

On the speckle pattern of a light field in a dispersion medium illuminated by a laser beam

A.P. Ivanov, I.L. Katsev

Abstract. A simple analytic model is proposed for analysis of the speckle pattern of laser radiation in a dispersion medium by the example of a biological tissue. It is shown that three components – unscattered, diffraction, and diffusion, can be distinguished in the radiation flux propagating through a turbid medium. The widths of their angular spectra differ by a few orders of magnitude. Taking into account the interrelation between the theory of light-field coherence in a scattering medium and the theory of radiation transfer, the expressions are obtained for the average sizes of speckle spots of three types formed by these components. These sizes are of the order of millimetre, micrometre, and fractions of micrometre. As laser radiation penetrates into a tissue, first large spots are observed, then middle-size spots dominate, and after that – very small spots appear. Other parameters of the speckle pattern are also presented.

Keywords: biological tissue, spotty structure, interference, absorption, laser beam, light field.

It is known that upon illumination of a diffusively reflecting screen by a laser beam, randomly distributed spots are observed, the so-called speckle pattern, which changes when the spatial (mode) structure changes. The presence of speckles means that gradient electromagnetic fields can appear in the laser radiation field. Upon illumination of a biological object by laser radiation, these fields can cause transformation at the cell level. Such a mechanism of laser radiation action on the human organism and some other possibilities of practical application of laser gradient fields were discussed by A.N. Rubinov and co-workers in a number of papers [1–4]. The analysis of speckles also allows one to estimate the size of particles, the velocity of their random motion, the viscosity of a medium, etc. [5–9].

In this paper, based on the known relation between the theory of light-field coherence in a scattering medium and the theory of radiation transfer (see, for example, [10–17]), we analysed the speckle pattern produced by the laser field in a scattering biological medium. We assume that the size of scattering particles is large compared to the wavelength,

so that the diffraction parameter $\rho_{\text{eff}} = 2\pi r_{\text{eff}}/\lambda$ is $30–100$, where r_{eff} is the effective radius of particles and λ is the radiation wavelength. This means that $r_{\text{eff}} \approx 3–10 \mu\text{m}$ for the visible range.

Let a horizontal semi-infinite layer of a dispersion medium be illuminated from above by a monochromatic laser beam with the angular divergence of the order of a few minutes normally incident on the layer. The laser-beam cross section is assumed sufficiently large for two reasons: first, this allows one to neglect the transverse spread of the beam at different penetration depths when estimating the energy parameters of radiation and, second, to use the relation between the theories of coherence and radiation transfer, the beam diameter should noticeably exceed the coherence radius of the light field [16, 17]. This condition is fulfilled, for example, for a laser emitting multimode radiation.

The attenuation of the unscattered component of the light field in a medium is described by the Bouguer law

$$E_0(\tau) = E^* e^{-\tau}, \quad (1)$$

where E^* is the irradiance produced by laser radiation on the layer surface; $\tau = \varepsilon z$ is the optical depth of the observation point in the layer; z is the geometric depth; $\varepsilon = \sigma + k$ is the attenuation factor of radiation in a unit volume of the medium; and σ and k are the scattering and absorption coefficients, respectively.

As will be shown below, the speckle pattern of the light field in a medium is related to the angular structure of radiation. If the medium does not scatter but only absorbs radiation, the total light flux in it is described by expression (1) for $\sigma = 0$. In this case, the angular structure of radiation does not change with depth and, therefore, the speckle pattern of the light field does not change. In a turbid medium, scattering of light occurs. Upon scattering from particles that are large compared to the wavelength, two components can be distinctly distinguished in the forward hemisphere of the phase function [18], which substantially differ in their angular scale: a sharp peak in the ‘forward’ direction (diffraction peak) with the half-width of the radiance distribution of the scattered radiation of the order of a degree and a much more diffuse component with the half-width of the order of a few tens of degrees. Because of this, upon multiple forward scattering at least up to optical depths $\tau \approx 3–5$, three radiation components can be distinguished [18]: unscattered component with the angular divergence of the incident beam of the order of a few minutes, the diffraction component with the divergence of

A.P. Ivanov, I.L. Katsev B.I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, prosp. F. Skariny 68, 220072 Minsk, Belarus; e-mail: ivanovap@dragon.bas-net.by, katsev@zege.bas-net.by

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the order of a few degrees, and the diffusion component with the divergence of a few tens of degrees. Although this notation is conventional, we will use it for brevity.

In the small-angle approximation of the radiation transfer theory [18], the sum of irradiances produced by the unscattered and diffraction radiation components on the horizontal area is

$$E_{0+1}(\tau) = E^* e^{-(\sigma-\sigma_1+k)z} = E^* e^{-(1-\Lambda_1)\tau}, \quad (2)$$

where $\sigma_1 = a_1\sigma$ is the scattering coefficient caused by the diffraction component of the phase function; a_1 is the fraction of the diffraction component in the total phase function; $\Lambda_1 = \sigma_1/\epsilon = a_1\Lambda$ and $\Lambda = \sigma/\epsilon$ are the single scattering albedo for the diffraction component and the total single scattering albedo, respectively. Upon scattering from large particles, $a_1 \sim 0.5$ [19, 20].

It follows from expressions (1) and (2) that the irradiance produced by the diffraction radiation component is

$$E_1(\tau) = E_{0+1}(\tau) - E_0(\tau) = E^* [e^{-(1-\Lambda_1)\tau} - e^{-\tau}]. \quad (3)$$

The total irradiance produced by the laser beam incident on the horizontal surface is described in the small-angle approximation [18] by the expression

$$E_\downarrow(\tau) = E^* e^{-(1-\Lambda F)\tau}, \quad (4)$$

where $F = (\sigma_1 + \sigma_2)/\sigma$ is the fraction of light scattered by a unit volume into the forward hemisphere; $\sigma_2 = a_2\sigma$; and a_2 is the fraction of the diffusion part of radiation scattered into the forward hemisphere in the total phase function.

Note that the upward laser flux produces the irradiance

$$E_\uparrow(\tau) = R E_\downarrow(\tau) \quad (5)$$

on the horizontal surface, where the reflection coefficient of the layer is

$$R = \exp \left[-\frac{4}{3} \left(\frac{1-\Lambda}{1-F} \right)^{1/2} \right]. \quad (6)$$

Expression (6) is written in the asymptotic approximation of small absorption [18] by using the correlation relation $(1-\bar{\mu})/3 = 1-F$ [18], where $\bar{\mu}$ is the average cosine of the phase function.

The irradiance produced by the downward radiation flux propagating at comparatively large angles, i.e., caused by the diffusion part of the phase function can be found in the same small-angle approximation by subtracting expression (2) from (4) to obtain

$$E_2(\tau) = E_\downarrow(\tau) - E_{0+1}(\tau) = E^* [e^{-(1-\Lambda F)\tau} - e^{-(1-\Lambda_1)\tau}]. \quad (7)$$

Therefore, the three components of the downward light flux – unscattered, diffraction, and diffusion, produce irradiances determined by expressions (1), (3), and (7). As pointed out above, the scales of the angular structure of these components substantially differ. We will assume that the radiance angular distributions of these components $I_i(\tau, \theta)$ ($i = 0, 1, 2$) are described by a Gaussian, i.e.,

$$I_i(\tau, \theta) = \frac{E_i(\tau)}{2\pi D_i(\tau)} e^{-\theta^2/[2D_i(\theta)]}, \quad (8)$$

where θ is the angle between the beam propagation direction and the normal to the layer; $E_i(\tau)$ is the irradiance produced by the i th component at the optical depth τ ; and $D_i(\tau)$ is the dispersion of the radiance angular distribution for the corresponding component.

The dispersion D_0 of the radiance angular distribution of the initial laser beam is related to the solid (Ω_{src}) and aperture (θ_{src}) angles by the expressions

$$D_0 = \frac{\Omega_{\text{src}}}{2\pi} = \frac{\theta_{\text{src}}^2}{2}. \quad (9)$$

It follows from [18] that the dispersion of the radiance angular distribution of the diffraction component of multiply scattered radiation at the optical depth τ is

$$D_1(\tau) = D_0 + \frac{\Lambda_1 \tau \beta_{21}}{2(1 - e^{-\Lambda_1 \tau})}, \quad (10)$$

where

$$\beta_{21} = \frac{\int \beta^2 x_1(\beta) \beta d\beta}{\int x_1(\beta) \beta d\beta} \quad (11)$$

is the average square of the scattering angle β of the diffraction component $x_1(\beta)$ of the phase function. Similarly, the dispersion of the radiance angular distribution of multiply scattered radiation caused by the diffusion part of the phase function, i.e., scattered forward at comparatively large angles is described by the expression

$$D_2(\tau) = D_1(\tau) + \frac{\Lambda_2 \tau \beta_{22}}{2(1 - e^{-\Lambda_2 \tau_2})}. \quad (12)$$

Here, the quantity β_{22} has the same meaning as β_{21} , but for the diffusion part of the phase function; $\Lambda_2 = \sigma_2/\epsilon_2 = \sigma_2/(\epsilon - \sigma_1)$ is the single scattering albedo for the light scattered forward at comparatively large angles and $\tau_2 = \tau(1 - a_1\Lambda)$. It can be easily shown that $\Lambda_2 \tau_2 = (F - a_1\Lambda)\tau$.

The expressions for the photometric (energy) parameters of the light field presented above were obtained within the framework of the radiation transfer theory, i.e., by neglecting the wave nature of light. However, as mentioned above, the theory of the light-field coherence and the radiation transfer theory are interrelated [10–17]. Let us use this relation to estimate some parameters of the speckle pattern of the laser radiation field in a scattering medium.

Consider the function of mutual spatial coherence of the light field $\Gamma(\mathbf{r}, \rho)$ [21] in the $z = \text{const}$ plane

$$\Gamma(\mathbf{r}, \rho) = \overline{u(\mathbf{r} - \rho/2) u^*(\mathbf{r} + \rho/2)}. \quad (13)$$

Here, $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$; $\rho = \mathbf{r}_2 - \mathbf{r}_1$; \mathbf{r}_1 and \mathbf{r}_2 are the coordinates of the observation points in the medium.

If the coherence function $\Gamma(\mathbf{r}, \rho)$ changes along \mathbf{r} slower than along ρ , then, as shown in [17], at least in the region of applicability of the small-angle approximation of the radiation transfer theory, the angular distribution of radiance $I(\mathbf{r}, \theta)$ is related to the coherence function by the Fourier transform

$$\Gamma(\mathbf{r}, \rho) = 2\pi \int_0^\infty I(\mathbf{r}, \theta) \exp \left(-i \frac{2\pi}{\lambda} \rho \theta \right) \theta d\theta. \quad (14)$$

Relation (14) is written for the case of the azimuthal symmetry of the field. It obviously follows from (14) that the width of the coherence function $\Gamma(\mathbf{r}, \rho)$ decreases with increasing the width of the radiance angular distribution $I(\mathbf{r}, \theta)$.

Let us use expressions (14) and (8) to find the correlation functions $\Gamma(\mathbf{r}, \rho)$ for all the three components of the light field considered above. We will restrict ourselves to the statistically homogeneous field, when $I(\mathbf{r}, \theta) = I(\theta)$ and $\Gamma(\mathbf{r}, \rho) = \Gamma(\rho)$. By substituting (8) into (14), we obtain

$$\begin{aligned} \Gamma_i(\rho) &= \exp \left[-\frac{(2\pi\rho/\lambda)^2 D_i}{2} \right] \\ &= \exp \left(-\frac{\rho^2}{2D_{\rho i}} \right), \quad i = 0, 1, 2, \end{aligned} \quad (15)$$

where ρ is the distance between the points being considered; D_i and $D_{\rho i}$ are dispersions of the components of the radiance angular distribution $I_i(\theta)$ and correlation function $\Gamma_i(\rho)$ of the field, which are related by the expression

$$D_{\rho i} = \left(\frac{\lambda}{2\pi} \right)^2 \frac{1}{D_i}. \quad (16)$$

Let us now estimate the speckle pattern of the light field. As an average size of spots in the speckle pattern for each component of the light field, it is reasonable to use the correlation radius r_{ci} , defined as the distance at which the correlation function $\Gamma_i(\rho)$ decreases by a factor of e, i.e.,

$$r_{ci} = (2D_{\rho i})^{1/2} \approx \frac{0.225\lambda}{D_i^{1/2}}. \quad (17)$$

Therefore, by substituting the values of D_i for the three components of the downward radiation flux into (17), we can find for them the average sizes of spots in the speckle pattern.

Let us estimate roughly the number of spots n_i per unit area for each i th component as the quantity inversely

proportional to the average spot area:

$$n_i \approx \frac{1}{\pi r_{ci}^2}. \quad (18)$$

Consider now quantitatively some features of the speckle pattern inherent in a light scattering medium such as the human skin. Table 1 presents some results of calculations. The optical characteristics $a(\lambda)$, $k(\lambda)$ and $\Lambda(\lambda)$ of the medium for the wavelengths 400 and 700 nm are taken (recalculated) from [22, 23] and the values $\beta_{21} = 10^{-3}$ and $\beta_{22} = 0.3$ from [18]. The aperture angle θ_{src} of the source was taken to be $0.0003 = 1'$, and the parameters $a_1 = 0.6$, $F = 0.95$. The above wavelengths were chosen because absorption of light by the human skin at them is strongly different. For $\lambda = 400$ nm, we have $\varepsilon = 668 \text{ cm}^{-1}$, $k = 45 \text{ cm}^{-1}$, $\Lambda = 0.932$, and $R = 0.211$, while for $\lambda = 700$ nm we have $\varepsilon = 126 \text{ cm}^{-1}$, $k = 0.33 \text{ cm}^{-1}$, $\Lambda = 0.997$, and $R = 0.721$.

It follows from Table 1 that at small depths only unscattered light is present in fact, which produces the irradiance E_0 , then the diffraction and diffusion components appear, producing the irradiances E_1 and E_2 , respectively. As the penetration depth increases, the unscattered light irradiance rapidly decreases, while the irradiance of the diffraction component increases. At the optical depth $\tau \approx 1.5$, the irradiance of the diffraction component already exceeds the unscattered irradiance and then begins to decrease slowly. The intensity of the diffusion component first increases slower than that of the diffraction component, but it continues to increase at least up to $\tau \approx 4$, so that it becomes greater than the irradiance of the diffraction component already at $\tau \approx 1.5$. Therefore, the relation between the contributions of different components (E_0 , E_1 , and E_2) depends on the penetration depth. At small depths, the contribution E_0 of unscattered light dominates. As the penetration depth increases, the E_1 , and E_2 components of scattered light become dominant, and then only the E_2 component remains. In this case, the total irradiance E_{\downarrow} produced by the downward flux decreases with the penetration depth.

Table 1. Parameters of speckles in a biological tissue.

λ/nm	τ	$z/\mu\text{m}$	Unscattered component			Diffraction component			Diffusion component			E_{\downarrow}/E^*	E_{\uparrow}/E^*
			E_0/E^*	$D_0/10^{-8}$	$r_{c0}/\mu\text{m}$	E_1/E^*	$D_1/10^{-3}$	$r_{c1}/\mu\text{m}$	E_2/E^*	D_2	$r_{c2}/\mu\text{m}$		
400	0	0	1	4.5	424	0	—	—	0	—	—	1	0.21
	0.5	7.5	0.61	4.5	424	0.20	0.57	3.76	0.14	0.16	0.22	0.94	0.20
	1	15	0.37	4.5	424	0.28	0.65	3.52	0.25	0.18	0.21	0.89	0.19
	1.5	22	0.22	4.5	424	0.29	0.73	3.32	0.32	0.19	0.21	0.84	0.18
	2	30	0.13	4.5	424	0.28	0.83	3.12	0.38	0.21	0.20	0.80	0.17
	2.5	37	0.08	4.5	424	0.25	0.92	2.96	0.42	0.22	0.19	0.75	0.16
	3	45	0.05	4.5	424	0.22	1.0	2.80	0.44	0.24	0.19	0.71	0.15
	3.5	52	0.03	4.5	424	0.19	1.1	2.67	0.46	0.25	0.18	0.67	0.14
700	4	60	0.02	4.5	424	0.15	1.3	2.54	0.46	0.27	0.17	0.63	0.13
	0	0	1	4.5	743	0	—	—	0	—	—	1	0.72
	0.5	40	0.61	4.5	743	0.21	0.58	6.54	0.16	0.16	0.39	0.97	0.70
	1	79	0.37	4.5	743	0.30	0.66	6.11	0.28	0.18	0.37	0.95	0.68
	1.5	119	0.22	4.5	743	0.32	0.76	5.72	0.38	0.19	0.36	0.92	0.67
	2	159	0.13	4.5	743	0.31	0.86	5.38	0.45	0.21	0.34	0.90	0.65
	2.5	198	0.08	4.5	743	0.28	0.96	5.08	0.51	0.23	0.33	0.88	0.63
	3	238	0.05	4.5	743	0.25	1.1	4.80	0.55	0.24	0.32	0.85	0.62
	3.5	278	0.03	4.5	743	0.21	1.2	4.56	0.59	0.26	0.31	0.83	0.60
	4	317	0.02	4.5	743	0.18	1.3	4.34	0.61	0.28	0.30	0.81	0.58

The angular distributions $I_i(\theta)$ of three radiance components of the downward flux and the upward radiance $I_{\uparrow}(\theta)$ are presented qualitatively in Fig. 1. The dispersion D_0 of the angular distribution of the unscattered light intensity does not change, naturally, with the penetration depth, remaining equal to 4.5×10^{-8} . Dispersions D_1 and D_2 increase with the optical depth, although slightly. It is important that D_1 exceeds D_0 by four orders of magnitude, while D_2 exceeds D_0 by seven orders of magnitude. For this reason, upon penetration into a layer by the optical depth of the order of $0.2 - 0.4$, where the diffraction and diffusion components become already noticeable, along with speckle spots of the order of fractions of millimetre, which are caused by the unscattered radiation component (Fig. 2a), spots appear of size of the order of micrometer (Fig. 2b) and smaller (Fig. 2c). The resulting speckle pattern is shown qualitatively in Fig. 2d. In the general case, it contains three characteristic spectra of spots caused by three radiation components in the medium. All the three components have Gaussian radiance distributions with substantially different widths. As the optical depth increases, the relation between the contributions of these components to the total irradiance changes, resulting in a gradual change in the structure of spots: first spots of millimetre size disappear and then of micron size. In the limit, the angular spectrum of multiply scattered light becomes almost diffuse and, therefore, the characteristic size of the speckle pattern becomes comparable with the radiation wavelength.

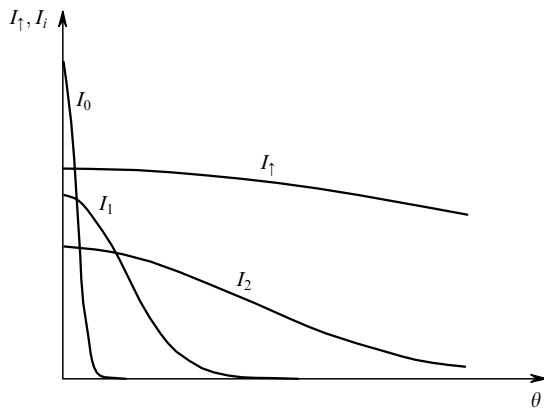


Figure 1. Angular structure of the upward (I_{\uparrow}) and three downward (I_i) components of the radiation flux.

It is natural that unscattered light produces a large-grain speckle pattern with the smallest average number of spots per unit area. For the diffusion component, on the contrary, the number of spots per unit volume is the greatest, exceeding by seven orders of magnitude the density of spots caused by unscattered light.

Recall that there also exists in a scattering medium the component of the upward radiation flux, which produces the irradiance E_{\uparrow} described by expression (5). This component has the most pronounced diffusion radiance angular distribution in the medium and, therefore, produces the speckle pattern with a very small correlation radius. Therefore, we can assume that this component produces almost uniform light background. The values of E_0 , E_1 , E_2 and E_{\uparrow} presented in Table 1 allow one to estimate the relation between different components and thereby the modulation

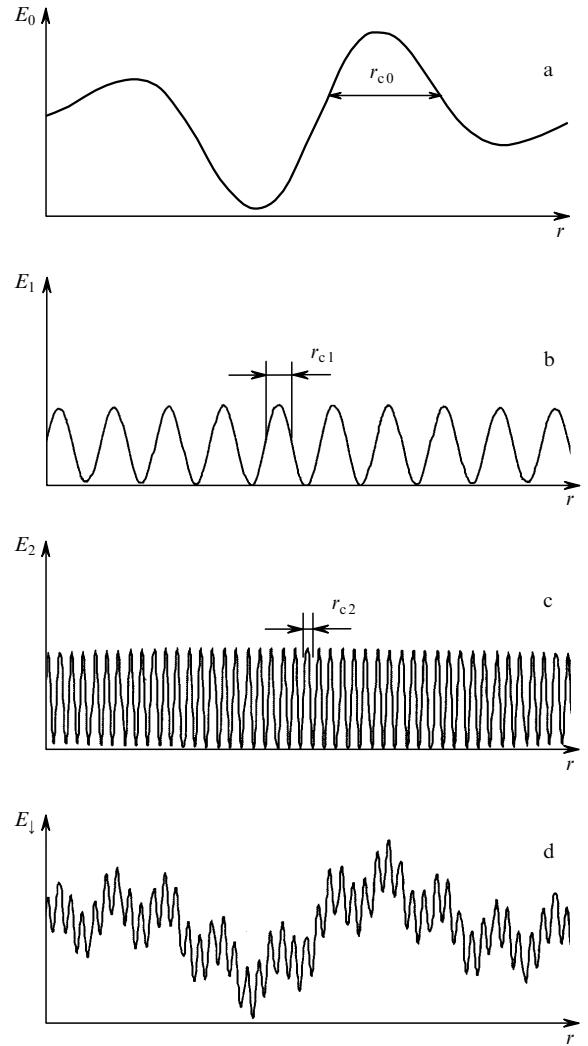


Figure 2. Speckle patterns of the individual components of the downward radiation flux (a–c) and of their sum.

depth of the light field, assuming that upon averaging over one or another spatial scales, the finer speckles produce an almost uniform light background.

So far we analysed the speckle pattern depending on the optical rather than the geometrical depth, because within the framework of the radiation transfer theory, the light fields in the medium depend on the optical depth. The attenuation factor ε in a biological tissue for $\lambda = 400$ nm is 5.3 times greater than for $\lambda = 700$ nm. Therefore, the geometrical penetration depth of light, all other factors being the same, should be 5.3 times larger for the second wavelength than for the first one. In reality, this excess is even greater because absorption of light in the red spectral region is much lower than in the violet region.

Note that such parameters as the average spot size r_{ci} and the concentration n_i of spots depend on the radiation wavelength λ . As the wavelength increases, r_{ci} increases, while n_i decreases.

As the size of scattering centres in the medium increases, the parameter β_{21} of the diffraction component decreases. This leads to a deeper penetration of the diffraction component into the medium and, hence, to an increase in the characteristic spot size r_{c1} . In this case, the parameter

β_{22} of the diffusion component of the phase function changes comparatively weakly.

Note in conclusion that all the results presented above were obtained for the case of a ‘frozen medium’, in which scattering centres are fixed, and for the spatial mode structure of the initial beam invariable in time. If these conditions are violated, the spatial speckle pattern will, naturally, vary in time.

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