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LASER BEAMS

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Laser heterodyning of Gaussian beams with partial spatial coherence

G.I. Kozin, A.P. Kuznetsov, M.O. Lebedinskii

***Abstract*. The characteristics of Gaussian beams with partial spatial coherence are considered theoretically. The conditions for efficient recording of laser radiation reflected from scattering surfaces upon laser heterodyning and intracavity radiation reception are analysed. Recommendations on the use of projecting telescopic systems are given. Theoretical conclusions are confirmed experimentally.**

Keywords: *laser, laser heterodyne, intracavity radiation reception, spatial coherence, reflection of light.*

It is known that laser heterodyning upon detecting weak radiation allows one to increase substantially the detection sensitivity and signal protection from background illumination. Intralaser radiation reception has also in fact the heterodyne nature. In this case, the detected signal is formed not in a photodetector but in the active medium of the laser and is manifested in variations in the power and laser frequency. However, the reception efficiency, which can be defined as the ratio of the power involved in the signal formation to the total power of incident radiation, upon heterodyne detection, unlike direct photodetection, is caused by the spatial coherence of radiation. It can be strongly violated after reflection of laser radiation from the rough surface of natural or artificial reflectors used in experiments. The spatial coherence is characterised by the coherence radius – the statistically average radius of coherence spots, i.e., areas on the wave front in which the phase shift can be considered regular. It is known that heterodyne detectors detect radiation only in one coherence spot [1]. This limits the input angular aperture which can be achieved with the help of a projecting optical system of lenses or mirrors. The detected signal can be enhanced by increasing power in a coherence spot. This can be achieved by decreasing the size of a light spot on a reflector [2, 3] by means of the transmitting optical system. Therefore, the transmitting and receiving projection systems should match the parameters of the received radiation and detector.

However, these recommendations are only qualitative. The modern concepts of the spatial coherence of reflected light are insufficient for quantitative estimates of the optimal parameters of projecting systems and the detection efficiency. The well-known van Cittert – Zernicke theorem and its subsequent modifications are valid only when a light beam of radius w contains many coherence spots of radius ρ_c, i.e., when $w \gg \rho_c$. Only far-field radiation is considered at a distance $l \gg w\rho_c/\lambda$ (λ is the radiation wavelength) from the reflector, where the mutual radiation intensity J is represented by a product of the intensity I by the complex function of spatial coherence μ. In the case of intracavity reception, we deal with the projection of the near-field reflection region made coincident with the near-field region of a laser beam, where the active medium is usually located. In this connection we analysed reflected laser radiation without the restrictions discussed above.

We determined the mutual radiation intensity from a surface with a plane on average wave front for the Gaussian field distribution V and the Gaussian degree of coherence (as the most probable):

$$V(0,\boldsymbol{r}) = \left(\frac{2P}{\pi w_0^2}\right)^{1/2} \exp\left[-\left(\frac{\boldsymbol{r}}{w_0}\right)^2\right],$$

$$\mu(\boldsymbol{\rho}) = \exp\left[-2\left(\frac{\boldsymbol{\rho}}{\rho_{c0}}\right)^2\right].$$

Here, $\boldsymbol{r}$ are the two-dimensional coordinates of a point on the emitting plane (Fig. 1) measured from the centre O of a light spot of radius w_0; the longitudinal coordinate $l = 0$; $\boldsymbol{\rho}$ are the relative coordinates of two points; ρ_{c0} is the coherence radius on the plane; and P is radiation power. The same function μ in the image of a light spot also takes place [4] in the soft aperture approximation for the entrance pupil with radius r_a of the projecting optical system, when the dependence of its transmission on the angle γ is approximated by the function $T(\gamma) = \exp[-2(\gamma/\gamma_a)^2]$, where $\gamma_a = \sqrt{2}r_a/l$. Based on the definition of the mutual intensity $J(Q_1, Q_2)$ at two observation points Q_1 and Q_2 and using the Kirchhoff theorem, we can assume that for small observation angles with respect to the normal to the emitting plane

$$J(Q_1, Q_2) = C\int\int \mu(\boldsymbol{\rho}) V(0,\boldsymbol{r}) V^*(0,\boldsymbol{r}+\boldsymbol{\rho}) \exp[ik(l_1 - l_2)]\mathrm{d}\boldsymbol{\rho}\mathrm{d}\boldsymbol{r},$$

$$C = \left[\frac{k}{2\pi(l_1 l_2)^{1/2}}\right]^2.$$

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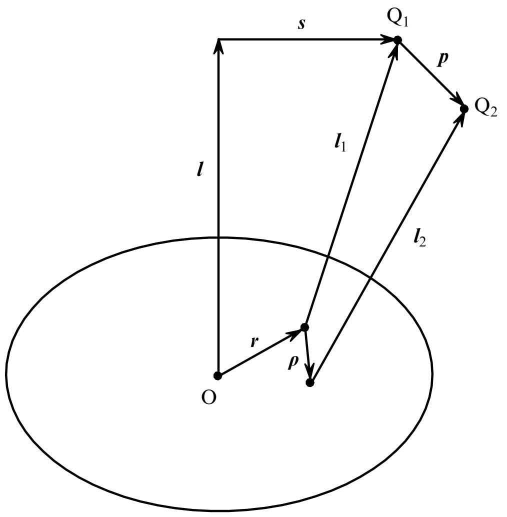

Figure 1. To the calculation of the mutual intensity of reflected radiation at two observation points (see explanation in the text).

Here, $\boldsymbol{l}_1$ и $\boldsymbol{l}_2$ are the vectors coinciding with the beams directed to the observation points Q_1 and Q_2; and k is the wave number. The vector $\boldsymbol{l}$ is the coordinate of the point Q_1 along the normal to the emitting plane; $\boldsymbol{s}$ is its coordinate along the observation plane; and $\boldsymbol{p}$ is the vector of displacement from point Q_1 to point Q_2. The product $V(0,\boldsymbol{r})V^*(0,\boldsymbol{r}+\boldsymbol{\rho})$ contains the exponential with the argument $r^2+(\boldsymbol{r}+\boldsymbol{\rho})^2=2(\boldsymbol{r}+\boldsymbol{\rho}/2)^2+\rho^2/2$. Because integration over $\boldsymbol{r}$ and $\boldsymbol{\rho}$ is performed within infinite limits, we can pass to the variable $\boldsymbol{r}+\boldsymbol{\rho}/2$, by selecting its contribution to the difference l_1-l_2. The length l_2 can be found from the vector equality $\boldsymbol{l}_2=\boldsymbol{l}+\boldsymbol{s}+\boldsymbol{p}-\boldsymbol{r}-\boldsymbol{\rho}$. Taking into account that $\boldsymbol{s}$, $\boldsymbol{r}$, and $\boldsymbol{\rho}$ are orthogonal to $\boldsymbol{l}$, we have

$$l_2=l\left[1+\frac{2\boldsymbol{lp}}{l^2}+\frac{(\boldsymbol{s}+\boldsymbol{p}-\boldsymbol{r}-\boldsymbol{\rho})^2}{l^2}\right]^{1/2}. \tag{1}$$

By expanding the root in a power series in the ratios of s, p, r, and ρ to l up to third-order terms inclusive, we obtain

$$l_2=l\left[1+\frac{2(\boldsymbol{lp})+(\boldsymbol{s}+\boldsymbol{p}-\boldsymbol{r}-\boldsymbol{\rho})^2}{2l^2}-\frac{(\boldsymbol{lp})^2+(\boldsymbol{lp})(\boldsymbol{s}+\boldsymbol{p}-\boldsymbol{r}-\boldsymbol{\rho})^2}{2l^4}+\frac{(\boldsymbol{lp})^3}{2l^6}\right].$$

By setting here $\boldsymbol{p}=0$, we find l_1. By subtracting l_1 from l_2, using the identity $l^2p^2-(\boldsymbol{lp})^2=[\boldsymbol{l},\boldsymbol{p}]^2$ and selecting the quantity $\boldsymbol{r}+\boldsymbol{\rho}/2$, we obtain

$$l_2-l_1=p_l+\frac{p_s^2+2(\boldsymbol{p}_s\boldsymbol{s})}{2l}+\frac{(\boldsymbol{r}+\boldsymbol{\rho}/2,\boldsymbol{\rho}-\boldsymbol{p}_s)}{l}-\frac{(\boldsymbol{\rho},\boldsymbol{p}_s+2\boldsymbol{s})}{2l}-\frac{p_l(\boldsymbol{p}_s+\boldsymbol{s}-\boldsymbol{r}-\boldsymbol{\rho})^2}{2l^2}, \tag{2}$$

where p_l и $\boldsymbol{p}_s$ are the components of the displacement vector along the normal to the emitting plane and in the plane, respectively. We will use the higher-order term (here third-order term), as usual, to estimate the applicability of conclusions made. Without this term, after integration over $\boldsymbol{r}+\boldsymbol{\rho}/2$, we have

$$J(\mathrm{Q}_1,\mathrm{Q}_2)=CP\exp\left\{-\mathrm{i}k\left[p_l+\frac{p_s^2+2(\boldsymbol{ps})}{2l}\right]\right\}\times\int\exp\left\{-\left[a\rho^2+b(\boldsymbol{\rho}-\boldsymbol{p}_s)^2\right]+\frac{\mathrm{i}k(\boldsymbol{\rho},\boldsymbol{p}+2\boldsymbol{s})}{2l}\right\}\mathrm{d}\boldsymbol{\rho},$$

where $a=(4n+1)/(2w_0^2)$; $b=[\pi w_0/(\lambda l)]^2/2$; $n=(w_0/\rho_{c0})^2$ is the number of coherence spots within a light beam on the emitting plane. After the passage to the variable $\boldsymbol{\rho}-[b/(a+b)]\boldsymbol{p}_s$, integration, and algebraic transformation, the expression for the mutual coherence takes the form

$$J(\mathrm{Q}_1,\mathrm{Q}_2)=C(\lambda l)^2\frac{2P}{\pi w^2}\exp\left\{-\frac{(\boldsymbol{s}+\boldsymbol{p}_s)^2}{w^2}-\left(\frac{s}{w}\right)^2-2\left(\frac{p_s}{\rho_c}\right)^2-\mathrm{i}k\left[p_l+\frac{p_s^2+2(\boldsymbol{p}_s\boldsymbol{s})}{2R}\right]\right\}, \tag{3}$$

where

$$w=w_0\left[1+\left(\frac{l}{l_0}\right)^2\right]^{1/2};\quad \rho_c=\rho_{c0}\left[1+\left(\frac{l}{l_0}\right)^2\right]^{1/2};$$

$$R=l\left[1+\left(\frac{l_0}{l}\right)^2\right];$$

$l_0=\pi w_0^2/[\lambda(4n+1)^{1/2}]$ is the near-field region boundary.

In the far-field region ($l\geqslant l_0$), we can assume that the radius of the regular part of the wave front is $R=l$, the radius of the light beam is $w=\gamma_r l$, and the radius of the coherence spot is $\rho_c=\theta_c l$, where

$$\gamma_r=\frac{\lambda}{\pi\rho_{c0}}\left(\frac{4n+1}{n}\right)^{1/2},\quad \theta_c=\frac{\lambda}{\pi w_0}\left(\frac{4n+1}{n}\right)^{1/2} \tag{4}$$

are the scattering and coherence angles. For $n\geqslant 1$, these expressions coincide with those obtained in [4].

In the case when n is of the order of unity or smaller, the applicability of the obtained expressions can be estimated from the following condition: it is necessary that the third-order term omitted in (2) should be much smaller than λ. One can see that this term increases from the central axis ($s=0$) to the light-beam periphery. To find its upper bound, we assume that $|\boldsymbol{p}_s+\boldsymbol{s}|=w$ and neglect the smaller variable $\boldsymbol{r}+\boldsymbol{\rho}$: $p_l w^2/l^2\ll\lambda$, which gives $p_l\ll l^2 l_0/(l^2+l_0^2)$. This shows that in the far- and near-field regions, but not very close to the emitting plane ($l\geqslant l_0$), the admissible value is $p_l\ll l_0$, while for $l\to 0$, it quadratically decreases to zero. In this case, the differences between l and l_1, l_2 in the expression for C can be neglected and the mutual intensity can be represented in the form

$$J(l,\Delta l,\boldsymbol{r},\boldsymbol{\rho})=V(l,\boldsymbol{r})V^*(l+\Delta l,\boldsymbol{r}+\boldsymbol{\rho})\mu(\boldsymbol{\rho}),$$

$$V(l,\boldsymbol{r})=\left(\frac{2P}{\pi w^2}\right)^{1/2}\exp\left\{-\left[\left(\frac{r}{w}\right)^2+\mathrm{i}k\left(l+\frac{r^2}{2R}\right)+\mathrm{i}f\left(\frac{l}{l_0}\right)\right]\right\}, \tag{5}$$

$$\mu(\boldsymbol{\rho})=\exp\left[-2\left(\frac{\rho}{\rho_c}\right)^2\right],$$

where $f(l/l_0)$ is the function varying slower than a linear function and equal to zero for $l = 0$. Here, the same variables $\boldsymbol{s} \to \boldsymbol{r}$ and $\boldsymbol{p}_s \to \boldsymbol{\rho}$ as for the emitting plane are introduced and the change of variable $p_l \to \Delta l$ was made. In the limiting case of specular reflection ($n = 0$), we have a 'pure' Gaussian beam with a waist on the mirror and the wave parameter $l_0 = \pi w_0^2/\lambda$. It is known that for this beam $f(l/l_0) = \arctan(l/l_0)$. Therefore, expressions (3) and (5) describe the characteristics of partially coherent Gaussian beams. One can see that $(w/\rho_c)^2 = (w_0/\rho_{c0})^2$, i.e., the number of coherence spots is the beam invariant. Together with the caustic waist radius w_0 and the wavelength λ, it determines the wave parameter l_0 and all the characteristics of the observed field: the beam radius w, the radius of curvature R of the regular part of the wave front and the coherence radius ρ_c.

However, the conditions of applicability of these expressions are determined not completely; in the absence of longitudinal movement, the third-order term in the expansion of $l_2 - l_1$ vanishes. The fourth-order term should be considered, which can be easily obtained from (1) assuming that $p_l = 0$. This term increases, as the third-order term, from the beam axis to its periphery and should be much smaller than λ. This leads to the conditions $\Delta l \ll l_0$ and $l^3 \gg w^3 \rho_c/\lambda$. The first condition means that the parameters ρ, w, R and the values of the function f are different in beam sections separated by large distances, and this should be taken into account. The second condition is stronger than the condition of small observation angles: $l \gg w$. It seems that these restrictions are caused by the deviations of the wave-front shape from parabolic and of the intensity distribution from a Gaussian at the beam periphery, which are especially noticeable near the waist. Because only a small part of the beam power is located at the beam periphery, these deviations can be neglected, in particular, in the estimate of the detected heterodyne signal.

When the received and heterodyne (reference) radiations are summed in a quadratic photodetector, each element of the photosensitive surface of the detector produces a current proportional to the square of the total field on this element. We define the heterodyne detection signal as a part of the total photocurrent expressed in the radiation power units, which is determined by a product of the fields being summed. If the frequencies of the fields are different, this part of the photocurrent proves to be alternate with the difference frequency and can be separated from the direct component and background illumination current. In the complex representation, the detected signal corresponds to the analytic signal

$$v = 2\int V V_g^* \mathrm{d}\boldsymbol{r},$$

where V and V_g are the analytic signals of the received and heterodyne radiations on the photodetector surface. For simplicity, we assume that the photodetector surface sensitivity is uniform and the surface is large compared to the cross sections of combined light beams. When random radiation is received, the power $p_{sig} = \langle v v^* \rangle$ of the detected signal averaged over the realisations of the received radiation should be considered. Then, we have

$$p_{sig} = 4\int\int J(l_{ph}, l_{ph}, \boldsymbol{r}, \boldsymbol{\rho}) V_g^*(l_{ph}, \boldsymbol{r}) V_g(l_{ph}, \boldsymbol{r} + \boldsymbol{\rho}) \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{\rho},$$

where l_{ph} is the longitudinal coordinate of the photodetector. For the laser heterodyne beam whose axis is coincident with the axis of the received beam, we obtain

$$V_g(\boldsymbol{r}) = \left(\frac{2P_g}{\pi w_g^2}\right)^{1/2} \exp\left\{-\left[\left(\frac{r}{w_g}\right)^2 + \frac{\mathrm{i}kr^2}{2R_g}\right]\right\},$$

where P_g is the heterodyne beam power; w_g is the radius of its cross section; and R_g is the radius of the wave-front curvature on the photodetector. Here, we omitted the phase dependence on the longitudinal coordinate of the photodetector because it disappears in the product of complex conjugate quantities. The dependence $p_{sig}(l)$ is manifested only through the radii of cross sections and wave fronts of two beams. Note also that the shape of a photosensitive surface does not play any role because its roughness Δl is usually much smaller than the wave parameter l_0 of both beams. By using (3) and (5), we can easily obtain the expression

$$p_{sig} = \frac{16PP_g}{\pi^2 w^2 w_g^2} \int\int \exp\{-[c\rho^2 + a[r^2 + (\boldsymbol{r} + \boldsymbol{\rho})^2]$$

$$- \mathrm{i}b[(\boldsymbol{r} + \boldsymbol{\rho})^2 - r^2]]\}\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{\rho},$$

$$a = \frac{1}{w^2} + \frac{1}{w_g^2}, \quad b = \frac{k}{2}\left(\frac{1}{R} - \frac{1}{R_g}\right), \quad c = \frac{2}{\rho_c^2}.$$

By making the change of variables $\boldsymbol{r} = x_1\boldsymbol{\xi} + x_2\boldsymbol{\eta}$, $\boldsymbol{\rho} = \boldsymbol{\xi} + \boldsymbol{\eta}$, we exclude the product $\boldsymbol{\xi\eta}$ in the exponential for $x_{1,2} = -[1 \pm (1 + 2c/a)^{1/2}]/2$. Therefore, the double integral is reduced to the product of two complex conjugate integrals of the type

$$\int \exp\left\{\left[-a - 2c + \mathrm{i}b\left(1 + \frac{2c}{a}\right)^{1/2}\right]\xi^2\right\}\mathrm{d}\xi$$

with the common factor $1 + 2c/a$. After integration, we obtain the expression for the detected signal power

$$p_{sig} = \frac{16PP_g}{w^2 w_g^2 (a^2 + 2ac + b^2)}.$$

The power is maximal over the parameter b when $b = 0$, i.e., when the wave fronts of the two beams are matched: $R = R_g$. Then, the power can be written in the form

$$p_{sig} = 4\varkappa PP_g, \quad \varkappa = \frac{4m}{(m+1)^2 + 4n(m+1)}, \tag{6}$$

where $\varkappa$ is the detection efficiency; $m = w^2/w_g^2$ is the ratio of the beam cross-section areas; and $n = w^2/\rho_c^2$ is the number of coherence spots in the received radiation. The maximum efficiency $\varkappa = 4/(m+1)^2$ is achieved for $m = (4n+1)^{1/2}$. One can easily see that these conditions are equivalent to the requirements of coincidence of the waists of the beams and the equality of their wave parameters, i.e., to the requirement $w_0 = w_{0g}(4n+1)^{1/4}$. If this relation is fulfilled for the waists, it is also fulfilled for any cross section of the beams. Therefore, the matching between a partially coherent Gaussian beam and a heterodyne beam is achieved simultaneously for the entire caustics of the heterodyne and, hence, for intracavity reception as well. In this case, the signal power is described by the expression

$$p_{\rm sig} = 4P_{\rm c}P_{\rm g}\frac{m-1}{m+1},$$

where $P_{\rm c} = P/n$ is the power in one coherence spot. One can see that for $n \to \infty$ the power $p_{\rm sig}$ monotonically increases, approaching asymptotically the value $4P_{\rm c}P_{\rm g}$, when the power of one coherence spot is involved in the signal formation. In this case, the detection efficiency decreases to zero. We assume here, that the optimal reception conditions correspond to $m = 3$, $n_{\rm a} = 2$, when the signal power is half the maximum power and $\varkappa = 1/4$. Therefore, we obtain three parameters related to the number of coherence spots upon heterodyne reception: the number of spots on the entrance pupil $n_{\rm a}$ of the projecting optical system, the number of spots in the heterodyne beam $n_{\rm a}/m = w_{\rm g}^2/\rho_{\rm c}^2$, and the fraction of radiation power of the coherence spot $(m-1)/(m+1)$ involved in the signal formation.

The number of coherence spots on the entrance pupil is determined by the aperture angle $\gamma_{\rm a}$ of a receiving telescope and the coherence angle $\theta_{\rm c} = 2\lambda/(\pi w_0)$: $n_{\rm a} = (\gamma_{\rm a}/\theta_{\rm c})^2$. The condition $n_{\rm a} = 2$ specifies the effective objective radius. The value of m is determined by the projection angle $u\gamma_{\rm a}$ and the divergence angle $\gamma_{\rm d}$ of the heterodyne beam: $m = (u\gamma_{\rm a}/\gamma_{\rm d})^2$. The condition $m = 3$ specifies the angular magnification u. It is easy to verify that the receiving telescope without the transmitting telescope has a low efficiency. If a laser source and heterodyne are located at the same distance from a reflector and have the same divergence of caustics, then $r_{\rm a} = 2w_{\rm 0g}$. However, even in the absence of the telescope, one fourth of the coherence spot is located within the heterodyne waist, although without matching the wave fronts of the input and heterodyne radiations. Nevertheless, the use of the transmitting telescope provides the amplification of the detected signal almost by a factor of four compared to the signal obtained in the absence of the telescope.

Consider the simplest case, when the laser source is used either as a receiver of reflected radiation or a source of the heterodyne beam in conventional heterodyning. In this case, one receiving–transmitting optical system can be used, usually a telescope. During signal transmission, the telescope transforms the laser caustics by reducing the divergence angle: $\gamma_{\rm d} \to \gamma_{\rm d}/u$. However, because of the aperture restriction, the divergence angle $\gamma_{\rm t}$ in the transformed caustics converging to a waist on a reflector proves to be smaller. In the soft aperture model, $1/\gamma_{\rm t}^2 = (u/\gamma_{\rm d})^2 + 1/\gamma_{\rm a}^2$. The radius of a light spot on the reflector is $w_{\rm s} = \lambda/(\pi\gamma_{\rm t})$, the coherence angle in the reflected radiation covering many coherence areas is $\theta_{\rm cs} = 2\lambda/(\pi w_{\rm s})$. The scattering angle $\gamma_{\rm rs} = 2\lambda/(\pi\rho_{\rm cs})$ and coherence radius $\rho_{\rm cs}$ are characteristics of the reflector surface. After the inverse transformation in the telescope, the coherence angle $\theta_{\rm ci}$ and scattering angle $\gamma_{\rm ri}$, taking the aperture restriction into account, are determined by the expressions $\theta_{\rm ci}^{-2} = (u\theta_{\rm cs})^{-2} + (u\gamma_{\rm a})^{-2}$ and $\gamma_{\rm ri}^{-2} = (u\gamma_{\rm rs})^{-2} + (u\gamma_{\rm a})^{-2}$. This gives $n_{\rm a} = (\gamma_{\rm ri}/\theta_{\rm ci})^2$ and $m = (\gamma_{\rm ri}/\gamma_{\rm d})^2$. Assuming that the scattering angle is considerably larger than the aperture angle ($\gamma_{\rm rs} \gg \gamma_{\rm a}$), we obtain from these relations

$$n_{\rm a} = 1 + \frac{m+1}{4}, \quad m = \left(\frac{u\gamma_{\rm a}}{\gamma_{\rm d}}\right)^2. \qquad (7)$$

This relation, specified in the transmission of radiation to the reflector, together with the matching condition for parameters of the received radiation and heterodyne $m = (4n_{\rm a} + 1)^{1/2}$, gives $n_{\rm a} = 2$ and $m = 3$. The detection efficiency should be estimated by taking into account the attenuation of the laser power $P_{\rm las}$ upon transmission of radiation to the reflector. The power on the reflector is $P_{\rm s} = P_{\rm las}/\{1 + [\gamma_{\rm d}/(u\gamma_{\rm a})]^2\} = P_{\rm las}m/(m+1)$. The power P on the entrance pupil depends, of course, on the radiation pattern for reflection from one or another surface. In the case under study, the power can be written in the form $P = P_0 m/(m+1)$, where $P_0 \propto m$ is the power that would be received by a telescope without aperture attenuation during the transmission of laser radiation to the reflector. By using (6) and (7), we obtain the expressions for the heterodyne signal and detection efficiency without matching conditions:

$$P_{\rm sig} = \frac{8P_0P_{\rm g}m^2}{(m+1)^2(m+3)} \propto \frac{m^3}{(m+1)^2(m+3)},$$

$$\varkappa = \frac{2m^2}{(m+1)^2(m+3)}.$$

These dependences are in fact the dependences on the area of the entrance pupil of the telescope and can be used to compare the results of calculations with experimental data. Of course, the maximum of $\varkappa$ is achieved for $m = 3$, corresponding to $n_{\rm a} = 2$.

The outlook for telescope application can be estimated from the effective radius of its objective. If the telescope ocular with the focal distance f_1 is located in the far-field zone of the laser caustics at the distance $z \gg f_1$ from the waist and the reflector is located at the distance f_2 from the objective with the focal distance $l \gg f_2$, then $u \simeq lf_1/(zf_2)$. Taking into account the condition $m = (u\gamma_{\rm a}/\gamma_{\rm d})^2 = 3$ and relations $\gamma_{\rm a} = \sqrt{2}r_{\rm a}/l$, $\gamma_{\rm d} = \lambda/(\pi w_0)$, $l_0 = \pi w_0^2/\lambda$, we obtain

$$r_{\rm a} = \left(\frac{3}{2}\right)^{1/2} w_0 \frac{f_2}{f_1}\frac{z}{l_0}, \qquad (8)$$

where w_0 is the waist radius of the laser caustics and l_0 is the boundary of its near-field zone. Usually, the telescope is located near the laser, and we can assume that $z \sim l_0$. Therefore, the receiving–transmitting telescope, for which $f_2/f_1 \gg 1$, provides the amplification of detected signals, which are proportional to $w_0^2(f_2/f_1)^2$, by several orders of magnitude compared to signals detected without the telescope, which are proportional to w_0^2.

In the case under study, a small number of coherence spots in the received radiation are determined by the angular aperture of the telescope. In some cases, for example, in the remote gas analysis of the atmosphere, artificial reflectors with a narrow radiation pattern oriented normally to the radiation direction are used. In this connection we also performed calculations for nearly mirror reflectors for which the coherence radius on the reflecting surface is of the same order of magnitude with the light spot radius. No considerable difficulties appear in this case. It should be taken into account that scattering and coherence angles (4) are determined, generally speaking, not only by the coherence radius and light-beam radius on the surface but also by the number of coherence spots. Because the relevant expression is cumbersome, we present the results by the curve (see the solid curve in Fig. 4). It is clear qualitatively that for a narrow radiation pattern of reflected radiation, the efficiency maximum is achieved for $m < 3$ because the optimal value of m is related to n. Therefore, for $m = 3$, reflection from surfaces of any type can be detected.

We estimated experimentally the possible efficiency of intracavity reception of reflected radiation by using a receiving – transmitting telescope. Radiation was received (Fig. 2) by 3.39-μm Zeeman He – Ne laser (*1*) with a gas-discharge tube of diameter 2.2 mm, which emitted two orthogonally polarised longitudinal TEM_{00} modes. The same laser was used as a radiation source. A quarter-wave plate, which mutually transformed circular polarisations of the emitted modes, was placed in the laser beam directed to the reflector. Due to the crossed action of reflected radiation on the laser modes, we observed the power modulation for each mode with the difference intermode frequency 5 MHz. For this purpose, a quarter-wave plate was placed in front of receiver (*5*), which transformed circular polarisations to linear, and polariser (*8*). The power modulation amplitude after narrowband amplifier (*6*) was measured with spectrum analyser (*7*). Reflector (*4*) made of a rolled duralumin was placed normally to the laser beam at a distance of 10 m from the laser. We used a receiving-transmitting telescope consisting of mirror (*2*) of diameter 20 cm with the focal distance $f_2 = 1.05$ m and mirror (*3*) of diameter 2 cm with $f_1 = 0.1$ m placed at a distance of 2 m from the output mirror of the laser. By varying the distance between the foci of telescope mirrors, probe radiation was focused on the reflector into a spot of diameter ~3 mm.

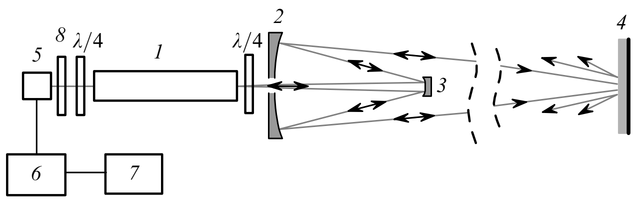

Figure 2. Scheme of the experimental setup: (*1*) He – Ne laser; (*2*, *3*) spherical telescope mirrors; (*4*) scattering reflector; (*5*) photodetector; (*6*) amplifier; (*7*) spectrum analyser; (*8*) polariser.

We measured the dependence of the modulation amplitude q on the radius of the entrance pupil of the telescope, which was varied with the help of removable apertures. The results are presented in Fig. 3. Figure 4 shows the experimental dependence of the efficiency of intracavity reception p_{sig}/r_a^2 on the radius r_a of the entrance pupil of the receiving – transmitting telescope (points). One can see that the experimental data well agree with the theoretical curve. The reception efficiency for the laser and telescope used in experiments is maximal for the aperture radius of the principal mirror equal to 3 cm (8). In this case, the area of the telescope pupil is three orders of magnitude greater than the laser beam cross section area. It should be taken into account that the radius r_a can be restricted by the applicability conditions. For example, when detecting reflected radiation with a narrow radiation pattern, it is necessary

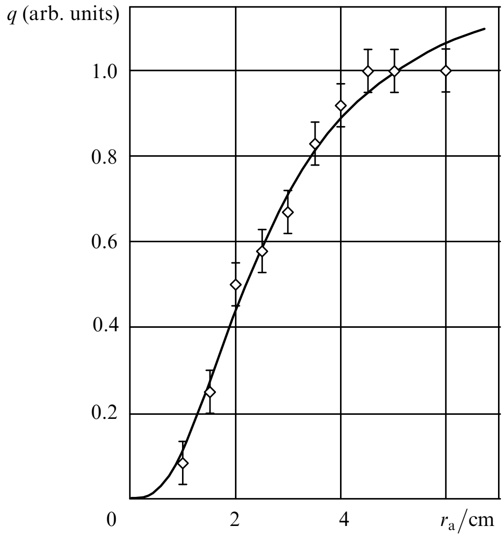

Figure 3. Dependence of the heterodyne signal q on the aperture radius r_a of a receiving – transmitting telescope upon intracavity reception.

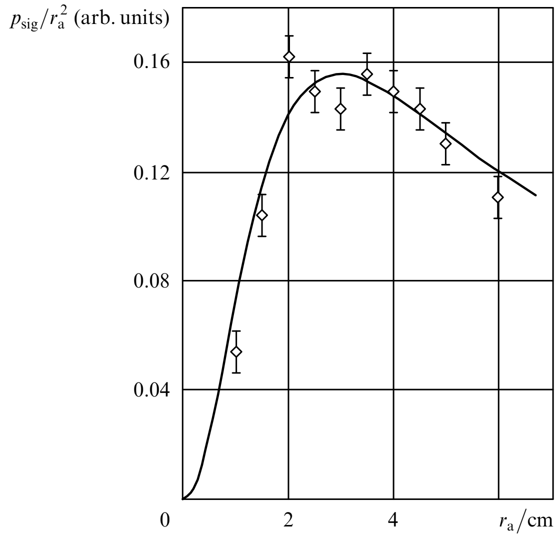

Figure 4. Dependence of the intracavity reception efficiency p_{sig}/r_a^2 on the aperture radius r_a of a receiving – transmitting telescope. Squares are the experiment, the solid curve is the calculation.

to take into account scattering from atmospheric inhomogeneities. The average inhomogeneity radius in a quiet atmosphere is ~10 cm [5], and the radius $r_a = 3$ cm can be considered rather large.

We verified the theoretical conclusion about the enhancement of the heterodyne signal with decreasing the spot size on the reflector. The spot size on the reflector was varied by changing the distance between the foci of telescope mirrors with the entrance pupil radius equal to 3 cm. Figure 5 shows the experimental dependence obtained. It confirms the assumption that the laser receives reflected radiation within the coherence angle. This angle increases inversely proportional to the decrease in the light-beam radius on the reflector. The effective radius of the telescope objective and the laser power modulation amplitude increase correspondingly.

Thus, we have extended the concept of Gaussian beams by taking the degree of their spatial coherence into account. The notion about the number of detected coherence spots

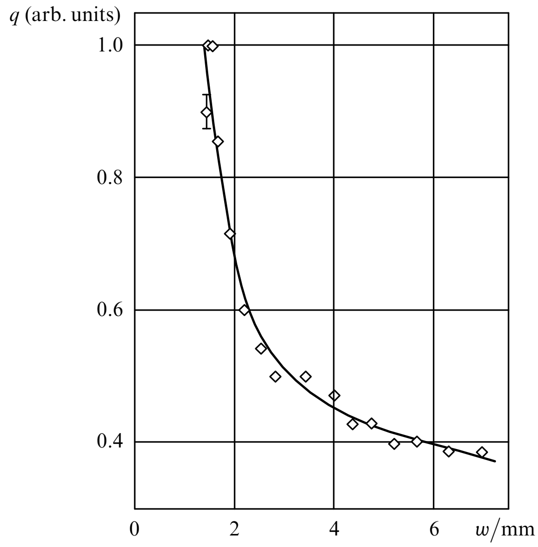

Figure 5. Dependence of the power modulation signal q for a laser receiver on the light-spot radius w on the reflector.

has been developed. We have shown that upon heterodyne detection, it is necessary to take into account the number of coherence spots on the entrance aperture of the telescope and in the heterodyne caustics, as well as a fraction of the coherence spot forming the detected signal. We have found out that the use of only a receiving telescope under calm atmospheric conditions provides the enhancement of the detected signal by a factor of four compared to a signal obtained without the telescope. The use of a receiving–transmitting telescope is much more efficient. The signal increases proportionally to the value of $(f_2/f_1)^2$, which can amount to $10^2 - 10^4$. In addition, by using the expression for the angular magnification u for $z \sim l_0$ presented before relation (8), we see that the spot radius on the reflector decreases proportionally to the ratio f_1/f_2. This allows one to reduce the aperture of reflectors by one–two orders of magnitude or to increase correspondingly the detection range. The results obtained in the paper can be used in the development of the methods and devices for remote diagnostics in the problems of gas analysis, aerosol control, telemetry, surface polarimetry, etc.

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