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Frequency shifts in a stabilised laser due to reflection of radiation from extracavity optical elements

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***Abstract*. The effect of radiation reflected from an external optical element on the performance of a stabilised laser with an intracavity absorption cell is studied experimentally and theoretically taking into account the nonlinear frequency pulling to the absorption line centre.**

Keywords: *frequency stabilisation, intracavity absorption cell, narrow resonances.*

1. Introduction

Upon laser frequency stabilisation by narrow optical resonances, one of the factors limiting frequency stability is the reflection (scattering) of radiation from extracavity optical elements to the laser cavity. As a rule, such an element is a photodetector whose photosensitive surface is oriented nearly normally to the laser radiation. The effect of an additional mirror on the laser radiation parameters has been studied in many papers (see, for example, [1]). The theory of frequency shifts in lasers stabilised by absorption and dispersion resonances in the presence of an additional mirror is presented in [2]. The results of calculations were in agreement with the model experiments performed by using a double-mode $He-Ne/CH_4$ laser.

In this work, we have studied in detail the effect of radiation reflected from an external optical element on the frequency stability of a laser stabilised by the first harmonic of the laser radiation power taking into account the nonlinear pulling of frequency to the absorption line centre. A simplified model is proposed to explain the experimental results.

2. Radiation intensity and frequency of a laser with an additional mirror

Figure 1 shows the equivalent optical scheme of a laser with an additional ('parasitic') reflecting mirror. For the homogeneously broadened gain line and a light beam homogeneous over its cross section, the so-called laser radiation reduced power passing through mirror (2) is described by the expression [3]

$$\kappa = \frac{\left[g_0 l_g + \ln\left(R_1 R_2'\right)^{1/2}\right] T_2 (R_1)^{1/2}}{\left[R_1^{1/2} + \left(R_2'\right)^{1/2}\right]\left[1 - \left(R_1 R_2'\right)^{1/2}\right]}, \qquad (1)$$

where l_g is the length of the amplifying medium; R_{1-3} are the reflection coefficients of mirrors (*1–3*); R_2' is the effective reflection coefficient of mirror (*2*) that depends on R_3 and the distance between mirrors (*2*) and (*3*); T_2 is the transmission coefficient of mirror (*2*); and g_0 is the unsaturated gain. For a weak reflection of radiation from mirror (*3*), when $T_2 R_3^{1/2} \ll R_2^{1/2}$, we can take into account only the first reflection from mirror (*3*), by neglecting multiple trips of the light beam between mirrors (*2*) and (*3*). Under this condition, we have

$$R_2' = R_2(1+\theta)^2, \qquad (2)$$

where

$$\theta = T_2 \left(\frac{R_3}{R_2}\right)^{1/2} \cos\varphi;$$

$$\varphi = \frac{4\pi L \nu}{c} \qquad (3)$$

is the phase difference between the light beams reflected from mirrors (*2*) and (*3*) to the laser resonator; L is the length of the external resonator; and ν is the laser radiation frequency.

By substituting (2) into (1), we arrive at the relation

$$\kappa = \kappa_0 \times \frac{1 + \theta\left[g_0 l_g + \ln(R_1 R_2)^{1/2}\right]^{-1}}{\left[1 + \theta R_2^{1/2}\left(R_1^{1/2} + R_2^{1/2}\right)^{-1}\right]\left\{1 - \theta (R_1 R_2)^{1/2}\left[1 - (R_1 R_2)^{1/2}\right]^{-1}\right\}}. \qquad (4)$$

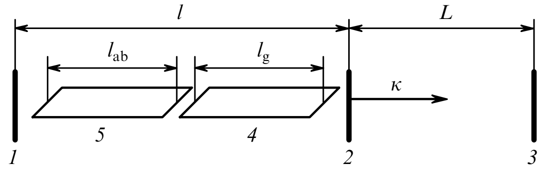

Figure 1. Equivalent optical scheme of a laser with a 'parasitic' mirror: (*1*) highly reflecting mirror; (*2*) output mirror; (*3*) additional ('parasitic') mirror; (*4*) amplifying medium; (*5*) absorbing medium.

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Here, κ_0 is the reduced laser radiation power behind mirror (*2*) in the absence of mirror (*3*). The quantity κ_0 is determined by (1), where the reflection coefficient R_2' is replaced by R_2. When the presence or absence of mirror (*3*) does not affect the lasing power noticeably, i.e., for $\theta \ll [1-(R_1R_2)^{1/2}]$, expression (4) assumes the form

$$\kappa = \kappa_0(1 + A\cos\varphi), \tag{5}$$

where

$$A = \left\{\frac{1}{g_0 l_g + \ln(R_1R_2)^{1/2}} + \frac{(R_1R_2)^{1/2}}{1-(R_1R_2)^{1/2}} - \frac{R_2^{1/2}}{R_1^{1/2}+R_2^{1/2}}\right\} \times\left(\frac{R_3}{R_2}\right)^{1/2} T_2.$$

One can see that the dependence of power on the phase difference φ in the approximation considered here is cosinusoidal, while the amplitude A is proportional to $R_3^{1/2}$. Note that the amplitude A of power modulation increases as the lasing threshold is approached.

On the other hand, the phase variation of the reflected signal can be interpreted as a change in the resonator length l, which leads to the laser frequency shift $\Delta\nu_0$, while the phase of the frequency oscillations is shifted by $\pi/2$ relative to the phase of power fluctuations:

$$\Delta\nu_0 = -\left(\frac{R_3}{R_2}\right)^{1/2} T_2 \frac{c}{2l}\sin\varphi. \tag{6}$$

Note, however, that this shift in the laser stabilisation regime will be compensated exactly in the same way as the variation in the laser resonator length, for example, due to the temperature drift.

3. Laser frequency stabilisation by the first harmonic of radiation power

Frequency stabilisation in a laser with a nonlinearly absorbing cell is accompanied, as a rule, by the modulation of the radiation frequency by the harmonic probe signal. For the modulation frequency Ω and frequency deviation $\delta\nu$ that are small compared to the resonance half-width Γ ($\delta\nu$, $\Omega \ll \Gamma$), the error signal $\Delta\kappa_s$ at the probe signal frequency for a Lorentzian nonlinear optical resonance with the amplitude B near the absorption line centre ($\Delta \ll \Gamma$) depends, in the absence of mirror (*3*), linearly on the laser frequency shift Δ relative to the absorption line centre:

$$\Delta\kappa_s = -2B\frac{\Delta}{\Gamma^2}\delta\nu\cos\Omega t. \tag{7}$$

The feedback 'forces' the laser frequency to shift towards the zero signal so that the mean frequency coincides with the frequency of the absorption line centre.

In the presence of mirror (*3*), 'parasitic' modulation appears in addition to the error signal at the modulation frequency, its sign and magnitude depending on the phase difference φ and the phase deviation. When L changes by half the radiation wavelength, the phase difference φ changes by 2π.

For a weak phase modulation ($\Delta\varphi \ll 2\pi$), the expression for the variable component of the laser radiation power associated with reflection from mirror (*3*) has the form

$$\Delta\kappa = -\Delta\varphi A\sin\varphi. \tag{8}$$

It follows from (3) that the phase shift is

$$\Delta\varphi = \varphi\left(\frac{\Delta L}{L} + \frac{\Delta\nu}{\nu}\right), \tag{9}$$

where ΔL and $\Delta\nu$ are the deviations of the external resonator length L and the laser radiation frequency ν from their initial values, respectively.

If mirror (*1*) is used for frequency modulation, there is no modulation of the distance between mirrors (*2*) and (*3*), i.e., $\Delta L = 0$, and the phase shift is

$$\Delta\varphi_1 = \varphi\frac{\delta\nu\cos\Omega t}{\nu}. \tag{10}$$

By substituting (10) into (8), we obtain the 'parasitic' amplitude modulation at the probe signal frequency Ω:

$$\Delta\kappa_1 = -A\sin\varphi\frac{\varphi\delta\nu}{\nu}\cos\Omega t. \tag{11}$$

In case of lock-in detection used to obtain the control signal, the 'parasitic' signal is added to the error signal. The resulting signal at the modulation frequency is

$$S_1 = -\left(2B\frac{\Delta_1}{\Gamma^2} + A\frac{\varphi}{\nu}\sin\varphi\right)\delta\nu\cos\Omega t. \tag{12}$$

Because the control system tends to set the laser radiation frequency in such a way that the error signal amplitude tends to zero, the expression for the laser frequency shift Δ_1 from the absorption line centre taking into account (3), will have the form

$$\Delta_1 = -\frac{2\pi A\Gamma^2 L}{Bc}\sin\varphi. \tag{13}$$

Note that in real frequency stabilisation schemes, the product AL varies little upon a variation of L, because an increase in the separation between output mirror (*2*) and the scattering surface [mirror (*3*)] decreases the effective reflection coefficient R_3 and the power modulation amplitude A.

Modulation of the laser frequency by mirror (*2*) leads simultaneously to a modulation of the distance between mirrors (*2*) and (*3*). If there is no frequency pulling towards the absorption line centre, the relation $\Delta\nu/\nu = -\Delta l/l$ is satisfied, where $\Delta l = -\Delta L$. This gives $\Delta L = l\Delta\nu/\nu$ and, using (9), we obtain

$$\Delta\varphi_2 = \varphi\frac{\delta\nu}{\nu}\left(1+\frac{l}{L}\right)\cos\Omega t. \tag{14}$$

By substituting this relation into (8), we obtain

$$\Delta\kappa_2 = -A\sin\varphi\frac{\varphi\delta\nu}{\nu}\left(1+\frac{l}{L}\right)\cos\Omega t. \tag{15}$$

In this case, the total signal is

$$S_2 = -\left[2B\frac{\Delta_2}{\Gamma} + A\frac{\varphi}{\nu}\sin\varphi\left(1+\frac{l}{L}\right)\right]\delta\nu\cos\Omega t. \tag{16}$$

By repeating the procedure used for deriving expression (13), we obtain the frequency shift

$$\Delta_2 = -\frac{2\pi A\Gamma^2 L}{Bc}\left(1+\frac{l}{L}\right)\sin\varphi \qquad (17)$$

upon frequency modulation by mirror (*2*).

A comparison of expressions (13) and (17) shows that due to reflection from mirror (*3*), the amplitude of the laser frequency fluctuation caused by the radiation frequency modulation by mirror (*1*) is $1 + l/L$ times lower than in the case of modulation of the radiation frequency by mirror (*2*). This effect is the stronger, the closer the scattering (reflecting) object to the output mirror.

The situation becomes more complicated when the laser frequency pulling is taken into account. In this case, $q\Delta\nu/\nu = -\Delta l/l$, where q is the coefficient of nonlinear frequency pulling towards the absorption line centre [4], and taking frequency pulling into account, expression (14) takes the form:

$$\Delta\varphi_2' = \varphi\,\frac{\delta\nu}{\nu}\left(1+\frac{lq}{L}\right)\cos\Omega t. \qquad (18)$$

By substituting this expression into (8), we obtain

$$\Delta\kappa_2' = -A\sin\varphi\,\frac{\varphi\delta\nu}{\nu}\left(1+\frac{lq}{L}\right)\cos\Omega t. \qquad (19)$$

Taking frequency pulling into account, the expression for the stabilised laser frequency fluctuation upon frequency modulation by output mirror (*2*) has the form:

$$\Delta_2' = -\frac{2\pi A\Gamma^2 L}{Bc}\left(1+\frac{lq}{L}\right)\sin\varphi. \qquad (20)$$

One can see that, due to reflection from mirror (*3*), the laser frequency fluctuation amplitude upon modulation by mirror (*1*) is lower than the fluctuation amplitude in the case of modulation by mirror (*2*) by a factor of $1 + lq/L$.

4. Experimental

Figure 2 shows the scheme of our experimental setup. Laser (*3*) ($\lambda = 3.39$ μm) under study consists of an amplifying He–Ne tube and an intracavity methane absorbing cell. The laser frequency was stabilised using automatic frequency control (AFC) unit (*14*) by the saturated absorption resonance at the methane $F_2^{(2)}P(7)\nu_3$ line. The error signal at a frequency of 15 kHz from photodetector (*13*) was fed to the lock-in detector (not shown in the figure) of the AFC system. The laser resonator had a length $l = 150$ cm, while the external resonator had a length $L = 60$ cm. The additional 'parasitic' mirror was in the form of glass plate (*9*) with a slight wedging to prevent two beams reflected from the front and back planes of the plate from falling simultaneously into the resonator.

The frequency shifts were measured by the standard technique. The radiation frequency from laser (*3*) was determined relative to heterodyne laser (*2*). Automatic frequency–phase control system (*5*) made it possible to lock the frequency of heterodyne laser detuned by 1 MHz with the frequency of stable laser (*1*). The signal of beats between heterodyne laser (*2*) and stabilised laser (*1*) was separated by photodetector (*7*). The frequency of laser (*1*) was stabilised by saturated absorption resonance of the methane $F_2^{(2)}P(7)\nu_3$ line using AFC unit (*4*). The error signal at the first harmonic of the modulation frequency of laser radiation was separated by photodetector (*6*).

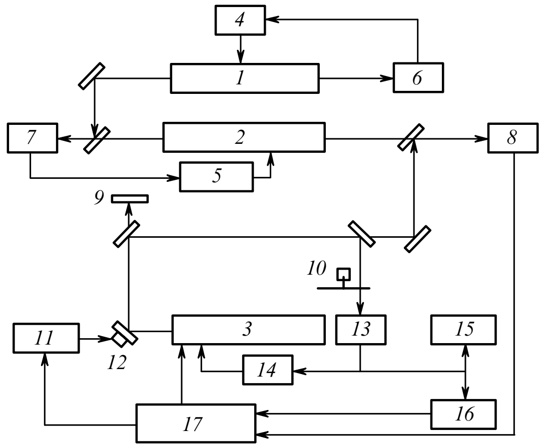

Figure 2. Scheme of the experimental setup: (*1*) frequency-stabilised laser; (*2*) auxiliary heterodyne laser; (*3*) laser under study; (*4*, *14*) automatic frequency control units; (*5*) frequency–phase control unit; (*6–8*, *13*) photodetectors; (*9*) glass plate (additional mirror); (*10*) mechanical light beam chopper; (*11*) piezoelectric element control unit; (*12*) fold mirror on the piezoelectric element; (*15*) oscilloscope; (*16*) selective nanovoltmeter; (*17*) measuring and computing complex.

The length of the external resonator was varied with the help of a piezoelectric element with fold mirror (*12*). The output radiation from the laser was incident on InSb photodetector (*13*) cooled by liquid nitrogen. During the recording of the resonance and power modulation by additional mirror (*3*), the laser beam was modulated by mechanical chopper (*10*) at a frequency of ~ 150 Hz to which selective nanovoltmeter (*16*) was tuned. Oscilloscope (*15*) was used for on-line control. Auxiliary mirror (*9*) was not used while recording the saturated absorption and nonlinear dispersion resonances. The radiation frequency from laser (*3*) was locked to the frequency of heterodyne laser (*2*) with the help of a frequency–phase locking unit (not shown in the figure). The feedback signal was determined by recording the dependence of the laser radiation power on the length L of the external resonator. For this purpose, a sawtooth voltage from power supply (*11*) was applied to the piezoelectric element of fold mirror (*12*).

Control of the experiment and automatic data collection were performed with measuring and computing complex (MCC) (*17*) to which signals from photodetector (*8*) were fed for measuring the difference between the frequencies of the heterodyne laser and the laser under study, and also the output signal from selective nanovoltmeter (*16*), which was proportional to the radiation power of the laser under study. The MCC also contained a frequency meter and a frequency detector for analysing the difference between the frequencies of the heterodyne laser and the laser under study (not shown in the figure).

5. Experimental results and discussion

The radiation power entering the resonator of the laser under study after reflection from the glass plate was set in our experiments at a level of $\sim 10^{-4}$ of the output laser power and was chosen in such a way as to provide a fairly strong signal on the one hand and to prevent 'jumps' between adjacent longitudinal laser modes on the other hand. The laser radiation power was measured with a photodetector under a 100-% modulation of the light beam by a mechanical chopper.

The laser radiation intensity and methane pressure were maintained at a constant level throughout the experiment to provide the constant amplitude B and half-width Γ of the resonance. Figure 3 shows the nonlinear resonance in methane. The shape of the nonlinear resonance is nearly Lorentzian, slight deviations being probably caused by the quadratic Doppler effect. The resonance half-width Γ is 78 kHz and its amplitude is 400 mV, which corresponds to a contrast of $\sim 10\,\%$. The frequency of the investigated laser is indicated relative to the frequency of a stable laser.

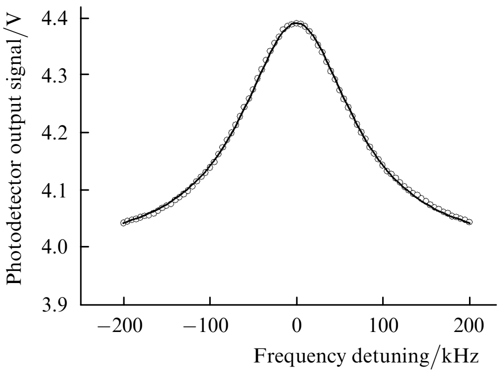

Figure 3. Saturated absorption resonance at the methane $F_2^{(2)}P(7)\nu_3$ line. The circles correspond to the experimental results, and the solid curve is the Lorentzian curve fitting the experimental values by the least squares method.

The frequency shift of the stabilised laser is connected with the magnitude and phase of the signal reflected from the external optical element. The signal phase was varied by varying the distance L by applying a sawtooth voltage to the piezoelectric element of the fold mirror. In this case, the signal reflected from the external optical element causes a sinusoidal modulation of the laser power upon a variation in the distance between the output mirror and the glass plate (Fig. 4). The amplitude of power modulation was 300 mV, which is smaller than 10^{-1} in relative units. A change in voltage by ~ 270 V corresponds to a variation in the external resonator length L by half the wavelength of the investigated laser radiation.

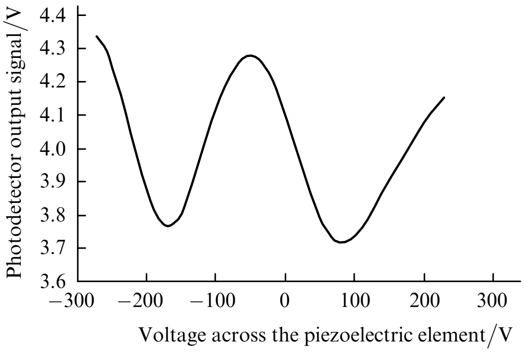

Figure 4. Dependence of the photodetector output signal (proportional to the laser radiation power) on the voltage across the piezoelectric element (length L of the external resonator).

The frequency pulling coefficient q was measured by modulating the laser frequency by mirror (*2*) and was found to be equal to 1.54 (Fig. 5). The amplitude of frequency deviation at the absorption line centre was 32.5 kHz.

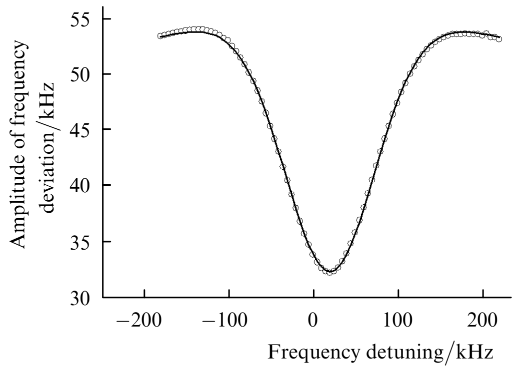

Figure 5. Saturated dispersion resonance (amplitude of the frequency deviation) at the methane $F_2^{(2)}P(7)\nu_3$ line. The circles correspond to the experimental results, and the solid curve is obtained by fitting the experimental values by the least squares method.

Figure 6 shows the results of measurements of the dependence of the stabilised laser frequency shifts on the external resonator length (voltage across the piezoelectric element of the fold mirror). Curve (*2*) with a greater amplitude of frequency modulation equal to 460 Hz corresponds to the case when the probe signal is supplied to the piezoelectric element of output mirror (*2*), while curve (*1*) with an amplitude of 187 Hz corresponds to the case when the probe signal is supplied to the piezoelectric element of highly reflecting mirror (*1*). The initial phases of the reflected signals are different and hence the phases of the sinusoidal dependences are also different. Also, the mean frequency of the investigated laser changes by 400 Hz as a result of frequency modulation by mirrors (*1*) and (*2*). The experimental ratio of the amplitudes of laser frequency fluctuations was ~ 2.5. The amplitude of frequency deviation in both cases was set at 32.5 kHz.

For a probe signal applied to mirror (*1*), the amplitude of frequency shift of the stabilised laser was calculated from

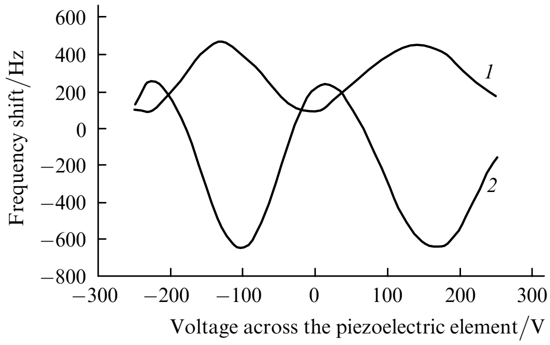

Figure 6. Frequency shift for a laser stabilised by the first harmonic of radiation power on the voltage across the piezoelectric element (length L of the external resonator) upon frequency modulation by mirrors (*1*) [curve (*1*)] and (*2*) [curve (*2*)].

Eqn (13) for the above-mentioned values of the parameters and was found to be 57 Hz. The calculated amplitude of the frequency shift increased to 278 Hz as a result of frequency modulation by mirror (*2*). The ratio of these amplitudes, which is equal to 4.85, is slightly higher than the experimental value.

The quantitative difference between the theoretical and experimental values can be attributed primarily to the approximation used in calculations: the frequency deviation $\delta\nu = 32.5$ kHz and modulation frequency $\Omega = 15$ kHz cannot be regarded as negligible compared to the resonance half-width $\Gamma = 78$ kHz.

6. Conclusions

Our study has shown that the adverse effect of laser radiation reflection from external optical elements on the frequency stability is weaker when modulation is performed using a highly reflecting mirror than in the case when a semi-transparent mirror adjoining the external optical element is used for this purpose. This difference is emphasised by the fact that a nonlinear frequency pulling towards the absorption line centre takes place.

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