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Natural convection in laser systems

A.I. Osipov, A.V. Uvarov

***Abstract*. The general type of boundary conditions in heat exchange problems, including heat exchange in laser systems, is discussed. The appearance of convection in a plane layer of a nonequilibrium gas with the volume energy release and its temperature dependence is briefly considered. Convection in a cylindrical system and in a system of coaxial cylinders with a cooled central part is considered in detail. Such systems simulate real laser devices. It is shown that the maximum temperature in the cylindrical system decreases due to convection, whereas the maximum temperature in the system of coaxial cylinders increases, i.e., the analysis of heat removal in a laser system reveals a very important role of convection.**

Keywords: *convection, heat removal, coaxial laser.*

1. Introduction

Analysis of the thermal regime in laser systems is an important problem of laser physics. While relaxation processes resulting in the heating of laser systems have been adequately studied, the investigation of thermal flows forming temperature fields is only beginning. At the same time, many problems in thermal physics, which are similar to those encountered in laser technologies, have been already solved. In particular, an important role of convection, which substantially enhances the efficiency of heat conduction, has been established.

The aim of this paper is to show that convection considerably affects heat removal in laser systems with diffusion cooling. Such problems are encountered, for example, in geophysics and nuclear energetics [1].

2. Heat removal in a nonequilibrium system

Historically, kinetic processes in a nonequilibrium gas were first studied under stationary homogeneous conditions. Although such a simplification was quite natural at the first stage of investigations, it is difficult to imagine that a nonequilibrium homogeneous system confined by walls can be in a stationary state. Indeed, the gas is heated due to relaxation processes, and a heat sink through the wall produces the temperature gradients, which increase with increasing the extent of nonequilibrium.

Consider region (*1*) of a nonequilibrium gas, which is separated from environment (*2*) by a thin wall (Fig. 1). If energy is released in region (*1*), the system of equations describing the state of the gas has the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\,\mathrm{div}\,\boldsymbol{v} = 0,$$

$$\rho\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \mathrm{grad}\,p - \eta\Delta\boldsymbol{v} - \frac{\eta}{3}\mathrm{grad}\,\mathrm{div}\,\boldsymbol{v} - \boldsymbol{f} = 0, \tag{1}$$

$$\frac{\gamma}{\gamma-1}\frac{\mathrm{d}T}{\mathrm{d}t} - \frac{T}{p}\mathrm{div}\,(\lambda\nabla T) - \frac{T}{p}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\eta}{2}\sum_{i,k}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\mathrm{div}\,\boldsymbol{v}\right)^2 = I,$$

where $\boldsymbol{v}$, T, p, and ρ are the gas velocity, temperature, pressure, and density; γ is the adiabatic index neglecting vibrational degrees of freedom; η and λ are the coefficients of viscosity and translational – rotational heat conduction; and $\boldsymbol{f}$ is the external force. Note that the third equation in (1) is an equation for energy, which describes translational – rotational degrees of freedom, and the energy flux I related to the heat capacity can also include the term appearing due to energy relaxation. If necessary, system (1) can be supplemented with corresponding relaxation terms. Calculations are often based on the stationary medium approximation. One can see from system (1) that this

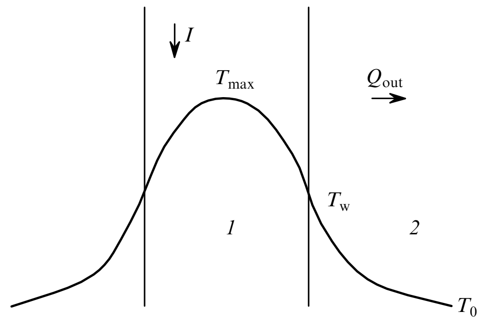

Figure 1. Scheme of heat removal in a laser system.

A.I. Osipov, A.V. Uvarov Department of Physics, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119992 Moscow, Russia; e-mail: uvarov@phys.msu.ru

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approximation can be used only in some special cases. For example, the direction of the density gradient in the gravitational field should coincide with the direction of the gravitational force, otherwise the equations of motion never vanish. However, a gas at rest can be unstable as well. An increase in gradients leads to the development of convection instability and passage to the convection regime.

System (1) for a gas at rest reduces to the equation

$$\frac{\gamma}{\gamma-1}\frac{\mathrm{d}T}{\mathrm{d}t}-\frac{T}{p}\mathrm{div}(\lambda\nabla T)=I, \quad (2)$$

which transforms to a simple Laplace equation in the stationary case and the constant heat conductivity. The same equation should be also considered for the environment (the only difference being that in the latter case $I = 0$). Usually, the following important simplification is employed. The approximate solution of the heat conduction equation for the environment is found, which allows one to determine the heat flow depending on the temperature difference $T_w - T_0$:

$$Q_{out} = \alpha(T_w - T_0), \quad (3)$$

where α is the heat transfer coefficient; T_w is the wall temperature; and T_0 is the environmental temperature. Figure 1 shows the approximate temperature profile. The relation between temperatures T_{max} (maximum temperature inside the volume), T_w and T_0 is determined by the dimensionless parameter $\mathrm{Bi} = \alpha L/\lambda$ called the Bio number (L is the characteristic linear size of the nonequilibrium gas region). If $\mathrm{Bi} \ll 1$ (poor external heat removal and high heat conduction inside the system), we can assume that $T_{max} - T_w \ll T_w - T_0$, i.e., $T_{max} \approx T_w$. In this case, $Q_{out} = \alpha(T_w - T_0) \approx \alpha(T - T_0)$, where $T \approx \mathrm{const}$. It is this approximation that was used in the thermal explosion theory of Semenov [2]. Thermal instabilities in discharges and gases are also calculated, as a rule, in this approximation [3]. However, in real systems, in particular, lasers the opposite condition $\mathrm{Bi} \gg 1$ is very often fulfilled, especially in the case of additional water cooling of the walls and a large heat transfer coefficient. If $\mathrm{Bi} \gg 1$, we can assume that $T_w \approx T_0$, but there exists the temperature profile inside volume (*1*) (Fig. 1). This approximation was used in the thermal explosion theory proposed by Frank-Kamenetskii [2], and it is this approximation, despite some complication of the problem, should be used for calculating instabilities in discharges and lasers. Of course, the second, alternative mechanism of energy transfer is possible, which is related to diffusion to the wall followed by heterogeneous relaxation. The relation between the efficiencies of these mechanisms is determined by the Lewis number, while the consideration of diffusion transfer additionally complicates the problem [4]. These mechanisms are not considered in this paper.

3. Plane layer of a nonequilibrium gas

A horizontal plane layer of a nonequilibrium gas is the simplest model for studying convection [5]. At the same time, this is also the simplest model of a laser system, but only in the case when the distance L between the walls is much smaller than the characteristic transverse dimensions of the system. For a horizontal plane layer, the solution of the problem is determined by the temperature profile and the temperature dependence of heat release. The temperature profile $T(z)$ is determined by the difference of temperatures on the planes and the energy release efficiency. When the temperature changes moderately, the function

$$\mathrm{Ra}(z) = -8\frac{g\beta L^3}{\nu\chi}\frac{\mathrm{d}T}{\mathrm{d}z}$$

determining stability (where g is the acceleration of gravity; β is the thermal expansion coefficient; χ and ν are the thermal diffusivity and kinematic viscosity coefficient, respectively, respectively) can be approximated by the linear dependence

$$\mathrm{Ra}(z) = \mathrm{Ra}_0 + \mathrm{Ra}_T z, \quad (4)$$

i.e., the semiparabolic temperature profile is considered. The number $\mathrm{Ra}_0 = -g\beta L^3\Delta T/(\nu\chi)$ (where ΔT is the difference of temperatures on the walls) is called the Rayleigh number. In the absence of energy release ($\mathrm{Ra}_T = 0$), the classical Rayleigh – Benard problem appears, in which convection begins when $\mathrm{Ra}_0 > 1708$. However, convection appears not only due to the difference in temperatures. The value of $\mathrm{Ra}_T = g\beta L^5 I/(\nu\chi^2)$ is determined by the amount of energy release, and convection also develops with increasing this amount.

In addition, as pointed out above, the stability of the system is also determined by the temperature dependence of the heat release rate. This rate is determined by the temperature dependence of the relaxation time and is characterised by the parameter $s = -T\tau^{-1}\mathrm{d}\tau/(\mathrm{d}T)$ [4]. It is the increase in the parameter s that results in the appearance of thermal nonequilibrium and thermal explosion in the Frank-Kamenetskii model. The possibility of a thermal explosion in the laser system strongly depends on the pump method and heat removal near the walls [4].

Figure 2 shows the critical surface separating stationary states (below the surface) and convective motions in the model of a constant vibrational energy of the unit mass. The

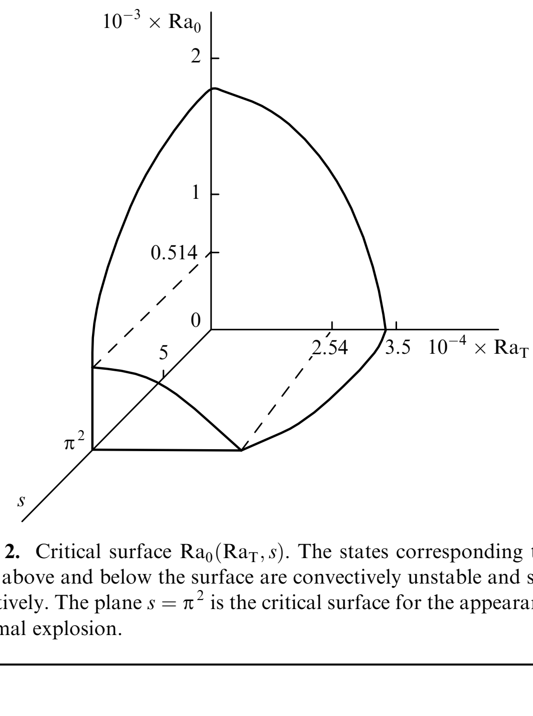

Figure 2. Critical surface $\mathrm{Ra}_0(\mathrm{Ra}_T, s)$. The states corresponding to the points above and below the surface are convectively unstable and stable, respectively. The plane $s = \pi^2$ is the critical surface for the appearance of a thermal explosion.

use of such a model is justified in a number of cases because a fast energy exchange occurs between electrons and vibrational degrees of freedom of molecules. Above the critical surface, the nonequilibrium medium is convectively unstable, and the $s = \pi^2$ plane corresponds to the parameters at which a thermal explosion occurs (a drastic increase in temperature) observed at $s > \pi^2$. Figure 2 well illustrates different types of instability which can appear in the system. While the intersection of the plane $s = \pi^2$ leads to a drastic increase in the translational temperature, which is extremely undesirable in laser systems, the intersection of another plane separating stable and unstable regimes gives rise to convection, which substantially improves heat removal.

The question of how much convection improves heat release is well studied for the classical Rayleigh – Benard problem, when $\mathrm{Ra_T} = 0$. It is found that the influence of convection increases with the Rayleigh number. The thermal flow, taking convection into account, can many-fold exceed this flow without convection. This means that the maximum temperature in the case of volume energy release can strongly depend on the convection conditions.

4. Coaxial laser

The influence of convection is distinctly manifested in a coaxial laser [6]. At the centre of the operating region of such a laser, where temperature is maximal, a cooled cylinder is located, which provides an additional heat removal, thereby decreasing temperature. In this case, the problems related to the characteristics of laser radiation appear, which are not considered here. As pointed out above, convection inevitably develops in such coaxial systems, which has long been studied at different temperatures [7]. In this case, as in a plane layer, the regime changes under certain conditions; however, not from a medium at rest to convection, but from slow convection to rapid three-dimensional convection. Calculations showed that even upon slow convection, the thermal flow changes very strongly compared to the case when convection is absent. Consider the problem in the approximation of a constant volume energy release.

We assume that the working medium fills the region between two coaxial cylinders of radii R_i and R_0 (Fig. 3), the relation between them being characterised by the dimensional quantity $\sigma = 2R_i/(R_0 - R_i)$. The cylinders are oriented horizontally, and the gravitational force is directed perpendicular to their axis. The temperature of the cylinders in maintained constant ($T_0 = T_i$), and we do not consider here the problem at different temperatures. A constant energy release occurs in the system with the volume efficiency Q independent of the coordinates of the medium. We will describe the system using the cylindrical coordinates (R, θ), the angular coordinate θ being measured from the vertical symmetry plane counter-clockwise.

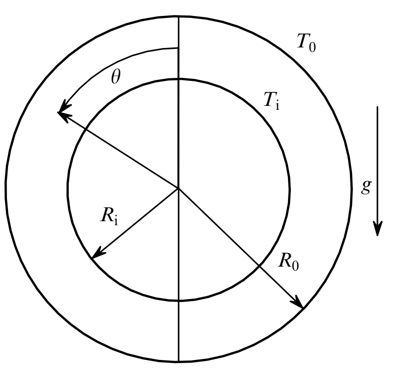

Figure 3. System of coaxial cylinders.

The system of hydrodynamic equations (1) in the cylindrical coordinates in the Oberbeck – Boussinesq approximation has the form

$$\nabla^2\psi = -\omega,$$

$$\nabla^2\omega = \frac{1}{\mathrm{Pr}}\left(\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial r} + \frac{v}{r}\frac{\partial\omega}{\partial\theta}\right) + \mathrm{Ra_T}\left(\sin\theta\frac{\partial\varphi}{\partial r} + \frac{1}{r}\cos\theta\frac{\partial\varphi}{\partial\theta}\right), \qquad (5)$$

$$\nabla^2\varphi = u\frac{\partial\varphi}{\partial r} + \frac{v}{r}\frac{\partial\varphi}{\partial\theta} - 1 + \frac{\partial\varphi}{\partial t},$$

where

$$\psi = \frac{\Psi}{\chi};\ r = \frac{R}{L};\ \varphi = \frac{T - T_0}{T_0 q};\ u = \frac{UL}{\chi};\ v = \frac{VL}{\chi};\ q = \frac{IL^2}{\chi T_0};$$

$L = R_0 - R_i$; U and V are the radial and angular velocity components, respectively; and Pr is the Prandtl number. The function Ψ is defined by the relations

$$U = R^{-1}\frac{\partial\Psi}{\partial\theta}, \quad V = -\frac{\partial\Psi}{\partial R}. \qquad (6)$$

The boundary conditions are formed taking into account two isothermal walls of cylinders and the vertical symmetry plane for $\theta = 0$ and $180°$. The function of flow along each of the walls and the symmetry plane is zero because flows of matter through the walls and the plane are absent. It is also assumed that the angular derivatives of the temperature and vorticity ω in the symmetry plane vanish. The vorticity along the walls takes the form

$$\omega = -\frac{\partial^2\psi}{\partial r^2}. \qquad (7)$$

Therefore,

$$\psi = \omega = \frac{\partial\varphi}{\partial\theta} = 0 \qquad (8)$$

in the symmetry plane, and

$$\psi = u = v = 0, \quad \omega = -\frac{\partial^2\psi}{\partial r^2}, \quad \varphi|_{r=r_i} = \varphi|_{r=r_0} = 0 \qquad (9)$$

on the inner and outer cylinders.

The problem is solved, taking convection into account, in the following way. An initial distribution of the temperature and function of flow is specified. The evolution of this distribution can be studied using equations (5) and boundary conditions (7) – (9), in particular, the limiting stationary regime can be obtained (if it exists). The problem can be solved numerically by the method of finite differences. The finite-difference scheme is standard, and the Poisson equation is solved by the method of variable directions by using the sweep method for each of them.

Typical isotherms and streamlines are sown in Fig. 4a. For comparison (Fig. 4b), the distribution for a typical thermal physical problem is presented in the absence of energy release when the inner cylinder is more heated ($T_i > T_0$).

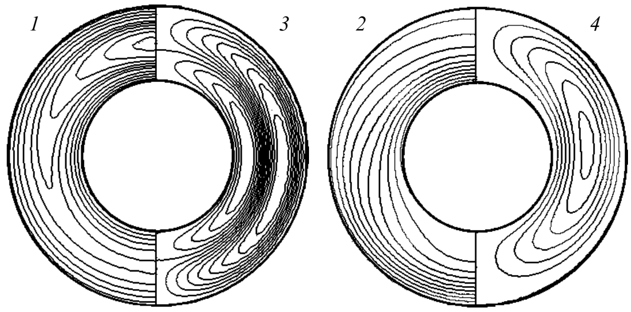

Figure 4. Isotherms (*1*, *2*) and streamlines (*3*, *4*) in the presence of volume energy release and $T_i = T_0$ (a) and in the absence of energy release for $T_i > T_0$ (b).

Note that in the absence of energy release in the two-dimensional regime, only one vortex is formed (in accordance with the direction of the temperature gradient), and the temperature gradient does not change its sign inside this vortex.

Consider now this system from the point of view of the outlook for developing a laser. In the absence of convection but upon energy release, the temperature distribution between coaxial cylinders for $T_i = T_0$ has the form

$$\varphi = -\frac{r^2}{4} + \left(r_0^2 \ln\frac{r}{r_i} - r_i^2 \ln\frac{r}{r_0} \right) \left(4 \ln\frac{r_0}{r_i} \right)^{-1}. \tag{10}$$

In the limit $\sigma \to 0$ ($r_0 \gg r_i$), we obtain the maximum value $\varphi_{max} = 0.25$, which corresponds to the same value of φ_{max} in the problem with energy release for a cylinder, when $\varphi = (r_0^2 - r^2)/4$. The coaxial geometry provides the decrease of the maximum temperature in the system by half compared to the cylindrical geometry. As σ increases, the maximum temperature rapidly decreases and already at $\sigma \geqslant 1$ it weakly differs from $\varphi_{max} = 0.125$. Therefore, if convection is neglected, the coaxial geometry of the laser is preferable.

When convection is taken into account, the situation is different. One can see from Fig. 4a that in this case the angular distribution of temperature is strongly inhomogeneous. Moreover, the maximum temperature in the presence of convection is greater than in its absence. Figure 5 shows the dependences of the maximum temperature difference achieved in the system on the modified Rayleigh number for different σ ($T_i = T_0$). One can see that the maximum temperature difference [$\varphi_{max} = (T_{max} - T_0)/(T_o q)$] for a system of coaxial cylinders increases with increasing Ra_T.

The value of T_{max} increases due to the simultaneous action of two vortices. Indeed, if we consider the upper region of the system of coaxial cylinders (Fig. 6) and compare energy fluxes in the presence and absence of

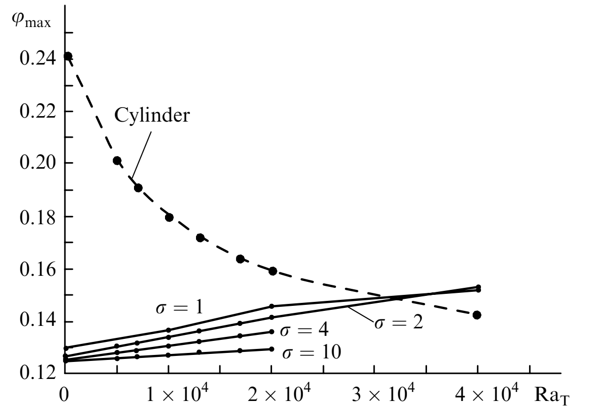

Figure 5. Dependence of the maximum temperature difference φ_{max} on Ra_T for different σ.

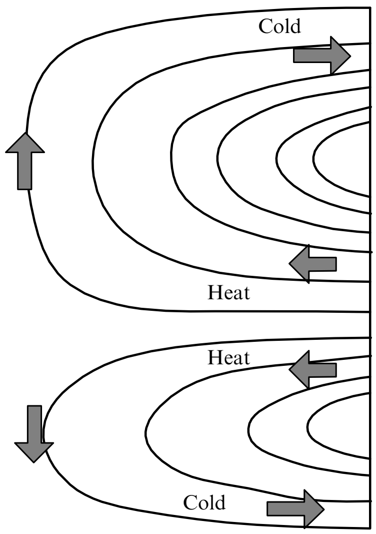

Figure 6. Convective flows at the upper point of a system of coaxial cylinders.

convection, we will see that in the presence of convection the additional energy transfer to the system occurs along the line dividing the vortices because incoming fluxes are more heated than the outgoing fluxes (Fig. 5). This results in an increase in the maximum temperature difference. The inhomogeneity of the angular distribution of temperature caused by the angular inhomogeneity of thermal flows to the wall can be characterised by the Nusselt number. If the surface is divided into two halves – for angles from 0 to π/2 and from π/2 to π, then for $\sigma = 2$ for $Ra_T = 3.5 \times 10^4$, which corresponds to Fig. 5, the Nusselt numbers are

$$\int_0^{\pi/2} r \frac{\partial\varphi}{\partial r}\bigg|_{r=r_0} = 1.52, \quad \int_{\pi/2}^{\pi} r \frac{\partial\varphi}{\partial r}\bigg|_{r=r_0} = 1.29,$$

$$\int_0^{\pi/2} r \frac{\partial\varphi}{\partial r}\bigg|_{r=r_i} = 0.92, \quad \int_{\pi/2}^{\pi} r \frac{\partial\varphi}{\partial r}\bigg|_{r=r_i} = 0.86,$$

respectively. Of course, the total flux is determined only by the energy release and is preserved irrespective of convection. The total flux normalised to the energy release [see (5)] depends only on σ and is equal to $\pi(1+\sigma)/2$ for two semicircles.

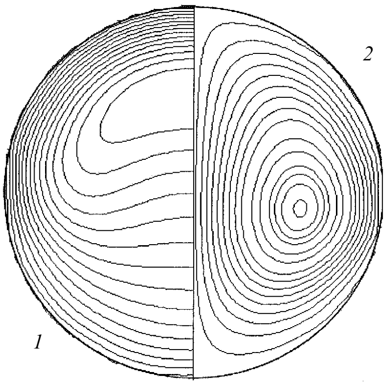

Figure 7. Isotherms (*1*) and streamlines (*2*) for a cylinder in the case of volume heat release, $Ra_T = 2 \times 10^4$.

The problem with the volume energy release in a cylinder is solved similarly. Figure 7 shows typical isotherms and streamlines, and Fig. 5 demonstrates the dependence of the maximum temperature difference φ_{max} on Ra_T. In the cylindrical system, this temperature decreases with increasing energy release, and for some value of Ra_T it becomes even lower than in the system of coaxial cylinders.

Note, however, that the results cannot be directly compared in this case because the value of Ra_T for a cylinder is normalised to the radius, whereas for a coaxial system it is normalised to the slit width, so that the cross-sectional area for the same Ra_T will be different. If the same energy release and the same cross section are considered, the corresponding value of Ra_T for coaxial cylinders will be substantially lower [namely, by a factor of $(1+\sigma)^{5/2}$]. However, a lower value of Ra_T means that the convection velocity is also substantially lower (note that for the same value of Ra_T, the convection velocity in the system of coaxial cylinders is substantially lower because of the presence of two vortices). Nevertheless, the two-dimensional convection in the cases of a cylinder and a system of coaxial cylinders leads to two substantially different results. In the first case, the maximum temperature strongly decreases, whereas in the second case, it somewhat increases. These variations are of the order of magnitude of the difference in the maximal temperatures in the absence of convection.

Therefore, the consideration of convection (which is always present in these systems) makes the cylindrical geometry of the laser preferable over the coaxial one.

The calculations presented in the paper concerned the two-dimensional model and moderate Rayleigh numbers. In addition, the energy release in the system was accounted for quite simply. However, even a simple model allows us to reveal the main features of the convective flow in the system of coaxial cylinders. As the energy release (i.e., Ra_T) increases, the structure of a convective flow changes, which should be taken into account by selecting the laser system geometry.

5. Conclusions

1. We have analysed the boundary conditions in the general heat exchange problem. It is shown that, along with standard condition (3), it is necessary to use, in the case of intense external cooling, the condition of the equality of temperatures of cylinder walls and a cooling medium, which produces gradients of parameters in a nonequilibrium gas.

2. Based on the mathematical model developed in the paper, the two-dimensional convection is described in the cylindrical system and system of coaxial cylinders. It is shown that, as the energy input is increased, the maximum temperature in a cylinder decreases, whereas in coaxial cylinders it increases, which makes lasers with the cylindrical geometry preferable.

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