Quantum Electronics **35** (4) 323 – 327 (2005)

PACS numbers: 42.55.Wd; 42.545.Rz; 42.60.Lh
DOI: 10.1070/QE2005v035n04ABEH002854

Temperature dependence of the radiation wavelength of a fibre laser

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***Abstract*. A model of a fibre laser is proposed which gives an analytic expression relating the laser wavelength to the main parameters of the laser. It is shown that there exists a region of the ambiguous dependence of the laser wavelength on the temperature and length of the active medium. Experimental results agree quantitatively with the theoretical model in a broad spectral range of laser radiation. Two-wave lasing without an intracavity selecting element is predicted and observed for the first time.**

Keywords: *fibre lasers, laser wavelength, luminescence cross section, absorption cross section, temperature.*

Single-mode fibre lasers based on silica doped with rare-earth ions produce high output powers in a broad spectral range upon multimode pumping [1]. The development of the elemental base of fibre optics stimulates the elaboration of new, more accurate physical and mathematical models describing various processes in such lasers [1 – 4].

One of the main goals of the development of technology of high-power fibre lasers is the efficient conversion of multimode pump radiation of laser diodes to the high-quality radiation of a fibre laser. Because the energy of the laser photon is lower than that of the pump photon, a part of the pump power is spent to the heating of the active medium of a fibre laser. In [2], the thermal regime of a fibre laser was considered and it was shown that for output powers below 100 W the heating of the laser fibre core can be neglected.

At present the output power of the most efficient fibre laser, a single-mode Yb^{3+}-doped fibre laser, amounts to 400 W [5], and the possibilities of the development of higher-power fibre lasers are far from exhausted [6 – 8]. The study of thermal effects in fibre lasers is of current interest because the temperature of the fibre of single-mode lasers emitting powers above 300 W can exceed the environment temperature by more than 100 °C [3]. The increase in the fibre temperature can substantially affect the parameters of an Yb^{3+}-doped fibre laser. It was shown experimentally [4] that the lasing threshold can increase by 25 % when the fibre temperature increases from 0 to 100 °C. The fibre laser tuning by varying its temperature was also experimentally demonstrated [4].

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Received 27 August 2004; revision received 20 January 2005
Kvantovaya Elektronika **35** (4) 323 – 327 (2005)
Translated by M.N. Sapozhnikov

The generally accepted model of stationary oscillation in an ion-doped solid-state laser (the McCumber theory [9]) allows one to calculate the laser wavelength. In this model, the gain in an active medium is calculated from the absorption and luminescence cross sections of active ions, and the laser wavelength is equal to the wavelength at which the maximum gain is observed when the lasing threshold is achieved. The McCumber theory was used in [1] for the numerical calculation of the radiation wavelength of an Yb^{3+} laser. As far as we know, so far no physical models that would allow the calculation of the lasing wavelength taking into account the temperature of active ions along with other parameters have been reported in the literature.

In this paper, we propose a laser model based on the McCumber approach, which describes an active medium in the laser resonator with the help of balance equations.

We consider the laser medium containing rare-earth ions whose energy levels are split into sublevels due to their interaction with surrounding atoms. The excitation and relaxation processes of active ions accompanied by absorption or emission of photons upon transitions between any two levels are described by the corresponding absorption and luminescence cross sections [9].

Consider the two-level model of active ions in the resonator (note that the three- or four-level system can be reduced to the two-level system by introducing the cross sections for transitions between the levels, as was done below for the two-level system). Let N_1 and N_2 be the populations of levels 1 and 2 of the laser transition and $\sigma_{12}(\lambda, T)$ and $\sigma_{21}(\lambda, T)$ be the absorption cross section for the 1 – 2 transition and luminescence cross section for the 2 – 1 transition, respectively. We assume that many longitudinal radiation modes and only one transverse mode are excited in the laser resonator, the coefficient of spatial overlap of the transverse mode with the active medium being unity. The medium is excited by an external pump source of power P. The fibre axis z is parallel to the radiation beam. The gain in the active medium at the wavelength λ per transit of radiation in the resonator is described by the expression

$$G = \exp g,$$
$$g(\lambda) = \int_0^L (N_2\sigma_{21} - N_1\sigma_{12})\mathrm{d}z, \tag{1}$$

where L is the active medium length. The concentration of active ions is $N = N_1 + N_2$. Then,

$$g = (\sigma_{12} + \sigma_{21})\int_0^L N_2(z)\mathrm{d}z - \sigma_{12}NL.$$

The upper-level population and the density of pump photons depend on time in the small-signal approximation (below the lasing threshold):

$$\frac{\partial N_2}{\partial t} = -N_2\sigma_{21\mathrm{p}}S_\mathrm{p}\upsilon + N_1\sigma_{12\mathrm{p}}S_\mathrm{p}\upsilon - \frac{N_2}{\tau},$$

$$\frac{\partial S_\mathrm{p}}{\partial t} = N_2\sigma_{21\mathrm{p}}S_\mathrm{p}\upsilon - N_1\sigma_{12\mathrm{p}}S_\mathrm{p}\upsilon,$$

where S_p is the pump photon density (the number of pump photons per unit volume of the active medium); τ is the lifetime of the excited ion; $\upsilon = c/n$ is the speed of light in the medium; n is the refractive index; $\sigma_{12\mathrm{p}}$ and $\sigma_{21\mathrm{p}}$ are the transition cross sections at the pump wavelength.

In the stationary case, $\partial N_2/\partial t = 0$, so that $\partial S_\mathrm{p}/\partial t = -N_2/\tau$ or $\partial S_\mathrm{p}/\partial z = -N_2/(\upsilon\tau)$, which gives

$$\int_0^L N_2\mathrm{d}z = \upsilon\tau\int_0^L \mathrm{d}S_\mathrm{p} = \upsilon\tau[S_\mathrm{p}(L) - S_\mathrm{p}(0)], \tag{2}$$

where $S_\mathrm{p}(0)$ and $S_\mathrm{p}(L)$ are the pump photon densities at the entrance and exit of the active medium, respectively. Therefore,

$$\int_0^L N_2\mathrm{d}z = f(P_\mathrm{a}),$$
$$g = (\sigma_{12} + \sigma_{21})f(P_\mathrm{a}) - \sigma_{12}NL, \tag{3}$$

where $f(P_\mathrm{a})$ is the function depending only on the absorbed pump power P_a.

Note that $f(P_\mathrm{a})$ for an ytterbium-doped fibre laser is determined by expression (2). It is obvious that not all the absorbed pump power is involved in the production of inversion, and therefore expression (2) cannot be directly used. However, the conclusion that the inversion integrated over the active-medium length is a function of the absorbed pump energy is correct in the general case.

Let us introduce the function $\varphi(\lambda) = g(\lambda) - g_0(\lambda)$, where $g(\lambda)$ is the gain in the active medium determined by expression (2) and $g_0(\lambda)$ is the cavity loss, which depends on the wavelength in the general case. Then, the conditions of lasing at the wavelength λ_0 are

$$\varphi(\lambda_0) = 0, \tag{4}$$

$$\left.\frac{\partial\varphi}{\partial\lambda}\right|_{\lambda_0} = 0. \tag{5}$$

Condition (5) means that the function $\varphi(\lambda)$ has a maximum at the laser wavelength while condition (4) corresponds to the lasing threshold. The absorption and luminescence cross sections of active ions are related by the McCumber formula [9]

$$\sigma_{12}(\lambda, T) = \sigma_{21}(\lambda, T)\exp\left[\frac{hc\lambda^{-1} - \mu(T)}{kT}\right]. \tag{6}$$

Here, $\mu(T)$ is the excitation potential; k is the Boltzmann constant; T is the active-medium temperature; and h is Planck's constant. The excitation potential $\mu(T)$ is the energy required for exciting an ion under the condition that the temperature of active ions in kept constant [9]. The potential $\mu(T)$ also determines the distribution of populations N_1 and N_2 in the state of thermal equilibrium.

By solving Eqns (3)–(6), we obtain the expression describing the laser wavelength as an implicit function of the basic laser parameters and the temperature of active ions:

$$g_0 = \left\{-\sigma_{21}^2 LN\frac{hc}{\lambda^2 kT}\exp\left(\frac{hc\lambda^{-1} - \mu}{kT}\right) + \frac{\partial g_0}{\partial\lambda}\left[1 + \exp\left(\frac{hc\lambda^{-1} - \mu}{kT}\right)\right]\right\}\left[\frac{\partial\sigma_{21}}{\partial\lambda} + \frac{\partial\sigma_{21}}{\partial\lambda}\exp\left(\frac{hc\lambda^{-1} - \mu}{kT}\right) - \sigma_{21}\frac{hc}{\lambda^2 kT}\exp\left(\frac{hc\lambda^{-1} - \mu}{kT}\right)\right]^{-1}. \tag{7}$$

Expression (7) is substantially simplified if the cavity loss is independent of the wavelength, i.e., in the absence of an intracavity selecting element. In addition, the second term in the last cofactor in square brackets in (7) can be neglected compared to the first term. Indeed, one can see from (6) that in the spectral regions where lasing can occur [i.e., for the wavelengths at which $\sigma_{12}(\lambda, T) < \sigma_{21}(\lambda, T)$], the condition $(hc\lambda^{-1} - \mu)/(kT) < 0$ is fulfilled. For lasing at the wavelength λ satisfying the condition $|hc\lambda^{-1} - \mu| > kT$ (the most typical case), we have $\exp[(hc\lambda^{-1} - \mu)/(kT)] \ll 1$.

As a result, we obtain after simple transformations

$$g_0 = \sigma_{21}LN\left[1 - \frac{\partial\sigma_{21}}{\partial\lambda}\frac{\lambda^2 kT}{\sigma_{21}hc}\exp\left(-\frac{hc\lambda^{-1} - \mu}{kT}\right)\right]^{-1}. \tag{8}$$

The solution of Eqns (7) or (8) for the known parameters of the laser and temperature of the active medium gives the laser wavelength. These equations are transcendental with respect to λ and cannot be solved analytically. However, they can be solved with respect to L and N, which allows one to analyse the dependence of the laser wavelength on these parameters. One can see that expression (8) does not contain $f(P_\mathrm{a})$.

So far we considered a two-level medium, which is described with the help of absorption and luminescence cross sections. Below, we will use the obtained equations to describe an Yb^{3+}-doped silica fibre laser. Consider the system of equations (Fig. 1) with the absorption and luminescence cross sections measured at room temperature for the Yb^{3+} ion in a silica fibre used in our experiments (Fig. 2). Note that, although the absorption and luminescence cross sections depend on the chemical composition of the fibre core, this is insignificant for the description of results of our paper.

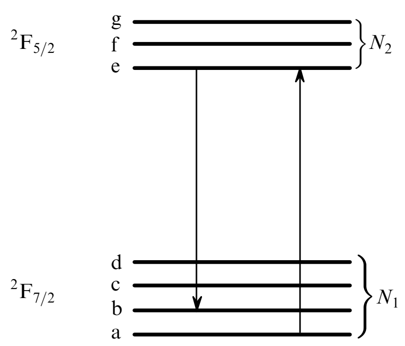

Figure 1. Energy level diagram of the Yb^{3+} ion in a silica fibre.

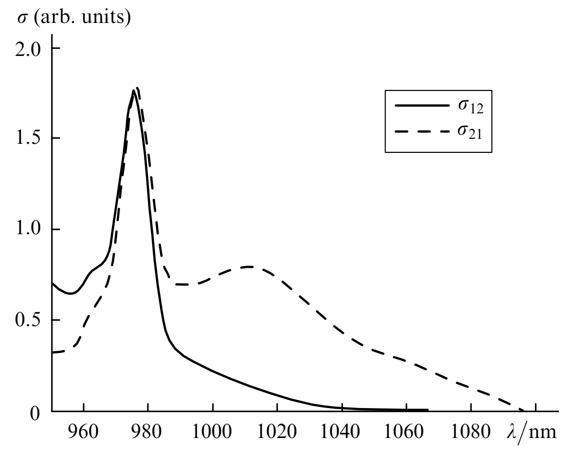

Figure 2. Absorption (σ_{12}) and luminescence (σ_{21}) cross sections for the Yb^{3+} ion in a silica fibre. The absolute values of the cross sections at a wavelength of 0.975 μm are 1.8×10^{-24} m^2.

The active levels of the Yb^{3+} ion are the $^2F_{5/2}$ and $^2F_{7/2}$ levels split into the Stark sublevels. The transition from the sublevel a to the sublevel e corresponds to a photon at 975 nm [1], and the energy of this transition is equal to the excitation potential $\mu(T)$. The upper-level lifetime is much longer than the establishment time of thermal equilibrium between the sublevels of the $^2F_{5/2}$ and $^2F_{7/2}$ levels. Therefore, we can assume that the Yb^{3+} ions always have the Boltzmann distribution among these sublevels. Note that the thermal-equilibrium establishment time should be compared with the reciprocal rate of the inversion decay due to stimulated transitions. However, these quantities can be made equal only at very high pump powers that cannot be realised in experiments. The luminescence cross section for the wavelengths exceeding 975 nm corresponds to transitions from the level e to the b, c, and d levels [1]. This cross section does not change substantially in the temperature range 200 – 400 K because the number of ions at the excited level e weakly depends on temperature according the Boltzmann distribution [4]. The temperature dependences of the homogeneous broadening of the luminescence cross section and the excitation potential in this temperature range are also weak [10]. Therefore, the cross section σ_{21} in (7) and (8) depends only on λ. Thus, temperature explicitly enters these equations, making possible to analyse the temperature dependence of the laser wavelength.

Equations (7) and (8) take into account the thermal distribution of active ions among Stark sublevels. The population of the sublevels b and c increases with temperature, resulting in the change of absorption at the laser wavelength [as follows from (6)]. As a result, the gain band shifts to the red and the laser wavelength increases.

Consider a single-mode Yb^{3+}-doped fibre laser with a broadband resonator (without a selecting element). The wavelength of this laser is determined by implicit function (8). The solid curve in Fig. 3 shows the temperature dependence of the laser wavelength calculated for the molar concentration of ytterbium ions $N = 8 \times 10^{-4}$ and the laser fibre length $L \sim 2.0$ m. The laser resonator is formed by a mirror with the reflectivity $R_1 \simeq 100\,\%$ and the fibre end with $R_2 = 4\,\%$. Figure 4 shows the typical dependences of the fibre length on the laser wavelength at temperatures 200, 300, and 400 K and the values of N, R_1, and R_2 indicated above. One can see from Figs 3 and 4 that the dependences of the laser wavelength on the fibre length and temperature have the S-like features. It follows from calculations that for $L \simeq 0.5$ m, lasing can occur at a wavelength of 980 nm. Note that high-power single-mode radiation at 980 nm is of considerable interest for applications in telecommunication, medicine, and industry.

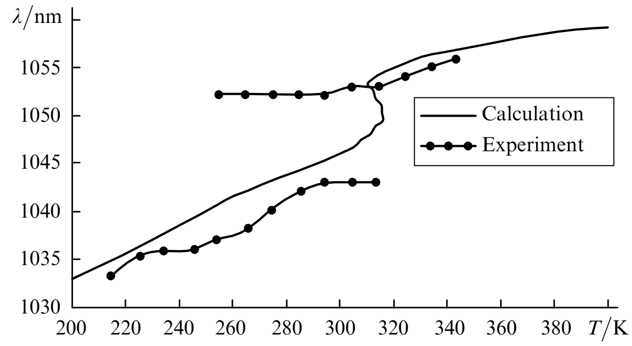

Figure 3. Calculated and experimental temperature dependences of the laser wavelength.

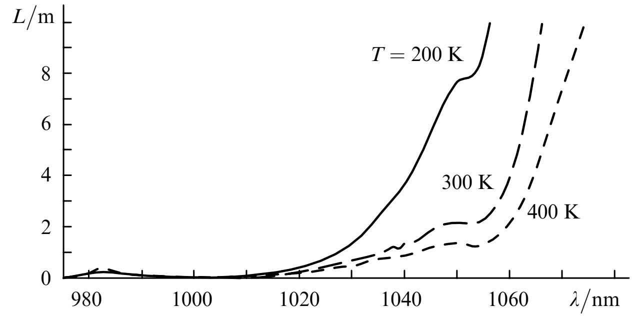

Figure 4. Dependences of the active medium length on the laser wavelength at different temperatures.

Because expression (8) does not contain explicitly the absorbed pump power, there exist the regions of parameters for dependences in Figs 3 and 4 where different threshold powers can correspond to one point. Consider the case when $\partial\sigma_{21}/\partial\lambda = 0$ in (8) (the derivative of the luminescence cross section is shown in Fig. 5), which corresponds to a local maximum of the cross section at a wavelength of 1015 nm and a minimum at wavelength of 995 nm (Fig. 2). Then, (8) takes the form

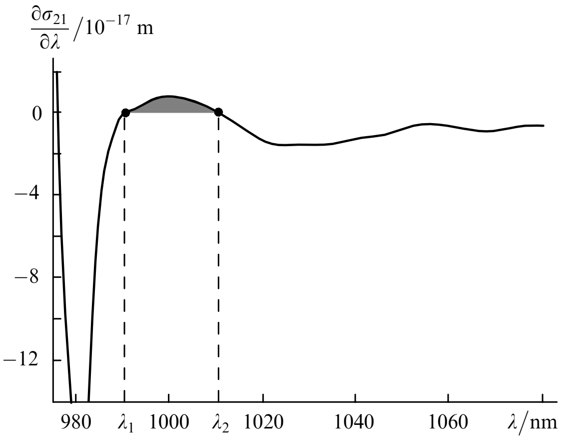

Figure 5. Derivative of the luminescence cross section for the Yb^{3+} ion in a silica fibre. Lasing cannot be obtained in the wavelength range $\lambda_1 - \lambda_2$ in a resonator without a selecting element.

$$g_0 = \sigma_{21} LN,$$

i.e., g_0 is equal to the gain of a completely inverted system. In the spectral range 995–1015 nm, $\partial\sigma_{21}/\partial\lambda > 0$, and hence $g_0 > \sigma_{21}LN$. For this reason, it is impossible to obtain lasing without using intracavity selecting elements in the spectral range 995–1015 nm, i.e., where the derivative of the luminescence cross section is nonnegative.

The properties of dependences in Figs 3 and 4 are explained by the shape of the luminescence cross section and its derivative with respect to the wavelength. The second term in (8) contains the derivative of the luminescence cross section in the numerator and the absorption cross section, expressed in terms of the luminescence cross section, in the dominator. As the wavelength increases, the absorption cross section deceases monotonically beginning from 975 nm, but the absolute value of the derivative of the luminescence cross section decreases more rapidly. Because the decrease in the absorption and the derivative of the luminescence cross sections dominates in turn (Fig. 5), the fraction changes nonmonotonically as a whole with the wavelength. This explains the appearance of ambiguities in Figs 3 and 4. It follows from Fig. 4 that temperature does not affect the position of characteristic features in the dependence $L(\lambda)$, although the shape of the dependence can change.

The model was experimentally tested for an Yb^{3+}-doped fibre laser with $L \simeq 2.0$ m and $N = 8 \times 10^{-4}$. The laser resonator was formed by the fibre end with the reflectivity $R_2 = 4\,\%$ and a dielectric mirror with $R_1 \simeq 100\,\%$ deposited on another fibre end. The width of the emission spectrum of the laser was about 3 nm; the wavelength of the maximum of the spectrum was considered the laser wavelength.

The scheme of the experimental setup is shown in Fig. 6. The laser fibre was placed in a thermostat, where its temperature could be varied from 200 to 400 K. The external heating of the fibre simulated processes developing in a high-power fibre laser due to the fibre core self-heating. We neglected the temperature gradient in the fibre core-cladding direction, which appears during the core self-heating, because this gradient is small near the core centre, being about 1 K per 10 μm [3] (the core diameter is $\sim$7 μm). The active fibre was end-pumped with a laser diode. The

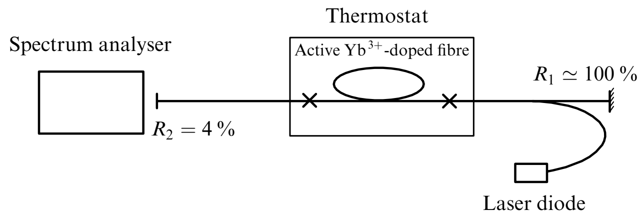

Figure 6. Scheme of the experimental setup. The crosses indicate splicings.

temperature dependence of the laser wavelength (Fig. 3) was measured with a spectrum analyser with a spectral resolution of 0.1 nm. The ambiguous dependence $\lambda(T)$ predicted by the model was manifested experimentally in simultaneous lasing at two wavelengths, the temperature range of two-wavelength lasing being broader than the predicted region of ambiguous dependence. Two-wavelength lasing depends on temperature in the following way. As temperature decreases from 320 to 310 K, a new weak peak at 1043 nm appears along with the main peak at 1052.5 nm in the emission spectrum. As temperature further decreases, the power of the main peak decreases, while that of the new peak increases, the powers of the peaks become equal at 293 K and then only the 1043-nm peak is observed at 250 K. Outside the region of two-wavelength lasing, the laser wavelength depends linearly on temperature with the slope 0.1 nm K^{-1}, in accordance with the theory.

The active medium length, the concentration of active ions, and the resonator loss enter expression (8) in the form NL/g_0. Therefore, it is reasonable to plot the dependence of NL/g_0 on the laser wavelength (Fig. 7). The calculated curve and all experimental points were obtained at room temperature; however, each point corresponds to different values of g_0 and L. It follows from Fig. 7 that the model calculation is in good agreement with the experiment.

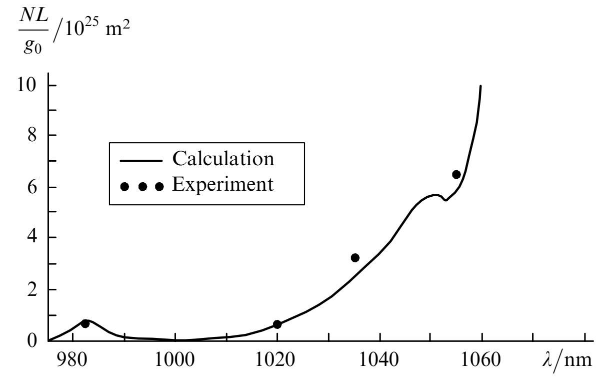

Figure 7. Dependence of the combination NL/g_0 of fibre laser parameters on the laser wavelength.

Thus, we have obtained the analytic formula relating the laser wavelength to the main parameters of the laser and the environment temperature. We have studied experimentally and theoretically the temperature dependence of the emission wavelength of the ytterbium-doped fibre laser, which has the region of the S-like ambiguity. The same ambiguity is present in the dependence of the laser wavelength on the active medium length. These dependences are determined by the shape of the luminescence cross section and its derivative with respect to the wavelength. The model predicts that lasing in a broadband resonator cannot be achieved in the wavelength range where the derivative of the luminescence cross section with respect to the wavelength is nonnegative. For the ytterbium laser, this range is from 995 to 1015 nm.

Our experimental results are in qualitative agreement with calculations. In the temperature region where the model predicts the S-like temperature dependence of the laser wavelength, we observed simultaneous lasing at two wavelengths in a resonator without a selective element. The experimental region of two-wavelength lasing is much broader than that predicted by the model. Outside the region of two-wavelength lasing, the temperature dependence of the laser wavelength is linear with a slope of 0.1 nm K^{-1}.

Note that the equations obtained within the framework of our model can be used for any laser medium described with the help of the luminescence and absorption cross sections. The model can be also applied to analyse the dependence of the threshold pump power on the parameters of the active medium and resonator and allows one to determine the laser parameters for obtaining lasing at the required wavelength [11, 12].

Acknowledgements. The authors thank V.P. Gapontsev and N.S. Platonov (IPG Photonics, USA), S.V. Popov (Imperial College, Great Britain), and I.E. Samartsev (NTO IRE–Polus) for discussions of the results and valuable comments.

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