

NONLINEAR OPTICAL PHENOMENA

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Interaction of screening solitons in cubic optically active photorefractive crystals

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Abstract. **The coherent interaction of Gaussian light beams polarised parallel to the $[1\bar{1}0]$ direction in a cubic optically active photorefractive crystal with the $(\bar{1}\bar{1}0)$ cut is studied in an electric field applied to the crystal in the $[00\bar{1}]$ direction. The effect of optical activity on the interaction of the light beams with the phase difference $\Delta = 0$, $\pi/2$, and π is theoretically studied. It is shown that, while a change in the intensity of a combined light beam at $\Delta = 0$ caused by the influence of optical activity in a 9-mm thick $Bi_{12}TiO_{20}$ crystal is comparatively small (less than 8 %), this change achieves 36 % in the crystal of thickness 18 mm. The interaction of orthogonally polarised beams in this crystal is considered. It is found that, although the optical activity of the crystal results in the appearance of the 'breathing effect', a soliton-like nature of the combined beam is preserved. The results obtained can be used for the address positioning of soliton-like light beams.**

Keywords: *cubic photorefractive crystal, optical activity, Gaussian beam, interaction of light beams, screening soliton, $Bi_{12}TiO_{20}$ crystal.*

1. Introduction

Spatial photorefractive screening solitons differ from solitons of other types (for example, Kerr solitons) in that they can exist at extremely low light powers (of the order of microwatts) [1, 2]. Due to the dependence of the photorefractive properties of a crystal on the light frequency, a spatial soliton can form a waveguide inside the crystal, which can transmit a higher-power light beam whose frequency corresponds to the lower photorefractive sensitivity [1, 3]. In addition, spatial solitons provide the propagation of extremely narrow light beams (a few micromeres in diameter) without diffraction, which are used in modern optical technologies. Of special interest is the study of the interaction of spatial solitons in photorefractive crystals [1 – 6] related to the development of devices for optical switching of light beams [7, 8].

Despite the fact that the soliton regime is realised, generally speaking, for light beams of a special type, which differ from Gaussian beams, experimental studies are performed, as a rule, with laser beams close to Gaussian beams. The propagation regime of such beams is quasi-soliton and the light beams are called soliton-like. The first experimental data describing the interaction of two parallel coherent soliton-like beams with the same polarisation in a $Bi_{12}TiO_{20}$ (BTO) crystal in an external electric field were reported in [5]. The experimental data were interpreted by neglecting the optical activity of the crystal. The interaction between one-dimensional bright photorefractive screening solitons in barium-strontium niobate was experimentally studied in [6]. The physical explanation of the nature of coherent and incoherent soliton 'attraction' and 'repulsion' was proposed in [1, 6].

The influence of optical activity on the quasi-soliton propagation of Gaussian beams in sillenite crystals has been studied in many recent papers [9 – 14], but, as far as we know, the effect of optical activity on the interaction between soliton-like beams in sillenites has not been investigated so far.

In this paper, we present the theoretical study and computer simulation of coherent two-beam interaction in cubic photorefractive crystals in a constant electric field in the quasi-soliton regime taking optical activity into account. We investigated the influence of the optical activity of the crystal on the interaction of light beams with the same liner polarisation when their initial phase difference is zero, $\pi/2$, and π. In addition, we considered the case of light beams linearly polarised in orthogonal directions. In this case, the crystal anisotropy produces different conditions for their propagation, so that one of the beams can control the transverse displacement of the other beam propagating in the quasi-soliton regime.

We also attempt to elucidate the physical mechanism of the action of optical activity on the interaction of soliton-like light beams in cubic crystals.

2. Theory

We will use a one-dimensional model of a Gaussian light beam. We assume that the drift-nonlinearity regime is realised in a crystal [15] (p. 50), which favours the formation of screening solitons. Then, the propagation of a light beam can be described by the equation for the vector envelope of

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the complex amplitude $\boldsymbol{A}(x, z)$ of the laser field obtained in the paraxial approximation,

$$\mathrm{i}\,\frac{\partial \boldsymbol{A}}{\partial z}+\frac{1}{2k_0n_0}\frac{\partial^2 \boldsymbol{A}}{\partial x^2}-\frac{k_0n_0^3}{2}\frac{I_\infty+I_\mathrm{d}}{I_\mathrm{d}+I(x,z)}\times(\boldsymbol{A}\hat{r}\boldsymbol{E}_0)+\mathrm{i}\rho[\boldsymbol{e}_z,\boldsymbol{A}]=0, \quad (1)$$

where k_0 is the length of the wave vector of the monochromatic light beam; n_0 is the unperturbed refractive index of the crystal; $\hat{r}$ is the electro-optic tensor of the third rank for the class 23 crystal; $I(x, z)$ is the light-field intensity of the beam; I_d is the dark irradiance (including in the general case the intensity of background radiation)*; I_∞ is the light-beam intensity for $x \to \pm\infty$; ρ is the specific rotary power of the crystal; $\boldsymbol{e}_z$ is the unit vector along the z axis. The z axis indicates the direction of light beams, and the x axis coincides with the direction of the external electric field $\boldsymbol{E}_0$ applied to the crystal. In particular cases, Eqn (1) transforms to the corresponding equations in [9, 11, 14].

Let us represent the vector $\boldsymbol{A}$ in the form $\boldsymbol{A} = A_x\boldsymbol{e}_x + A_y\boldsymbol{e}_y = (2\eta_0 I_\mathrm{d}/n_0)^{1/2}(u\boldsymbol{e}_x + \upsilon\boldsymbol{e}_y)$ [16], where $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the unit vectors of the coordinate system; A_x and A_y are the projections of the vector $\boldsymbol{A}$ on the axes x and y; u and υ are the dimensionless polarisation components of the normalised vector of the light-beam envelope; and $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$. Let us pass now to the dimensionless variables $s = x/x_0$ and $\xi = z/(kx_0^2)$, where x_0 is an arbitrary spatial size and $k = n_0k_0$. After the introduction of the dimensionless parameters $\beta = (n_0^4k_0^2x_0^2r_{41}E_0)/2$ and $\delta = n_0k_0x_0^2\rho$, where r_{41} is the component of the electro-optic tensor, we can obtain the system of differential equations for the polarisation components u and υ:

$$\mathrm{i}\,\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial s^2}-\beta(1+\gamma)\frac{\mu_1 u+\mu_2\upsilon}{1+|u|^2+|\upsilon|^2}-\mathrm{i}\delta\upsilon=0,$$
$$\mathrm{i}\,\frac{\partial \upsilon}{\partial \xi}+\frac{1}{2}\frac{\partial^2 \upsilon}{\partial s^2}-\beta(1+\gamma)\frac{\mu_2 u+\mu_3\upsilon}{1+|u|^2+|\upsilon|^2}+\mathrm{i}\delta u=0, \quad (2)$$

where $\gamma = I_\infty/I_\mathrm{d}$;

$$\mu_1 = 3\sin\theta\cos^2\theta;$$
$$\mu_2 = \cos\theta\left(1-3\sin^2\theta\right); \quad (3)$$
$$\mu_3 = \sin\theta\left(1-3\cos^2\theta\right);$$

θ is the angle measured clockwise from the $[1\bar{1}0]$ direction to the external-electric field vector [14]. Here, the light-field intensity is described by the expression $I(x, z) = (n_0/2\eta_0)\times(|A_x|^2 + |A_y|^2) = I_\mathrm{d}(|u|^2 + |\upsilon|^2)$ taken from [9].

The system of normalised coupled equations (2) differs from the system of equations in [9, 10] by an arbitrary orientation of the external electric field in the $(\bar{1}\bar{1}0)$ plane with respect to the crystallographic coordinate system. Equations (2) also differ from the corresponding equations in [14] first of all in that they can be used to describe the propagation of not only bright solitons but also dark solitons in cubic optically active photorefractive crystals.

3. Numerical simulation of the interaction of Gaussian beams in a sillenite crystal

Let us simulate numerically by using Eqns (2) the interaction of one-dimensional Gaussian soliton-like beams in a cubic optically active BTO crystal of symmetry class 23 with the $(\bar{1}\bar{1}0)$ cut plane to which an external electric field is applied. We assume in (2) that $\gamma = 0$ because bright solitons will be considered below ($I_\infty = 0$).

We will use in calculations the parameters of the crystal close to those in the first experiment [5]: $n_0 = 2.25$, $r_{41} = 6.175 \times 10^{-12}$ m V^{-1}, and $\rho = 6.3$ deg mm^{-1} ($\lambda = 0.6328$ μm). The electric field strength E_0 is set to be 9 kV cm^{-1}. We assume that the input beams have a Gaussian profile over the x axis, linear polarisation along the y axis, and the half-width $x_0 = 19.5$ μm $[I(x) = I_0\exp(-x^2/x_0^2)]$. The beams are separated by a distance of 39 μm, $\theta = 90°$, and the crystal thickness is $d = 18$ mm.

Consider first the interaction of light beams with the same v-polarisation, when the phase difference Δ between them is zero (Fig. 1a). By comparing the normalised intensities of a combined beam at the middle of the crystal ($z = 9$ mm) without consideration of optical activity [curve (*1*)] and taking it into account [curve (*2*)], we see that the

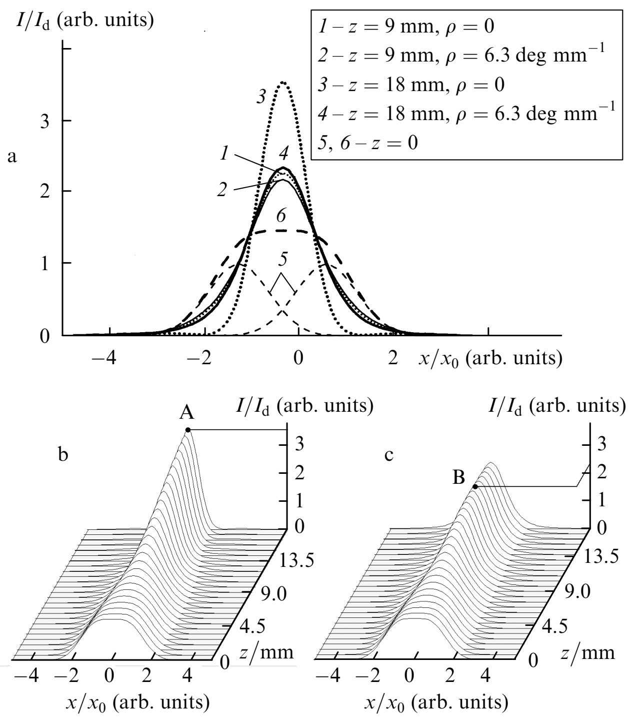

Figure 1. Interaction of Gaussian light beams in a BTO crystal under the experimental conditions [5] for the same initial phases of the beams ($\Delta = 0$): the intensity distributions of the beams in the middle of the crystal by neglecting optical activity (*1*) and taking it into account (*2*); on the output face of the crystal by neglecting optical activity (*3*) and taking it into account (*4*); on the input face of the crystal separately for each of the beams (*5*) and after the coherent summation of the beams (*6*) (a); as well as the intensity distributions of the beams over the coordinates x, z by neglecting optical activity (b, $\rho = 0$) and taking it into account (c, $\rho = 6.3$ deg mm^{-1}).

*The dark irradiance I_d [4, 9, 16] is defined as some conditional light intensity at which the generation of ionised donors would occur as in the case of thermal generation ($sI_\mathrm{d} = \beta$, where s is the photoionisation cross section and β is the degree of thermal generation of ionised donors [17]). The background radiation intensity is the intensity of the uniform background illumination [6].

influence of optical activity is weak (the intensity changes less than by 8 %) and the neglect of optical activity in the interpretation of experimental data in [5] was justified. At the same time, by comparing the results of numerical calculations of the light intensity at the output of the crystal without consideration of optical activity [curve (*3*)] and taking it into account [curve (*4*)], we see that optical activity strongly reduces the self-focusing of the combined beam during the 'attraction' of the beams, i.e., the influence of optical activity for a 15-mm thick crystal proves to be considerable (for $z = d = 18$ mm, the intensity changes by ~ 36 %)

Figures 1b and c demonstrate the combining process of coherent light beams of the same polarisation in a photorefractive crystal. It follows from the data presented in Fig. 1b that the maximum intensity and minimum halfwidth of the combined beam are not achieved in the range of crystal thicknesses considered (these extreme values are achieved for $z \approx 19.6$ mm). At the same time, optical activity causes the displacement of the intensity maximum of the combined beam to the point B with the coordinate $z = 13.6$ mm, the intensity of this maximum being considerably lower than that for $z = 13.6$ mm in Fig. 1b. Optical activity reduces self-focusing due to optical rotation of light beams because their initial polarisation was selected optimal in the absence of optical activity. For this reason, the maxima of curves (*1*) ($I_{max}/I_d \approx 2.3$) and (*3*) (~ 3.6) in Fig. 1a are higher than the maxima of curves (*2*) (~ 2.2) and (*4*) (~ 2.4), respectively.

If the phase difference of the interacting beams is $\Delta = \pi$, interference is destructive, and a region with almost zero intensity is formed by the overlapped beams (near the point $x = 0$ in Fig. 2a), which gradually increases due to the diffraction spreading of the beams. This leads to the deformation of the wave front of the beams and explains their mutual repulsion during propagation. Due to the nonlinear properties of the crystal, the refractive index increases in the external electric field in the regions of the maximum light-field intensity compared to other regions. In these regions, the self-focusing of the beams is observed, which prevents their spreading, so that the mutual 'repulsion' of the beams weakens. Because, as mentioned above, optical activity causes the deviation of the vector $\boldsymbol{A}$ from the direction in which nonlinear properties are manifested most strongly, the 'repulsing' beams in Fig. 2c (taking optical activity into account) are more diffuse than in Fig. 2b (neglecting optical activity). Note also that, when optical activity is taken into account, the mutual 'repulsion' of light beams slightly decreases: the intensity maxima are separated by a smaller distance than in the case of neglected optical activity (Fig. 2b).

Consider the case when the phase difference between the input light beams is $\Delta = \pi/2$. In this case, because of the interaction between soliton-like beams, the energy of one of the beams transfers to another (Fig. 3a). As a result, the first beam experiences self-focusing, while the second one spreads. The optical activity of the crystal leads to the spread of both light beams and the distance between their intensity maxima increases. The effect of optical activity on self-focusing is well demonstrated in Figs 3b and c.

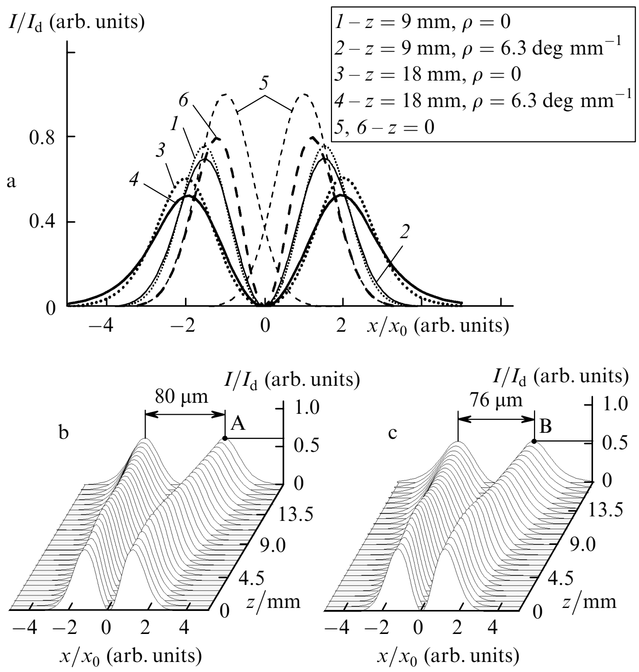

Figure 2. Same as in Fig. 1, for the initial phase difference $\Delta = \pi$.

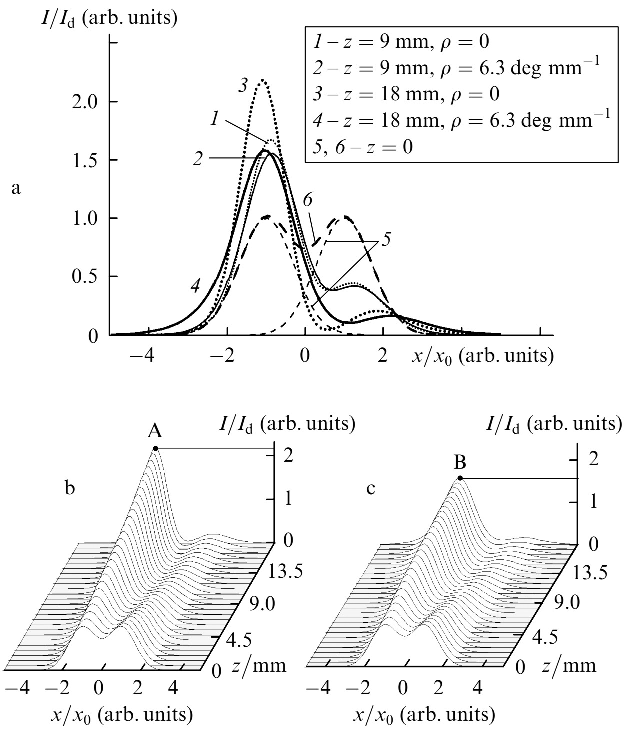

Figure 3. Same as in Fig. 1, for the initial phase difference $\Delta = \pi/2$.

The interaction geometry, parameters of light beams, and the distance between them described above and corresponding to experiment [5] are not optimal from the point of view of clearness of the physical interpretation of the interaction. Indeed, we cannot observe the combination of the interacting beams because they have been already overlapped at the input to the crystal (see Figs 1b, c) due to a small separation between them. In addition, the selected crystal length does not allow the study of the periodic spatial pulsations (focusing and defocusing) of the combined beam in the case of 'attraction' of the beams (Figs 1b and c) because either the intensity maximum of the combined beam has not been achieved yet in the considered regions of the crystal (Fig. 1b) or only one maximum is present (point B in Fig. 1c).

In this connection, to represent more clearly the picture of quasi-soliton interaction, we consider two parallel light beams of half-width $x_0 = 15$ μm and separated by a distance of 40 μm propagating in a BTO crystal. The rest of the beam and crystal parameters as well as the parameters of an external electric field, except its strength modulus, remain the same.

Because the shape of a Gaussian beam differs from that of a real soliton beam, we can speak only about the soliton-like propagation of the beam even when optical activity is neglected. From the existence curve for a soliton beam with the half-width $x_0 = 15$ μm [6], we find the external electric-field strength modulus $E_0 \simeq 15.2$ kV cm^{-1}, but for a Gaussian beam we assumed that $E_0 \simeq 14$ kV cm^{-1}. In this case, the intensity distribution for a Gaussian beam propagating along the z axis in the case $\rho = 0$ remains almost constant.

Let us present the physical interpretation of spatially periodic intensity pulsations of a combined light beam upon the coherent interaction of two Gaussian beams of the same intensity in a thick crystal (Fig. 4) when the phase difference Δ between the input beams is zero. In this case, the nonlinear interaction of the light beams results in the formation of one beam with an increased maximal intensity ($I_{\max}/I_{\rm d} = 2$). This leads to the displacement of the combined beam on the existence curve [6] and reduces the width of the soliton-like beam. The external electric-field strength proves to be insufficient to compensate for the diffraction divergence of this beam and the beam begins to spread. After the beam propagates over some distance, its width increases and the electric-field strength again becomes sufficient for producing beam self-focusing, and the described process repeats (Fig. 4a).

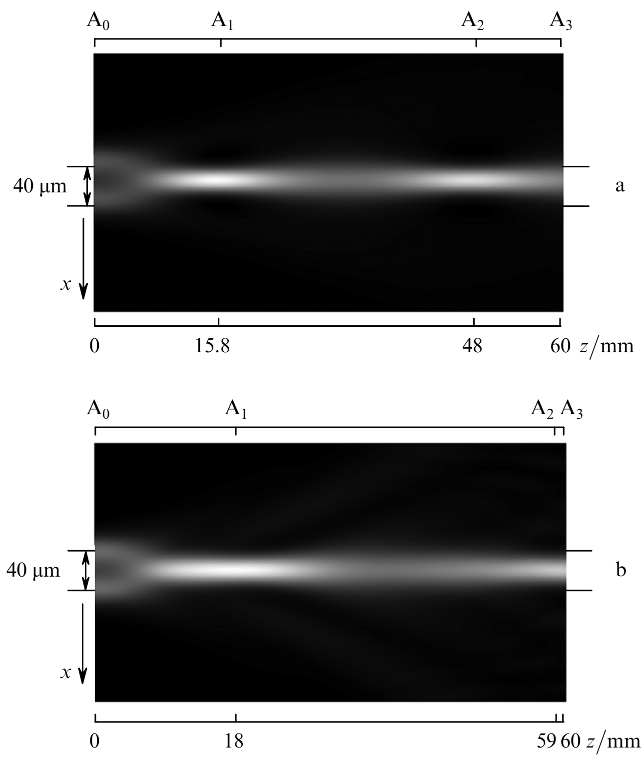

Figure 4. Distributions of the light intensity over the coordinates x and z during the interaction of Gaussian beams propagating in the quasi-soliton regime in a BTO crystal for the initial phase difference of the beams $\Delta = 0$. The light intensities at the characteristic points $I = 2.91I_{\rm d}$ (A_1), $2.6I_{\rm d}$ (A_2), and $1.7I_{\rm d}$ (A_3) are obtained by neglecting optical activity ($\rho = 0$) (a), while the intensities $I = 2.876I_{\rm d}$ (A_1), $1.9I_{\rm d}$ (A_2), and $1.8I_{\rm d}$ (A_3) are obtained taking optical activity into account (b).

Theoretical calculations show that, if the optical activity of a photorefractive crystal is taken into account ($\rho \neq 0$), the distance between the intensity maxima of the combined beam increases, while the intensities decrease (Fig. 4b). This is explained by the fact that due to optical rotation, the polarisation of the interacting beams deviates from the optimal polarisation corresponding to the appearance of the quasi-soliton regime. In the case of orthogonal polarisation of the beams, the self-focusing properties of the crystal weaken and, hence, the period of spatial oscillations increases. This explanation is only qualitative because it neglects the ellipticity of the natural waves in the optically active crystal produced due to the electro-optic effect.

Let us study the interaction of Gaussian beams in a BTO crystal ($\theta = 90°$) in the case of their orthogonal linear polarisation at the crystal input (parallel to the external electric-field-strength vector $\boldsymbol{E}_0$ and perpendicular to it) by using Eqns (2), the rest of the beam parameters, except the electric-field-strength modulus, remaining the same. Because the beam polarised orthogonally to $\boldsymbol{E}_0$ is in better conditions for the soliton-like propagation (the external electric field required for such propagation is lower), it passes to the soliton-like regime with increasing the electric-field strength earlier than another beam polarised parallel to $\boldsymbol{E}_0$. The latter beam plays the role of a guiding beam, determining the propagation direction of the first beam. In optics, this effect was called a 'logical gate of angular deviation' [18] because it allows one to perform logical operations by the spatial switching of a light beam in different directions. It is interesting to study this phenomenon in sillenite crystals because their have the fast response and high photosensitivity.

Figure 5a shows the results of the interaction of orthogonally polarised one-dimensional Gaussian beams of the same intensity (the upper beam is polarised perpendicular to the direction of the external electric field, while the lower one – parallel to it) propagating through a photorefractive BTO crystal in the case when the phase difference of the beams at the crystal input is zero and $E_0 = 18$ kV cm^{-1}. One can see that a one-dimensional beam is bent under the action of another beam to the side of the latter. The optical activity of the BTO crystal was neglected in this case.

The interaction of one-dimensional orthogonally polarised Gaussian light beams, taking optical activity into account, for $E_0 = 20$ kV cm^{-1} is shown in Figs 5b, c. One can see that the influence of optical activity on the propagation of a deflected soliton-like beam is manifested in the appearance of the breathing effect [11] – the periodic energy transfer resulting in a decrease in the light-beam intensity. However, the soliton-like propagation of the one-dimensional beam is preserved. To observe the breathing effect more distinctly, we used a crystal of doubled thickness (Fig. 5c), the rest of the crystal and beam parameters being as in Fig. 5c.

Note that the change in the electric-field direction to the opposite one has no effect on the result of interaction, while

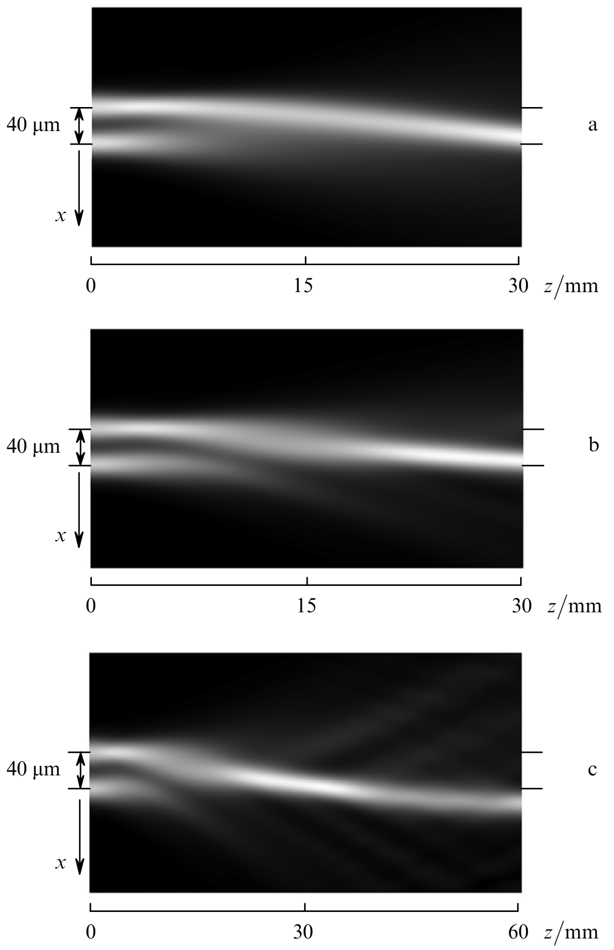

Figure 5. Distributions of the light intensity over the coordinates x and z during the interaction of orthogonally polarised light beams in a photorefractive BTO crystal obtained by neglecting the optical activity of the crystal ($\rho = 0$) for the crystal thickness $d = 30$ mm (a) and taking it into account ($\rho = 6.3$ deg mm^{-1}) for $d = 30$ (b) and 60 mm (c).

the change in the sequence of beam positions leads to the change in the direction of deflection of the soliton-like beam.

4. Conclusions

We have shown that even in a BTO crystal having a comparatively small specific rotary power (6.3 deg mm^{-1} at a wavelength of 0.6328 μm), the maximum intensity of a combined light beam formed due to the coherent interaction between two one-dimensional Gaussian beams of the same polarisation with the same initial phases can decrease by 36 % under the action of optical activity in a 18-mm thick crystal.

The presence of optical activity impairs the conditions for achieving the soliton-like regime, so that upon the mutual 'repulsion' of the light beams ($\Delta = 0$), they spread stronger, while for $\Delta = \pi/2$, the energy exchange between the interacting beams occurs weaker. Upon the interaction between orthogonally polarised one-dimensional Gaussian beams, one of them is deflected under the action of another. Despite the destructive influence of the optical activity of the crystal, the soliton nature of the deflected beam is preserved. Therefore, the interaction between two Gaussian beams in sillenite crystals is promising for the realisation of optical address positioning of light beams.

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References

1. Shih M., Segev M. *Opt. Lett.*, **21**, 1538 (1996).
2. Mamaev A.V., Saffman M., Zozulya A.A. *J. Opt. Soc. Am. B*, **15**, 2079 (1998).
3. Krolikowski W., Denz C., Stepken A., Saffman M., Luther-Davies B. *Quantum Semiclass. Opt.*, **10**, 823 (1998).
4. Singh S.R., Christodoulides D.N. *Opt. Commun.*, **118**, 569 (1995).
5. Garcia-Quirino G.S., Iturbe-Castillo M.D., Vysloukh V.A., Sanchez-Mondragon J.J., Stepanov S.I., Lugo-Martinez G., Torres-Cisneros G.E. *Opt. Lett.*, **22**, 154 (1997).
6. Meng H., Salamo G., Shih M., Segev M. *Opt. Lett.*, **22**, 448 (1997).
7. Swartzlander G.A. Jr. *Opt. Lett.*, **17**, 493 (1992).
8. Akhmediev N., Ankiewics A. *Opt. Commun.*, **100**, 183 (1993).
9. Singh S.R., Christodoulides D.N. *J. Opt. Soc. Am. B*, **13**, 719 (1996).
10. Krolikowski W., Akhmediev N., Andersen R.D., Luther-Davies B. *Opt. Commun.*, **132**, 179 (1996).
11. Fazio E., Babin V., Bertolotti M., Vlad V. *Phys. Rev. E*, **66**, 016605-1 (2002).
12. Fazio E., Ramadan W., Belardini A., Bosco A., Bertolotti M., Petris A., Vlad V.I. *Phys. Rev. E*, **67**, 026611-1 (2003).
13. Fazio E., Ramadan W., Bertolotti M., Petris A., Vlad V.I. *J. Opt. A: Pure Appl. Opt.*, **5**, S119 (2003).
14. Shepelevich V.V., Kowarschik R., Kiesling A., Matusevich V., Golub A.A. *Kvantovaya Elektron.*, **33**, 446 (2003) [*Quantum. Electron.*, **33**, 446 (2003)].
15. Petrov M.P., Stepanov S.I., Khomenko A.V. *Fotorefraktivnye kristally v kogerentnoi optike* (Photorefractive Crystals in Coherent Optics) (St. Petersburg: Nauka, 1992).
16. Christodoulides D.N., Carvalho M.I. *J. Opt. Soc. Am. B*, **12**, 1628 (1995).
17. Kukhtarev N., Markov V., Odoulov S., Soskin M., Vinetskii V. *Ferroelectrics*, **22**, 949 (1979).
18. Blair S., Wagner K. *Appl. Opt.*, **38**, 6749 (1999).