i.e., for $\lambda_r = \lambda$, the normalised intensity of the focal spot is smaller than its diffraction-limited value. As the reference radiation wavelength decreases, the intensity fluctuations increase, resulting in a rapid decrease in the correction efficiency. The decrease in the reference radiation wavelength λ_r from λ to 0.7λ results in the decrease in the Strehl parameter almost by half. The increase in the reference radiation wavelength from λ to 2λ leads to an increase in the Strehl number. As the reference radiation wavelength is further increased, the efficiency of adaptive correction begins to decrease, however, quite slowly. As λ_r is increased up to 6λ and 8λ, the Strehl parameter decreases approximately by half and by a factor of three, respectively.

6. Conclusions

The following conclusions can be made based on the results of our study:

(i) The efficiency of phase correction of turbulent distortions decreases approximately by half when the normalised variance of intensity fluctuations (scintillation index) β_0^2 increases from zero to unity. In this region of values of β_0^2, the correction efficiency is virtually independent of the ratio between the aperture diameter and coherence radius. As intensity fluctuations further increase, the correction efficiency begins to depend on the aperture diameter. For $\beta_0^2 \sim 3$, the correction efficiency decrease by an order of magnitude and more and the parameter Sr tends to the value obtained in the system without correction.

(ii) Because the value $\beta_0^2 = 3$ approximately corresponds to the bound of applicability [3] of the smooth perturbation method (SPM), we can assume that the SPM applicability is also related to the appearance of dislocations. Note that the radiation intensity at dislocations is zero and the logarithm of the amplitude χ tends to infinity, and the SPM is in fact the perturbation method for the field logarithm.

(iii) The use of a longer-wavelength reference radiation leads to a weak change in the correction efficiency for $\lambda_r = 1\lambda, .., 3\lambda$. As the reference radiation wavelength is further increased, the correction efficiency slowly decreases, which cannot be already avoided because this is simply caused by diffraction rather than by intensity fluctuations (which decrease with increasing λ) or by an inappropriate choice of a detector of distortions. As the wavelength increases, the phase distortions more rapidly transfer to the amplitude distortions, and this does not enhance intensity fluctuations because the phase distortions themselves decrease with increasing the wavelength.

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