PACS numbers: 42.68.Wt; 42.25.Kb
DOI: 10.1070/QE2005v035n11ABEH002860

Optimal detection of partially spatial coherent radiation by using optical quantum amplifiers

G.I. Kozin, A.P. Kuznetsov

Abstract. **Conditions for the efficient detection of reflected laser radiation by means of optical quantum amplifiers are considered theoretically. It is shown that in the case of a strongly disturbed spatial coherence, there is no need to take it into account and a simple geometrical consideration is sufficient. The expressions relating the effective radius of the input aperture and the angular magnification of a projecting optical system with the laser radiation divergence and amplifier parameters are obtained. The theoretical results are confirmed by experiments.**

Keywords: *optical quantum amplifier, spatial coherence, light reflection, radiation detection.*

The problems of remote laser measurements and diagnostics often involve the detection of radiation reflected from remote natural or artificial reflectors with the distortion of spatial coherence. A direct photodetection of a low-power radiation can be complicated because of the background and photodetector noise. In particular, this complicates the control of gas leaks in main gas pipelines by means of differential He – Ne lidars at the wavelengths of 3.3922 and 3.3912 μm [1]. The severe restrictions imposed on the mass and dimensions of lidars mounted in helicopters do not allow the use of receiving mirrors of diameter exceeding 30 cm. In this case, due to the low intensity of a detected signal, the measurements should be performed at dangerously small heights of 30 – 50 m.

The noise immunity and sensitivity of a lidar can be increased by detecting radiation preliminary amplified by an optical quantum amplifier (OQA). The radiation detection efficiency can be defined as the ratio of the detected radiation power, i.e., of the OQA output power by neglecting amplification to the radiation power incident on the OQA. Generally speaking, the detection efficiency depends on the spatial coherence of radiation.

It is known [2, 3] that the number of detected so-called coherence spots (speckles) is determined by the linear and angular aperture of a detector. This restricts the total angular aperture of the detector, which can be determined by the lens or mirror projecting optical system. The intensity of a detected signal can be increased by increasing power in a coherence spot. For this purpose, the size of a light spot on a reflector should be made as small as possible by means of a transmission optical system [4, 5]. Thus, the transmission and detection projecting systems should match the characteristics of radiation with the OQA parameters to provide the detection of the maximum number of coherence spots with the maximum radiation power in each of them.

The aim of our paper was to determine the angular magnification of the receiving projecting system providing the efficient detection of radiation by an amplifier at the minimal input aperture. A light spot of radius w from a laser with the wavelength λ produces usually many coherence spots of radius ρ_c on non-reflecting surfaces. According to the Van Cittert – Zernicke theorem and subsequent additions to it, the reciprocal radiation intensity in the far-field zone at the distance $l \gg w\rho_c/\lambda$ is determined by the product of the reflected radiation intensity I and the complex degree of spatial coherence μ.

We have shown [6] for an arbitrary geometry that the spatial coherence μ in the analysis of reflected radiation can be treated as the real function of the angular displacement $\boldsymbol{\theta}$ with respect to the observation direction and the intensity I – as the function of the observation angle γ. They are related by the Fourier transform with the characteristics of radiation reflected from a surface. In the case of small angular displacements,

$$\mu(\boldsymbol{\theta}) = P_0^{-1} \int I(\boldsymbol{r}) \exp(-\mathrm{i}k\boldsymbol{\theta}\boldsymbol{r})\mathrm{d}\boldsymbol{r}, \quad (1)$$

where $\boldsymbol{r}$ are the coordinates of points of the reflecting surface in the plane perpendicular to the observation axis; $I(\boldsymbol{r})$ is the visible intensity distribution in a spot; $P_0 = \int I(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$ is the power of reflected radiation; and k is the wave number.

In remote laser diagnostics, as a rule, a part of radiation reflected back is detected. In the case of a Gaussian beam, the intensity is $I(\boldsymbol{r}) = (2P_0/\pi w^2)\exp[-2(r/w)^2]$ and the spatial coherence is $\mu(\boldsymbol{\theta}) = \exp[-2(\theta/\theta_c)^2]$ (where $\theta_c = 2\lambda/\pi w$ is the coherence angle). Because the spot radius $w = \theta_d l$ is determined by the divergence $\theta_d = \lambda/\pi w_0$, where w_0 is the radius of the laser beam waist, the coherence angle $\theta_c = 2w_0/l$ is much smaller than the aperture angle $\gamma_a = r_a/l$ of the projecting system with the input radius $r_a \gg w_0$. Therefore, the transformed coherence degree $\mu(\boldsymbol{\theta}/u)$ at the output of the projecting system with the angular magnifi-

G.I. Kozin, A.P. Kuznetsov Moscow Engineering Physics Institute (State University), Kashirskoe sh. 31, 115409 Moscow, Russia;
e-mail: lasgas@pisem.net, kozingen@mail.ru

Received 7 September 2004; revision received 8 September 2005
Kvantovaya Elektronika **35** (11) 1009 – 1012 (2005)
Translated by M.N. Sapozhnikov

cation u does not change. Its inverse Fourier transform converts the intensity distribution on the reflector to the image with the linear magnification u^{-1}:

$$I'(\boldsymbol{r}) = u^2 I(u\boldsymbol{r})\frac{P_a}{P_0},$$

where P_a is the power of collected radiation. The light spot radius in the image is $w' = w/u = \lambda l/(\pi w_0 u)$.

Another interesting integral expression [6] relates the angular intensity distribution $I(\gamma)$ to the degree of spatial coherence $\mu(\boldsymbol{\rho})$ on the reflector observed through the input aperture. Because of the remoteness of the reflector and smallness of the coherence radius on its surface, the scattering angle $\gamma_c \sim \lambda/\rho_c$ is usually much greater than the aperture angle γ_a of the receiving projecting system. We can assume that the observed intensity I of reflected radiation is independent of the observation angle within γ_a and is zero for $\gamma > \gamma_a$. Therefore, upon observation from any point, we can assume that the reflecting surface emits independently and is oriented normally to the observation axis. Then, we obtain for small angles γ with respect to this axis (in our case, the laser beam axis)

$$I(\gamma) \sim \int \mu(\boldsymbol{\rho}) \exp(\mathrm{i}k\boldsymbol{\gamma\rho})\mathrm{d}\boldsymbol{\rho}, \qquad (2)$$

where $\boldsymbol{\rho}$ is the relative coordinate of two points of the reflecting surface in the plane perpendicular to the axis.

By using the inverse Fourier transform of the function $I(\gamma/u)$ (2), we can find the degree of spatial coherence of the image. It can be easily shown that for a circular aperture it is described the Airy function

$$\mu'(\boldsymbol{\rho}) = \frac{2J_1(x)}{x}, \quad x = \frac{kur_a\rho}{l},$$

where $J_1(x)$ is the first-order Bessel function. Its first zero determines the coherence radius of the image $\rho'_c = 0.61\lambda l/ur_a \ll w'$. The number of coherence spots in the image remains large, and the reciprocal intensity is still represented by the product of the intensity and the degree of spatial coherence.

The maximum radiation power received by the OQA is achieved for the maximum aperture angle of the projecting system $\gamma_a \leqslant \gamma_t/u$, which is restricted by the aperture angle γ_t of the OQA tube, and in the case of the maximum linear magnification u^{-1}, at which a signal completely fills the input aperture of the tube of radius r_t: $w/u \leqslant r_t$. These conditions determine the optimal angular magnification and the corresponding aperture angle

$$u = \frac{w}{r_t}, \quad \gamma_a = \frac{r_t\gamma_t}{w}. \qquad (3)$$

The number of coherence spots detected in this case is $N = (\gamma_t/u\theta_c)^2$. By using the relation $\theta_c \sim 2\lambda/\pi w$ and expression (3), we obtain $N = (\pi r_t\gamma_t/2\lambda)^2$. The radius r_t and the tube length L are adopted to satisfy the condition of low diffraction losses $r_t^2/\lambda L \sim 1$. It was shown in [2, 7] that reflections from the tube walls play an important role during radiation transfer in the tube. The angular function of single-pass transmission in the OQA tube can be approximated by the Gaussian

$$T(\gamma) = \exp\left[-2\left(\frac{\gamma}{\gamma_t}\right)^2\right], \quad \gamma_t = \left[2r_t\frac{(n^2-1)^{1/2}}{(n^2+1)L}\right]^{1/2}, \qquad (4)$$

where n is the refractive index of the wall material. The parameter γ_t achieves the maximum value $\sim(r_t/L)^{1/2}$ for $n^2 = 3$. Therefore, for amplifiers with quartz or glass tubes ($n \approx 1.5$), we have $N \approx 2r_t/\lambda \gg 1$. This estimate also follows from results obtained in [2]. Therefore, the angular distribution of the radiation intensity at the OQA output can be also considered by neglecting its coherence.

Thus, the detection of light with single-pass OQAs shows that the laser radiation intensity with a strongly disturbed spatial coherence can be considered by neglecting the degree of coherence. It is sufficient to know the scattering diagram in the geometrical optics approximation. However, the consideration of the degree of spatial coherence allows one to understand better radiation transfer processes and estimate their dependence on optical elements.

The radiation power at the OQA output is calculated by integrating over the input OQA aperture and angles within the projection angle $u\gamma_a$ taking into account the transmission function $T(\gamma)$ according to the intensity distribution on the reflector, the power collected by the projecting system, and its linear magnification. As a result, we obtain the transmission coefficient k of the tube, determining the detection efficiency, and the dependence of the output power $P \sim k\gamma_a^2$ of the optical amplifier on the same parameters

$$k = \left\{1 - \exp\left[-2\left(\frac{ur_t}{w}\right)^2\right]\right\} \times\left\{1 - \exp\left[-2\left(\frac{u\gamma_a}{\gamma_t}\right)^2\right]\right\}\frac{\gamma_t^2}{2(u\gamma_a)^2}. \qquad (5)$$

One can see that the detection efficiency monotonically falls with increasing γ_a, although the output power increases. Therefore, there is no point in increasing strongly the input pupil radius of the projecting system. According to (5), the value $k \approx 0.4$ is optimal from the point of view of the acceptable detection efficiency and a reasonably small size of the projecting system. In this case, 87 % of the maximum power that can be detected by the OQA at $\gamma_a \to \infty$ is detected. If the projecting system consists of one lens or collecting mirror with the focal distance $f \ll l$, then, by using the relations $u \simeq l/f$ and $w = \gamma_d l$, we obtain from (3)

$$r_a = f\gamma_t, \quad f = \frac{r_t}{\gamma_d}. \qquad (6)$$

These results were verified in experiments. We studied the dependences of the transmission coefficient of OQA tubes on their radius and length. The scheme of the experimental setup is shown in Fig. 1. Radiation from 4-mW He–Ne laser (*1*) at 3.3922 μm with a divergence of 2×10^{-3} rad was modulated by chopper (*2*) and directed to normally oriented reflector (*4*) placed at a distance of 10 m. The radiation reflected backward was collected by mirror (*3*) of radius $R_1 = 5.5$ cm with a central aperture of radius $R_2 = 1$ cm and the focal distance $f = 45$ cm. Such mirrors are used in mirror telescopic systems. Fine-grained reflectors with a broad scattering diagram and metal reflectors with a narrow diagram were used. The coefficient k for these conditions was calculated as in (5) upon approximation of the scattering diagram by a Gaussian. The radiation

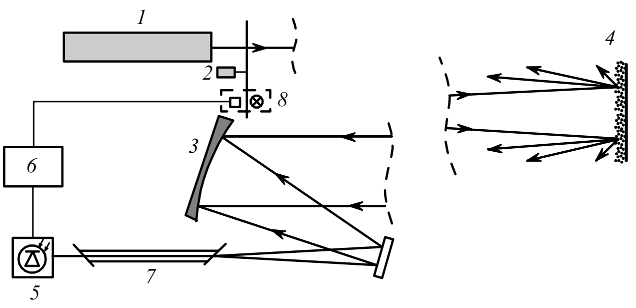

Figure 1. Scheme of the experimental setup for studying the efficiency of laser radiation detection with an OQA; (*1*) He–Ne laser; (*2*) chopper; (*3*) spherical mirror; (*4*) reflector; (*5*) photodetector; (*6*) lock-in detector; (*7*) OQA; (*8*) optical pair.

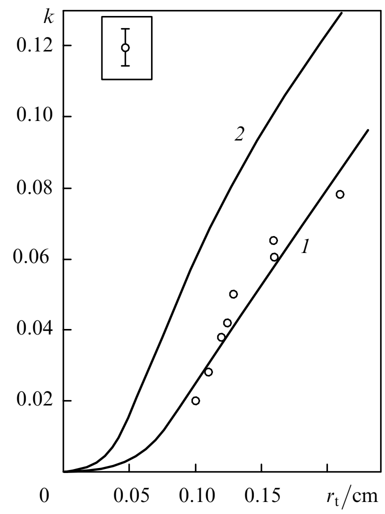

Figure 3. Dependences of the transmission coefficient k of the OQA tube on its diameter r_t for $L = 60$ cm. Theoretical curve (*1*) approximates the experimental data. Curve (*2*) is the dependence $k(r_t)$ calculated for a mirror without an aperture.

power was measured by photodetector (*5*) at the tube input and output by using lock-in detector (*6*). The reference signal at a modulation frequency of 2 kHz was formed by means of optical pair (*8*) placed on both sides of chopper (*2*). The passband of the detection system used in measurements was 1 Hz. The transmission coefficient of the tube was estimated from the ratio of the signals.

Figures 2 and 3 show the experimental dependences $k(L)$ for $r_t = 1.35$ mm and $k(r_t)$ for $L = 60$ cm. Good agreement of the experimental measurements with theoretical curves was obtained for the angle of scattering from the reflector surface $\gamma_r = 1.25 \times 10^{-2}$. For comparison, the theoretical dependences obtained for mirrors without apertures are also presented.

We also verified the validity of the relation $r_a = f\gamma_t$ in experiments with the OQA with a tube of length 73 cm and the inner diameter 2.2 mm filled with a He : Ne = 5.6 : 1 mixture at a pressure of 1.5 Torr and a discharge current of 26 mA. As a reflector, a rotating disc was used, which allowed us to average the detected signals over a statically homogeneous reflecting surface. The optimal focal distance f of the collecting mirror for the OQA tube radius and the divergence of laser radiation used in experiments was 50 cm. We used a mirror without an aperture with $f = 80$ cm and radius 5 cm. The mirror radius was varied by mounting diaphragms of different diameters in front of it. According to our calculations, the effective radius of the collecting mirror should be 2.6 cm.

Figure 4 presents the experimental dependence of the detected radiation power on the square of the aperture radius. The experimental values obtained by using a fine-grained polishing paper well agree with theoretical curve (*1*) for $\gamma_t = 4.4\times\ 10^{-2}$. The aperture angle of the OQA tube calculated from (4) was 3.2×10^{-2}. This discrepancy is explained by the influence of amplification in the OQA. As the angle γ_a increases, the path length and gain per transit increase, resulting in the increase in the angle γ_t. For this reason, $r_a = 3.5$ cm in the experiment. It is obvious that when a telescope is used, the value of r_a can achieve tens of centimeters.

Therefore, in practice the amount of light collected in the OQA is quite comparable with that collected upon direct

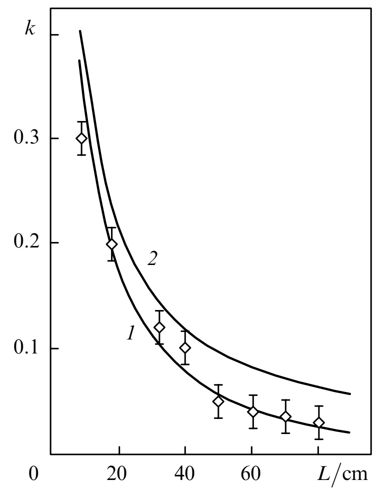

Figure 2. Dependences of the transmission coefficient k of the OQA tube on its length L for $r_t = 1.35$ mm. Theoretical curve (*1*) approximates the experimental data. Curve (*2*) is the dependence $k(L)$ calculated for a mirror without an aperture.

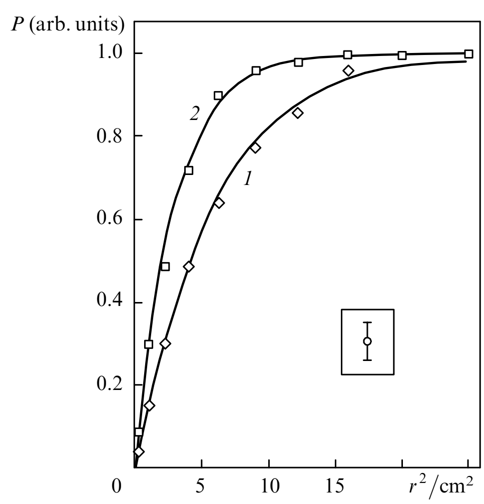

Figure 4. Radiation power P reflected from fine-grained paper (*1*) and duralumin (*2*) detected with the OQA as a function of the square of the radius r of a receiving mirror.

photodetection. The experimental data obtained for a duralumin reflector agree with the theoretical dependence [curve (2)] for $\gamma_r = 3.5 \times 10^{-3}$. The transmission coefficient of the tube $k = 0.15 \pm 0.01$ was measured from the ratio of signal intensities measured without a diaphragm at the input and output of the OQA with a quenched discharge. This value is in good agreement within the measurement error with that calculated from (5).

Figure 5 presents the experimental dependences of the radiation power at the OQA input and output on the effective power reflection coefficient $\rho_{\rm eff}^2$ of a diffusion reflector to the aperture angle. The measured values are normalised to the output power of the laser source. The output power of the OQA was measured directly during a discharge, and the input power was estimated from the output power in the absence of discharge taking into account the measured attenuation coefficient of the tube. The reflection coefficient was determined by the ratio of the radiation power at the OQA input detected in the absence of discharge to the output laser power, divided by k. The value of $\rho_{\rm eff}^2$ was reduced by means of calibrated light attenuators.

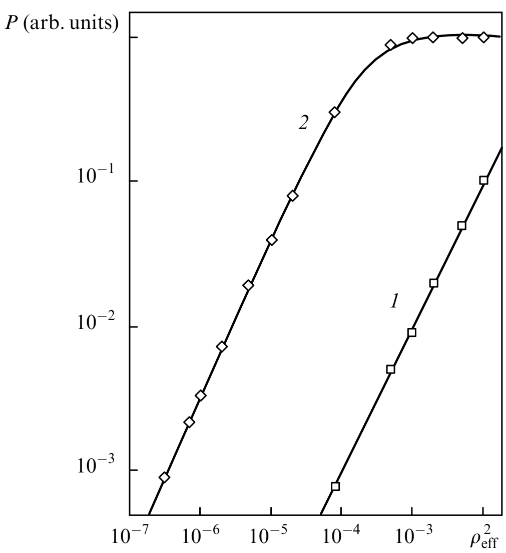

Figure 5. Radiation power P detected at the input (1) and output (2) of the OQA as a function of the effective power reflection coefficient $\rho_{\rm eff}^2$.

The dependence of the power at the OQA input on $\rho_{\rm eff}^2$, as expected, is linear. The dependence of the power at the OQA output is nonlinear for large $\rho_{\rm eff}^2$, which is explained by the gain saturation in the OQA. The ratio of powers in the linear region for both curves for the same value of $\rho_{\rm eff}^2$ gives the unsaturated gain K of the OQA $K = kG = (2.0 \pm 0.2) \times 10^2$, where G is the gain of the active medium. A comparison of K and k shows that $G = (1.5 \pm 0.2) \times 10^3$, in accordance with the known data for the active medium of this type ($\sim$44 dB m^{-1}).

Thus, our estimates and experimental results have shown that the use of high-gain OQAs can enhance the photodetection sensitivity by a few orders of magnitude. In particular, a He–Ne OQA with a collecting mirror of diameter 7 cm can increase the detected signal by an order of magnitude compared to the method of direct photodetection with a collecting mirror of diameter 30 cm. In the tests of gas pipelines and other areas on the presence of saturated hydrocarbons in air, this will allow flights at least at a standard height of 100 m. It can be expected that this height can be increased by an order of magnitude by using transmitting and receiving telescopes. The results of this paper can be also useful for the development of laser radars and rangers, eliminating the necessity of analysing radiation speckles. In this case, however, a great number of coherence spots should be provided in a light spot on the reflector and all optical elements, which excludes a tight focusing of light on reflectors or the use of nearly mirror reflectors.

The theoretical conclusion of the paper can appear paradoxical. The concept of spatial coherence has been used to show that it is insignificant in the case of a weak spatial coherence of the detected radiation. However, there is nothing strange in it. It is well known that the radiation of thermal sources is well described in the geometrical optics approximation.

References

1. Butikov Yu.A., Chura N.I., Shirochenskii S.I. *Sovremennye distantsionnye methody i apparatura kontrolya utechek iz magistral'nykh truboprovodov* (Modern Remote Methods and Equipment for the Leak Control in Main Pipelins) (Moscow: IRTs Gazprom, 1995).
2. Belonuchkin V.E., Eskin N.I., Kozel S.M., et al. *Kvantovaya Elektron.*, **4**, 1318 (1977) [*Sov. J. Quantum Electron.*, **7**, 742 (1977)].
3. Ustinov N.D., Matveev I.N., Protopopov V.V. *Metody obrabotki opticheskikh polei v lazernoi lokatsii* (Methods for Optical Field Processing in Laser Ranging) (Moscow: Nauka, 1983).
4. Massey G.A. *Appl. Opt.*, **4**, 7 (1965).
5. Kozel S.M., Lokshin G.R. *Radiotekh. Elektron.*, **20**, 1666 (1975).
6. Kozin G.I., Kuznetsov A.P. *Kvantovaya Elektron.*, **25**, 1079 (1998) [*Quantum Electron.*, **28**, 1050 (1998)].
7. Baikov S.S., Korolev F.A. *Zh. Prikl. Spektr.*, **10**, 441 (1969).