OPTICAL SOLITONS

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Excitation of two-dimensional soliton matrices by fundamental Gaussian beams

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***Abstract*.** **The excitation of two-dimensional periodic structures of fields of the first and second radiation harmonics due to the modulation instability of fundamental Gaussian beams is studied in a medium with a quadratic nonlinearity. The distances are found at which soliton matrix structures with a specified period are formed and destroyed. Optical gratings formed due to nonlinear aberration of broad Gaussian beams are considered.**

Keywords: *nonlinear optics, soliton matrices, interaction of radiation with matter, Gaussian beams.*

1. Introduction

The formation of periodic optical structures in nonlinear media attracted the attention of researchers about half-century ago, although the effects of spatial decomposition of nonlinear waves have been studied for a long time, as a rule, in cubic nonlinear media [1 – 3]. The wave structures formed in the medium are periodic because a rather narrow spectrum of spatial perturbations has a high gain in the focusing nonlinear medium. In Kerr media, due to strong self-focusing a two-dimensional beam decomposes into thin and intense light filaments, which can cause, for example, the damage of a crystal under study. To avoid this, researchers used various methods to reduce focusing such as phase modulation [4 – 9], polarisation modulation [10], etc. Such structures were also studied in photorefractive [11 – 14] and quadratic nonlinear [15 – 17] media. Sometimes a scheme of crossed beams was used to initiate modulation instability at a prescribed spatial frequency [17]. In this case, an intense light beam interferes with a weak inclined beam, thereby producing amplitude modulation with a spatial frequency proportional to the beam inclination angle. Such a scheme allows the generation of spatial wave structures of the field with a required periodicity.

The analytic description of the formation of nonlinear structures is complicated. However, the development of modulation instability of weakly perturbed stationary solutions can be described within the linearised model of plane stationary waves [18]. Numerical methods were used to study the decomposition of one-dimensional or elliptic beams and the effect of the beam diffraction and self-focusing, nonlinear aberrations and noises on the formation of contrast structures [19 – 22]. The dependence of the number of subbeams in a grating on the size and intensity of the incident beam was investigated in [15, 20]. However, the spatiotemporal parameters and dynamics of nonlinear soliton gratings have not been studied so far. These questions are especially important because any restricted soliton gratings in a homogeneous medium are subjected to the disintegration due to their initially inhomogeneous profile, diffraction, and mutual focusing [22]. In this paper, we study the excitation of two-dimensional periodic structures – matrices appearing due to modulation instability of Gaussian beams at the fundamental frequency of exciting radiation and analyse the evolution of soliton structures formed in this process.

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2. Formulation of the problem and basic equations

Consider a system of beams at the fundamental frequency and second harmonic of exciting radiation in a quadratic nonlinear medium. The evolution of their complex envelopes $A_j(x, y, z)$ ($j = 1, 2$) during the propagation of waves along the z axis is described by the equations

$$\frac{\partial A_1}{\partial z} + \mathrm{i}D_1\frac{\partial^2 A_1}{\partial x^2} + \mathrm{i}D_1\frac{\partial^2 A_1}{\partial y^2} + \mathrm{i}\gamma A_1^* A_2 = 0,$$
$$\frac{\partial A_2}{\partial z} + \mathrm{i}D_2\frac{\partial^2 A_2}{\partial x^2} + \mathrm{i}D_2\frac{\partial^2 A_2}{\partial y^2} + \mathrm{i}\Delta k A_2 + \mathrm{i}\gamma A_1^2 = 0, \qquad (1)$$

where x and y are the transverse coordinates; z is the longitudinal coordinate; $D_j = (2k_j)^{-1}$ are diffraction coefficients; $\Delta k = k_2 - 2k_1$ is the phase mismatch; and γ is the nonlinearity coefficient. Let us assume that a high-power Gaussian beam at the fundamental frequency is incident on a quadratic nonlinear medium, the beam amplitude profile being harmonically perturbed with the spatial frequency $\kappa = \kappa_x = \kappa_y$. The second harmonic at the input of the medium is absent. Thus, we have

$$A_1(x, y, 0) = A_s(x, y)[1 + \delta\cos(\kappa x)\cos(\kappa y)],$$
$$A_2(x, y, 0) = 0, \qquad (2)$$

where $A_s = A_{s0}\exp[-(x^2+y^2)/a^2]$ is the Gaussian profile with the peak amplitude A_{s0} and width a; and δ is the relative depth of the amplitude modulation of the beam.

Numerical simulation was performed by using in (1) the normalisation: $x/a = 10$, $D_1 = a^2/4l_d$, $D_2 = D_1/2$. Equations (1) with boundary conditions (2) were solved numerically by the pseudo-spectral method [23] for $D_1 = 0.25$, $\Delta k = 0$, $D_2 = D_1/2 = 0.125$, $\gamma = 0.25$, $A_s = 15$, $\delta = 0.15$, $a = 10$, and the length of the nonlinear medium equal to 10. The longitudinal component z was measured in units of the beam diffraction length $l_d = k_1 a^2/2$ $(a = 1)$.

3. Analytic estimate of the length of formation of wave matrices

Consider the perturbed stationary profile

$$A_j(x,y,z) = (E_j + U_j)e^{-i\Gamma_j z}, \tag{3}$$

where E_j is the amplitude of a stationary plane wave; $\Gamma_2 = 2\Gamma_1 = 2\Gamma$ is the nonlinear addition to its wave number; U_j is the amplitude of the perturbed wave; and $|U_j| \ll |E_j|$. In addition,

$$U_j = U_{j0}\cos(\kappa_x x)\cos(\kappa_y y)\exp(Gz). \tag{4}$$

By substituting solutions (3) and (4) into system (1) and neglecting nonlinear terms of the second-order smallness in U_j and E_j, we obtain

$$\begin{aligned}&(G - i\Gamma + iD_1\kappa^2)U_1 + i\gamma E_1^* U_2 + i\gamma U_1^* E_2 = 0,\\ &(G - i2\Gamma + iD_2\kappa^2 + i\Delta k)U_2 + i2\gamma E_1 U_1 = 0,\end{aligned} \tag{5}$$

where $\kappa^2 = \kappa_x^2 + \kappa_y^2$. From the condition of solubility of system (5) for the four unknowns U_1, U_1^*, U_2, and U_2^*, we obtain

$$\begin{aligned}&|g_1|^2|g_2|^2 - 2|B_1|^2|g_1|^2 - |B_2|^2|g_2|^2\\ &\quad - 2|B_1|^2 g_1 g_2 + 4|B_1|^4 = 0,\end{aligned} \tag{6}$$

where $g_j = iG + \Theta_j$; $B_j = \gamma E_j$; $\Theta_1 = \Gamma + 2D\kappa^2$; and $\Theta_2 = 2\Gamma + D\kappa^2 - \Delta k$. This equation expresses implicitly the increment of modulation instability as a function of the spatial modulation frequency and the field amplitude in a given medium, or $G' = \mathrm{Re}(G) = G'(\kappa, E_1, E_2)$. It is obvious that a contrast grating will be formed at the distance z_b, when

$$\delta\exp(G' z_b) \approx 1, \tag{7}$$

where $\delta = |E_1|/|U_{10}|$. Therefore, we obtain the estimate

$$z_b(\kappa) = -\frac{\ln\delta}{G'(\kappa)} \tag{8}$$

for the distance z_p of excitation of a contrast structure in a nonlinear medium.

4. Numerical simulation of excitation of periodic structures

First we studied the excitation of two-dimensional soliton gratings by performing numerical experiments with modulated Gaussian beams (2) by varying the modulation frequency at a fixed beam width. In these experiments, we determined the modulation frequencies at which a grating was formed in the medium. The criterion of the grating formation was the visual observation of a contrast periodic wave structure at the first harmonic. Note that parametric interaction causes the simultaneous appearance of a similar periodic structure at the second harmonic as well. The subbeams in the grating are close by their properties to solitons, so that this structure can be called a soliton structure with good reason. However, unlike solitons themselves, such structures are not always stable. During propagation, the subbeams interact with each other and radiation, and a regular periodic structure can become chaotic. For example, limited gratings formed of the beams are often transformed at large distances into one or several solitons. In planar waveguides, a periodic disappearance and recovery of a regular structure was observed during its propagation [24]. However, we did not observe the recovery of the regular structure in our experiments. This is probably explained by the fact that radiation distorting a grating in one-dimensional structures proves to be locked between the grating nodes, which is not the case for two-dimensional structures. By analysing the appearing soliton matrices, we obtained the dependence of the spatial range of existence of a regular contrast grating along the z axis on the modulation frequency of a high-power beam (Fig. 1). Figure 1 shows the region of existence of the grating and also presents the estimate for different modulation frequencies and the coordinate of formation of a contrast matrix in a quadratic nonlinear medium made using (8).

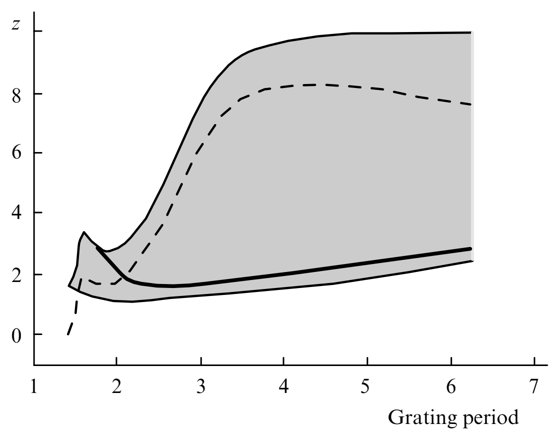

Figure 1. Region of existence of the matrix soliton structure formed due to the modulation instability of a fundamental Gaussian beam (hatched); the dependence of the spatial 'lifetime' of the grating on its period (dashed curve) and the estimate of the coordinate of formation of a contrast structure from expression (8) (thick curve).

Note that we have found an interesting fact of the appearance of new nodes and an increase in the spatial frequency of the grating due to energy redistribution at large modulation periods. The distance between new nodes and neighbouring nodes is smaller than the grating period by a factor of $\sqrt{2}$. The new grating is turned with respect to the initial grating by 45°, as was observed in our experiments (Fig. 2). The decrease in the grating period is favoured by the following conditions. The grating period should be much greater than the soliton size, and the beam power at one period of such a structure should be sufficient for the formation of new solitons. These conditions were fulfilled

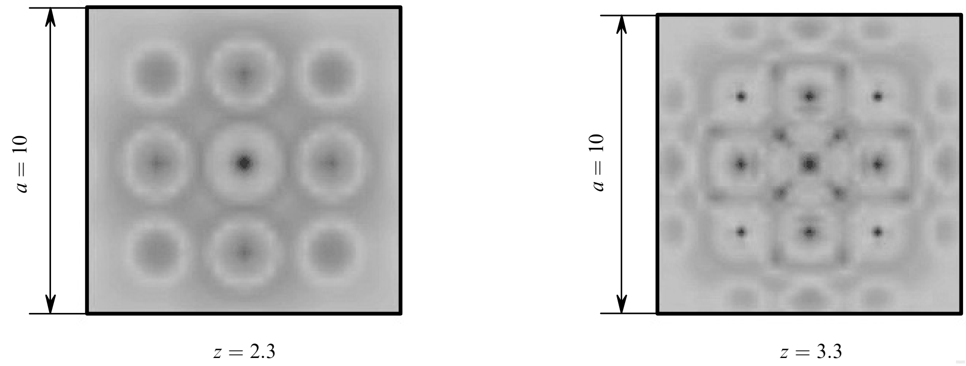

Figure 2. Periodic structures formed from a Gaussian beam with the modulation frequency $\kappa = 2.5$ at distances $z = 2.3$ and 3.3 for the beam width $a = 10$.

in our experiments on the period reduction in the frequency range $\kappa = 1 - 2$, where this effect was distinctly observed.

Figure 2 shows sections for two different coordinates z demonstrating a change in the frequency of the periodic structure. Because the grating is limited, an increase in its spatial frequency was accompanied by a small displacement of the nodes of the initial grating from edges to the centre. This can be caused by the influence of the mutual attraction of solitons because all the subbeams propagate initially in phase.

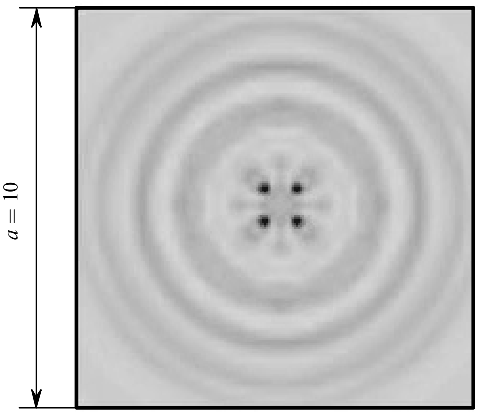

Figure 3. Periodic structure formed from an unmodulated Gaussian beam with the width $a = 10$. The grating period is 1.6.

Finally, we performed numerical experiments with unmodulated Gaussian beams. Because no seed for modulation instability existed, the beam began to decompose only at the distance $(8 - 10)l_{\rm d}$ due to nonlinear aberration. We have found that a structure being formed was also periodic, as in the case of modulated beams. Note that in this case the profile, width, and amplitude of the beam affect, generally speaking, the generated structure. We observed different periodic structures for Gaussian beams with the widths $a = 10$ (Fig. 3) and 100 (Fig. 4). The same effect was observed in experiments with modulated beams with frequency $k > 4.5$. In this case, the aberration grating was formed at different lengths, but always with the same period.

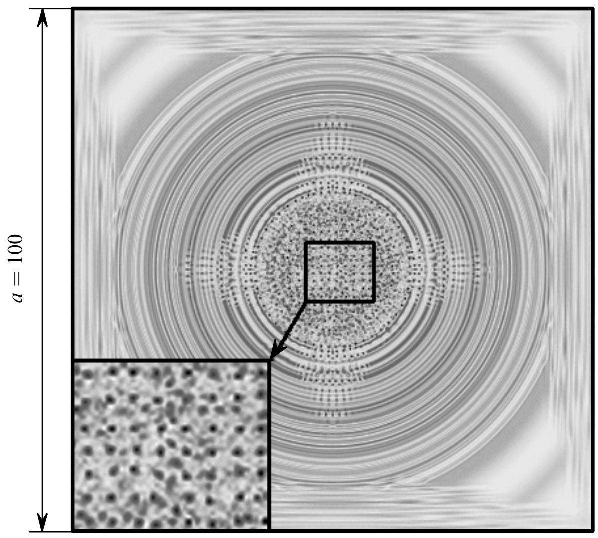

Figure 4. Periodic structure formed from an unmodulated Gaussian beam with the width $a = 100$. The grating period is 2.2.

5. Conclusions

We have studied excitation of two-dimensional periodic structures consisting of Gaussian beams at the fundamental radiation frequency. The gratings formed due to modulation instability have the periodicity corresponding to the initial beam modulation. However, in the case of large modulation periods or intense beams, the grating period can be decreased by a multiple factor (equal to $\sqrt{2}$ for matrix structures), when the power per a spatial element of the grating is sufficient for generating new nodes between existing subbeams. Because these nodes are located symmetrically, the changed structure remains regular. Note also that the dynamics of the grating period change can be reversed, when the period increases by a multiple factor. In this case, the power per a grating node should be lower than the power of a soliton capable to reside at the node. Finally, we have performed an interesting experiment on excitation of periodic soliton structures from a Gaussian beam without a modulating seed. Such a grating is formed with the size and periodicity determined only by the beam shape and power.

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