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# Effect of a permanent magnetic field on quasi-periodic and chaotic oscillation regimes in solid-state ring lasers

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Abstract. Features of nonlinear dynamics of solid-state ring lasers in quasi-periodic and chaotic oscillation regimes in a magnetic field are studied experimentally. Windows of periodic and quasi-periodic regimes within the region of dynamic chaos in a monolithic ring laser with periodically modulated pumping are observed experimentally for the first time. It is shown that an applied magnetic field changes the boundaries of the existence of various nonstationary regimes. It is established that, under certain conditions, lasing regimes can be switched in magnetic fields with strengths of  $\sim 1$  Oe.

**Keywords**: solid-state ring laser, nonlinear dynamics, dynamic chaos, amplitude and frequency nonreciprocity.

#### 1. Introduction

An interest in detailed studies of the nonlinear dynamics of emission from solid-state ring lasers is associated with the fact that monolithic ring lasers (ring chip lasers) pumped by laser diodes are widely used in modern fundamental laser physics and laser engineering. In the recent years, much attention has been given to studies of nonstationary oscillation regimes in autonomous and nonautonomous ring lasers. Studying such regimes and their evolution by changing the control parameters allows one to understand more comprehensively the physics of interaction of light waves in ring lasers.

One of the most important achievements in the field of nonlinear laser dynamics is the discovery and investigation of such an interesting phenomenon as the dynamic (deterministic) chaos [1-3]. The paradoxical character of regimes of dynamic chaos consists in the fact that chaotic oscillations in lasers appear in the absence of any external random factors, and their characteristics are fully determined by the initial conditions.

Despite numerous theoretical and experimental studies in this field, the nonlinear dynamics of ring lasers in regimes of dynamic chaos and quasi-periodic regimes has not been studied comprehensively by now (see, e.g., reviews [4, 5] and references therein). This is associated with the more complex

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Received 13 September 2004 *Kvantovaya Elektronika* **35** (1) 7–12 (2005) Translated by A.S. Seferov dynamics and a larger number of control parameters of ring lasers in comparison to linear lasers. Note that studies of dynamic chaos using analytic methods are actually impossible, while numerical methods allow one to analyse only particular individual cases, thus restricting the possibilities of generalising and predicting the features of the nonlinear dynamics at other values of the laser parameters. Another important factor is that conducting detailed studies is a complex procedure and requires the development of highly stable experimental setups free of the effect of external perturbations.

Detailed theoretical studies of the nonlinear dynamics and their comparison to experimental data form a basis for the development and improvement of the mathematical model of solid-state lasers. It is precisely a thoroughly arranged physical experiment that allows one to solve the problem of an adequate mathematical simulation for an actual nonlinear system and to determine experimentally the limits of its application.

One of pressing problems is a study of various mechanisms and conditions for the appearance of dynamic chaos in solid-state ring lasers. Note that deterministic chaotic lasing regimes in autonomous single-mode ring lasers could not be revealed for a long time. A dynamic chaos in a solidstate unidirectional ring laser was observed experimentally for the first time in [6], where it was shown that, in the presence of the resonator frequency nonreciprocity, there exists a region of laser parameters within which a resonance appears between two branches of relaxation oscillations. This resonance leads to their instability and the appearance of deterministic chaos. One more mechanism of the appearance of dynamic chaos in a bidirectional solid-state ring laser was found in [7, 8]. This mechanism is related to a parametric resonance between self-modulation and relaxation oscillations.

Studies have shown (see e.g., review [5]) that the regions of the laser parameters, for which chaotic lasing regimes are observed in solid-state ring lasers, are significantly wider when laser parameters are modulated: in this case, it becomes easier to attain parametric resonances and conditions ensuring the excitation of relaxation oscillations.

The simplest method for achieving nonstationary and chaotic regimes in solid-state ring lasers is to modulate the pump power. In this case, the output radiation characteristics depend on a number of control parameters (the frequencies of self-modulation and relaxation oscillations, the excess of the pump level over the threshold, the resonator amplitude and frequency nonreciprocities, and the pump-modulation depth and frequency).

The aim of this work was to study the features of the nonlinear dynamics of ring lasers appearing in chaotic and quasi-periodic oscillation regimes in solid-state ring lasers upon changes in the control parameters. Much attention was given to studies of the effect of an external magnetic field on the lasing dynamics.

## 2. Experimental setup

We studied a diode-pumped solid-state monolithic ring Nd: YAG laser. The monolithic block had a spherical input face and three total-internal-reflection faces. The geometrical perimeter of the resonator was 2.6 cm, and the resonator nonplanarity angle was 80°. The laser was pumped by a laser diode with a power of up to 500 mW. The temperature of the chip-laser active element was maintained to within an accuracy of 0.1 °C. Its design is described in more detail in [9, 10].

The pump power was modulated using a transformer, whose secondary winding was included in the power-supply circuit of the laser diode and the primary winding was connected to an ac voltage generator. The voltage across the primary winding determined the value of a control parameter (the pump-power modulation depth h). During the experiments, the pump power was modulated at frequencies  $\omega_{\rm p}/2\pi=17-50$  kHz, and the modulation depth could be varied from 0 to 50 %.

An external magnetic field was produced by a microelectromagnet located near the chip laser. The strength of the magnetic field H could be as high as 500 Oe. The radiation of counter-propagating waves from the ring laser was incident on LFD-2 photodetectors; their signals were fed to an ASK-3151 digital oscilloscope and were then processed in a computer. The intensities and spectra of the counter-propagating waves  $I_1$ ,  $I_2$  and  $J_1$ ,  $J_2$ , respectively, were simultaneously recorded in these experiments.

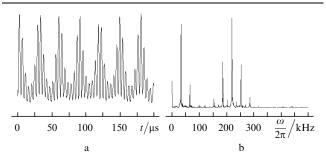
## 3. Experimental results

In studies of the nonlinear oscillation dynamics in quasiperiodic and chaotic regimes in ring chip lasers pumped by periodically modulated diodes, it was established that a number of windows of periodic and quasi-periodic regimes exist within the region of dynamic chaos. Such windows may exist both in the presence and absence of an external magnetic field. When a magnetic field is applied, the structure of windows changes and additional windows can also appear.

We studied the temporal and spectral characteristics of the output radiation of the bidirectional ring chip laser as functions of the pump-modulation frequency  $\omega_{\rm p}/2\pi$  and depth h, the relative excess of the pump level over the lasing threshold  $\eta$ , and the magnetic-field strength H. For a zero magnetic field (H=0) and nonmodulated pump power (h=0), a self-modulation regime of the first kind was observed in a ring chip laser [11]. Its radiation intensity spectrum consisted of a single spectral component corresponding to beats between two optical components at a frequency  $\omega_{\rm m}/2\pi=210$  kHz. At a relative excess of the pump level over the threshold pump power (excess factor)  $\eta=0.07$ , the fundamental relaxation frequency was  $\omega_{\rm r}/2\pi=65$  kHz.

It was found experimentally that the character of lasing at H=0 substantially depends on the frequency  $\omega_p/2\pi$ , modulation depth h, and excess factor  $\eta$ . In the region of

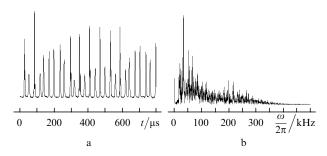
pump-modulation frequencies under study, a quasi-sinusoidal lasing regime QS-1 is observed in the chip laser at very small modulation depths ( $h \approx 0.01$  %). Hereafter, we use the classification of nonstationary lasing regimes in ring lasers proposed in [5]. Figure 1 shows the temporal and spectral characteristics in the QS-1 regime, which is characterised by self-modulation oscillations with a periodic envelope at a frequency  $\omega_p/2\pi$ . The most intense components in the lasing spectrum are those with the self-modulation ( $\omega_m/2\pi = 210$ kHz) and pump-modulation frequencies.



**Figure 1.** (a) Temporal and (b) spectral characteristics of radiation from a chip laser operating in the QS-1 regime at  $\omega_p/2\pi = 33$  kHz and h = 0.01 %.

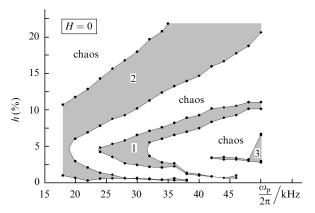
When the modulation depth increases to a certain critical value  $h_{cr}^{(1)}$ , a regime of dynamic chaos arises in the laser. The characteristic features that allow a lasing regime to be identified as a dynamic chaos are the appearance of irregular radiation-intensity pulsations and the presence of an almost continuous oscillation spectrum. Irregular radiation-intensity pulsations also manifest themselves in the structure of the Poincare cross sections [3]. In some cases, chaos was identified using Lyapunov exponents, which were determined by a numerical simulation of the laser emission dynamics. Typical characteristics of the output radiation in a chaotic regime are shown in Fig. 2. The temporal structure of the lasing intensity in this regime is a sequence of pulses with chaotically changing amplitudes and a repetition period close to the pump-modulation period. An intense component at a frequency  $\omega_p/2\pi$  can be distinguished in the spectrum that is virtually continuous. The  $h_{\rm cr}^{(1)}$  value strongly depends on  $\omega_{\rm p}/2\pi$  and  $\eta$ ; in particular,  $h_{\rm cr}^{(1)}=0.5\,\%$  at a modulation frequency  $\omega_{\rm p}/2\pi = 33$  kHz for  $\eta \leqslant 0.1$ .

Figure 3 shows the regions of existence of various lasing regimes at  $\eta=0.07$  in the  $(\omega_{\rm p}/2\pi,h)$  parameter plane. The region where the QS-1 quasi-sinusoidal regime exists is not



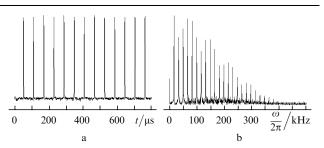
**Figure 2.** (a) Temporal and (b) spectral characteristics of radiation from a chip laser operating in the regime of dynamic chaos at  $\omega_{\rm p}/2\pi=33\,$  kHz and  $h=0.05\,\%$ .

shown in Fig. 3, since it almost coincides with the frequency axis. Figure 3 shows that, as h increases, a number of windows appear in the region of existence of dynamic chaos. Within these windows, no chaos is present and various nonstationary periodic and quasi-periodic lasing regimes are observed. Note that the mutual arrangement of windows and their widths depend significantly on the excess factor  $\eta$ . For  $\eta > 0.25$ , the periodicity windows in the region of chaotic oscillations disappear.

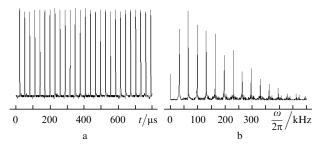


**Figure 3.** Regions of existence of different lasing regimes at  $\eta = 0.07$  in the  $(\omega_p/2\pi, h)$  parameter plane in the absence of a magnetic field. The windows of periodicity (regions 1, 2, 3) are shown in grey.

Figure 4 shows the temporal structure and spectrum for lasing in the windows of existence of the QPI-2T periodic pulse regime (region 1 in Fig. 3), for which the laser-pulse repetition period is equal to the double pump-modulation period. This regime differs from the regime of generation of periodic pulses observed earlier in [12] in the presence of a magnetic field, in which one period contained two pulses with different peak intensities. The difference in these intensities monotonically increases with the magnetic field. In this work, we observed a regime of repetitive pulses with a single pulse in a period equal to the double pump-modulation period. Apart from this window, there is also another window (region 2), in which a regime of repetitive pulses with a period equal to the pump-modulation period is observed (the QPI-1T regime [5]). Typical temporal and spectral characteristics of the output radiation in this regime are shown in Fig. 5. In addition to the windows mentioned above, there exists the third window (region 3) in a quite narrow frequency range near  $\omega_{\rm p}/2\pi \approx 45$  kHz, where the pulse repetition period is equal to the threefold modulation period.

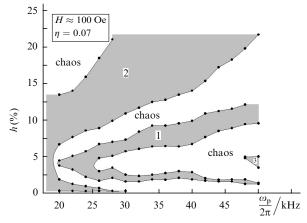


**Figure 4.** (a) Temporal structure and (b) spectrum of chip-laser radiation in the window of periodicity 1 (the QPI-2T regime) at  $\omega_{\rm p}/2\pi=33$  kHz, h=6%, and  $\eta=0.07$ .



**Figure 5.** (a) Temporal structure and (b) spectrum of chip-laser radiation in the window of periodicity 2 (the QPI-1T regime) at  $\omega_p/2\pi = 33$  kHz and h = 13 %.

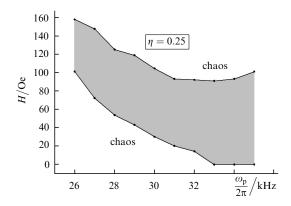
When magnetic fields of up to 500 Oe are applied to the active element, the regions of existence of different lasing regimes change significantly their boundaries. This is clearly seen from the comparison of Figs 3 and 6. The latter shows the regions of existence of different lasing regimes for  $H\approx 100$  Oe. In particular, the windows of existence of the QPI-2T and 1T regimes shift to lower pump-modulation frequencies and become somewhat wider. At H=0, the windows with the QPI-2T regime appear at  $\omega_p/2\pi \leqslant 25$  kHz, while at H=100 Oe, they appear already at  $\omega_p/2\pi \leqslant 20$  kHz. As to the window with the periodic regime with the threefold period, it shifts towards higher modulation frequencies.



**Figure 6.** Regions of existence of different lasing regimes in the  $(\omega_p/2\pi, h)$  parameter plane at  $H \approx 100$  Oe and  $\eta = 0.07$ .

It has been found that the structure and position of windows in the domain of dynamic chaos substantially depend on the excess factor. At  $\eta \geqslant 0.25$  and H=0, no periodicity windows are present, while the application of a magnetic field leads to their appearance. The appearance of such a window for the QPI-2T periodic regime upon application of a magnetic field H is shown in Fig. 7 in the  $(\omega_p/2\pi, H)$  parameter plane at  $\eta = 0.25$ . As the magnetic-field strength increases, the dynamic-chaos regime transforms into the QPI-2T periodic pulse regime, which, upon a subsequent increase in H, is again replaced by a dynamic chaotic regime.

Hence, the above data allow us to conclude that the mutual position and dimensions of the regions of existence of different lasing regimes substantially depend on both the magnetic-field strength and the excess factor. It follows from our results that certain values of the laser parameters exist at



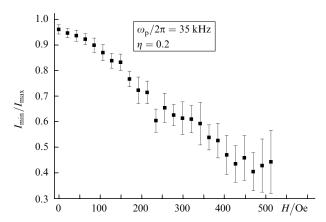
**Figure 7.** Regions of existence of different lasing regimes in the presence of a stationary magnetic field H in the  $(\omega_p/2\pi, H)$  parameter plane at  $\eta = 0.25$ .

which the radiation characteristics are rather sensitive to the magnetic-field strength. A transition from one lasing regime to another occurs at certain critical H values. If the initial point is taken close to the boundary of a transition, then even a very small change in the magnetic field (<1 Oe) may considerably modify the character of lasing.

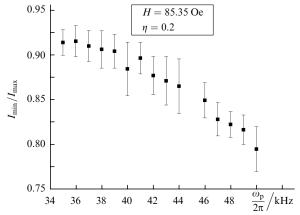
We have studied the modulation depth of radiation pulsations as a function of the magnetic-field strength in the QPI-2T quasi-periodic lasing regime. In the absence of a magnetic field, light is emitted in each direction in the form of a sequence of pulses (with almost equal amplitudes) with a period  $T_p = (\omega_p/2\pi)^{-1}$ . Applying a magnetic field to the active element leads to a double pulse repetition period. In this case, two laser pulses with amplitudes  $I_{\text{max}}$  and  $I_{\text{min}}$  are present in each pump-modulation period. The ratio  $I_{\min}/I_{\max}$  as a function of magnetic-field strength H at  $\omega_{\rm p}/2\pi=35$  kHz is plotted in Fig. 8. As H rises, this ratio falls monotonically from 1 to 0. The ratio  $I_{\min}/I_{\max}$  also depends on the pump-modulation frequency (Fig. 9). Applying a magnetic field to the active element of the ring chip laser also leads to the appearance of unequal mean intensities of counter-propagating waves.

## 4. Discussion

As is known, the application of a stationary magnetic field to the active element of a monolithic ring chip laser is



**Figure 8.** Ratio  $I_{\rm min}/I_{\rm max}$  in the QPI-2T regime as a function of the stationary magnetic-field strength at  $\omega_{\rm p}/2\pi=35$  kHz and  $\eta=0.2$ .



**Figure 9.** Ratio  $I_{\min}/I_{\max}$  in the QPI-2T regime as a function of the pump-modulation frequency  $\omega_{\rm p}/2\pi$  at H=85.35 Oe and  $\eta=0.2$ .

accompanied by the appearance of frequency and amplitude nonreciprocities of the ring resonator (see, e.g., [13]).

The effect of the frequency nonreciprocity on the lasing regimes in ring lasers is determined by two physical mechanisms. One of them is a change in the frequency of self-modulation oscillations at  $H \neq 0$ , which may lead to the fulfilment or violation of the conditions for parametric resonances in both autonomous and nonautonomous ring lasers. The second mechanism is related to the removal of the frequency degeneracy of counter-propagating waves in the laser and the appearance of an additional degree of freedom (or a control parameter). In particular, this additional parameter can be modulated in order to control lasing regimes.

The use of the amplitude nonreciprocity for controlling the characteristics of a ring laser is based primarily on the creation of asymmetric conditions for generating counterpropagating waves in the ring laser (such an asymmetry is, in particular, a necessary condition for the onset of a dynamic chaos at parametric resonances in an autonomous laser). The amplitude nonreciprocity of the resonator also results in the appearance of unequal fields of the counterpropagating waves and, consequently, in a change in the resonator dynamics. Changes in the resonator amplitude nonreciprocity can be used, for example, to control the frequency of laser self-modulation oscillations for attaining the conditions for a parametric resonance.

The following effects appearing upon application of a magnetic field can also be mentioned: the appearance of a difference between the phases of self-modulation oscillations of counter-propagating waves; and changes in the linear and nonlinear coupling (due to backscattering and on inverse-population gratings induced in the active medium, respectively) of counter-propagating waves resulting from changes in their polarisation.

The frequency  $\Omega$  and amplitude  $\Delta$  nonreciprocities of the resonator appearing upon application of a magnetic field to the chip laser under study can be estimated using the expressions

$$\Omega = \omega_1 - \omega_2 = k_1 H,\tag{1}$$

$$\Delta = \frac{1}{2} \left( \frac{\omega_1}{Q_1} - \frac{\omega_2}{Q_2} \right) = k_2 H + \Delta_0, \tag{2}$$

where  $k_1$  and  $k_2$  are the coefficients depending on the orientation of the magnetic field relative to the resonator loop, its nonplanarity, and other parameters;  $\Delta_0$  is the amplitude nonreciprocity at H=0; and  $Q_1$  and  $Q_2$  are the resonator quality factors at frequencies  $\omega_1$  and  $\omega_2$ . Coefficients  $k_1$ ,  $k_2$ , and  $k_3$  can be calculated for a particular resonator using the Jones matrix formalism.

When a magnetic field is applied, the frequency and amplitude nonreciprocities appear simultaneously; it is therefore difficult to determine their individual effects on the nonlinear laser dynamics. For this purpose, it is desirable to use both experimental and numerical simulation data.

The numerical simulation of nonlinear dynamics of oscillation in solid-state ring lasers was based on a standard model described by the following system of equations for complex amplitudes of counter-propagating waves  $\tilde{E}_{1,2}$  and spatial harmonics of inverse population  $N_0$  and  $N_{\pm}$  [5]:

$$\begin{split} \frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} &= -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1} \\ &+ \frac{\sigma l_{\mathrm{a}}}{2T}(1 - \mathrm{i}\delta)\left[N_{0}\tilde{E}_{1,2} + N_{\pm}\tilde{E}_{2,1}\right], \\ T_{1}\frac{\mathrm{d}N_{0}}{\mathrm{d}t} &= N_{\mathrm{th}}\left[1 + \eta + h\cos(\omega_{\mathrm{p}}t)\right] \end{split} \tag{3}$$

$$T_1 \frac{dN_+}{dt} = -N_+ [1 + a(|E_1|^2 + |E_2|^2)] - N_0 a E_2 E_1^*.$$

 $-N_0[1+a(|E_1|^2+|E_2|^2)]-N_+aE_1E_2^*+N_-aE_2E_1^*,$ 

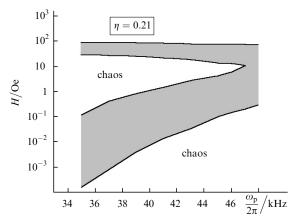
Here,  $\omega/Q_{1,2}$  are the resonator bandwidths for the counterpropagating waves; L is the length of the ring-resonator perimeter; T=L/c is the round trip transit time of light in the resonator;  $T_1$  is the longitudinal relaxation time;  $l_a$  the length of the active element;  $a=T_1c\sigma/(8\hbar\omega\pi)$  is the saturation parameter;  $\sigma=\sigma_0/(1+\delta^2)$  is the laser-transition cross section;  $\delta=(\omega-\omega_0)\Delta\omega_{\rm g}$  is the relative shift of the lasing frequency from the centre of the gain line;  $\Delta\omega_{\rm g}$  is the width of the gain line;  $\tilde{m}_{1,2}=m_{1,2}\exp{(\pm i\theta_{1,2})}$  are the complex coefficients of coupling of counter-propagating waves through backscattering  $(m_{1,2}$  and  $\theta_{1,2}$  are the magnitudes and phases of the coupling coefficients);  $N_{\rm th}(1+\eta)/T_1$  is the pump rate;  $N_{\rm th}$  is the threshold inverse population; and

$$N_0 = \frac{1}{I_a} \int_0^{I_a} N dz, \quad N_{\pm} = \frac{1}{I_a} \int_0^{I_a} N \exp(\pm i2kz) dz$$

are the complex amplitudes of the spatial harmonics of the inverse population N. It was assumed in the calculations that the shift of the lasing frequency from the centre of the gain line is small compared to the linewidth.

The results of the numerical simulation have shown that the modulation depth of radiation pulsations as a function of the magnetic-field strength in the QPI-2T quasi-periodic lasing regime is in qualitative agreement with the experimental results shown in Figs 8 and 9. Calculations have shown that a difference in the mean values of the intensities of counter-propagating waves experimentally observed in the QPI-2T regime in magnetic fields of up to 500 Oe can be explained by the appearance of an amplitude nonreciprocity of the ring resonator. The effect of the frequency nonreciprocity on the mean intensities is negligibly small in such magnetic fields.

The possibility of the formation of windows of periodicity in the region of dynamic chaos upon application of a magnetic field to the active element was studied by numerical simulations. As an example, Figure 10 shows the regions of dynamic chaos and windows of periodicity in the  $(\omega_p/2\pi, H)$  parameter plane at  $\eta = 0.21$  and a pumpmodulation depth h = 0.27 %. The values of the frequency of self-modulation oscillations  $\omega_{\rm m}/2\pi$  in the absence of a magnetic field and the frequency of relaxation oscillations  $\omega_{\rm r}/2\pi$  used in the calculations were 170 and 65 kHz, respectively. The structure of regions of dynamic regimes found in the numerical calculation qualitatively agrees with that observed experimentally (Fig. 7). At low magnetic fields  $(H \leq 1 \text{ Oe})$ , the calculation data, however, differ from the experimental results, since, in this case, the numerical calculation yields an additional window of periodicity at modulation frequencies  $\omega_{\rm p}/2\pi \leqslant 48$  kHz. This difference from the experiment can be determined by insufficiently homogeneous magnetic fields used in the experiment and by the presence of the geomagnetic field and parasitic magnetic fields in the laboratory.



**Figure 10.** Regions of dynamic chaos and windows of periodicity in the  $(\omega_p/2\pi, H)$  parameter plane at  $\eta = 0.21$ .

The results of the numerical simulation show that the standard model of a solid-state ring laser cannot adequately describe the chaotic dynamics at low modulation frequencies  $(\omega_p/2\pi \le 30 \text{ kHz})$  and at small factors of excess of the pump level over the threshold  $(\eta \le 0.1)$ . In this region, it is important to take into account spontaneous emission noises [14].

### 5. Conclusions

Windows of periodic and quasi-periodic lasing regimes in the region of dynamic chaos in a ring chip laser with periodically modulated pumping were observed experimentally for the first time in this study. The effect of a stationary magnetic field on the structure of the regions where various lasing regimes exist was studied. A new repetitively pulsed regime (QPI-2T) different from those studied earlier was discovered.

This study has revealed that the presence of magneto-optical properties of active elements in Nd: YAG ring lasers leads to a number of interesting features in their lasing dynamics. It is shown that the use of the frequency and/or amplitude nonreciprocity, which appear in a monolithic ring laser upon application of a magnetic field, is an efficient way to control the ring-laser nonlinear dynamics. The radiation characteristics in nonstationary lasing regimes in solid-state monolithic ring lasers are rather sensitive to the laser parameters, and using magnetic fields ensures their efficient changes. The efficiency of using magnetic fields to control the regimes of dynamic chaos is especially high.

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