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# Suppression of the collapse of two-dimensional light beams in one-dimensional refractive-index gratings

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***Abstract*. The propagation of light beams in a nonlinear cubic medium with the refractive index periodically modulated along one transverse coordinate is considered. The profiles of soliton beams are found and their stability is studied. It is shown that the refractive-index modulation causes the collapse suppression and soliton stabilisation almost within the entire region of their existence.**

***Keywords***: *solitons, cubic nonlinearity, periodic modulation of the refractive index.*

The periodic transverse modulation of the refractive index can substantially affect the properties of spatial optical solitons formed in a nonlinear medium due to an exact balance between diffraction and nonlinear self-focusing. In such periodic systems, the so-called discrete solitons can be generated, which are supported by the waveguide channels of the periodic structure [1]. This class of self-consistent soliton states possesses a number of unique features which are not inherent in solitons in homogeneous nonlinear media. Note among them the possibility to control the concentration of the light-field energy in different channels of the waveguide structure and fabricate ultrafast switches and optical couplers [2]. The advantages of using spatially modulated media are especially distinct in the case of harmonic refractive-index gratings. Gratings of this type can be used to construct optical schemes with the controllable 'degree of discreteness' because they can operate in the regimes of weak and strong coupling between adjacent channels of the waveguide structure, which is determined by the depth and period of modulation of the refractive index of the medium [3, 4].

One-dimensional and two-dimensional solitons in nonlinear media with the periodic modulation of the refractive index were recently observed in photorefractive crystals [5 – 8]. Periodic modulation in such crystals can be induced with the help of a few interfering plane light waves whose intensities and angles of intersection determine the depth and period of the refractive-index grating. Note that the method of optically induced refractive-index modulation opens up wide possibilities for studying the channeling and interaction of soliton beams in induced waveguide structures of different configurations, in particular, in waveguides induced by non-diffracting Bessel beams [9]. Apart for fundamental solitons, simplest harmonic gratings support stable coupled states of several solitons, or a soliton train, thereby providing the possibility for manipulating by complex multihump soliton structures. Along with the method of optically induced refractive-index modulation, there exist a number of technological methods, which are, however, less convenient because they do not allow one to reconstruct the initially formed structure of a sample [2].

Up to now, the main attention has been paid to the study of two-dimensional solitons in two-dimensional refractive-index gratings in media with a cubic and saturable nonlinearity as well as of one-dimensional solitons in one-dimensional gratings [10 – 14]. In this connection it is interesting to find out whether stable two-dimensional solitons can exist in nonlinear cubic media when the refractive index is modulated only in one transverse direction. Recall that in the absence of refractive-index modulation, two-dimensional solitons in cubic media are unstable and they diffract or collapse depending on the incident energy flux and soliton width [15].

In this paper, we analyse numerically in detail the properties and stability of such solitons.

The propagation of intense light beams in a nonlinear cubic focusing medium with the transversely modulated refractive index is described by the Schrödinger type equation for the dimensionless slowly varying complex amplitude $q(\eta, \zeta, \xi)$ of the light-wave field

$$\mathrm{i}\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) - pR(\eta)q - q|q|^2. \quad (1)$$

Here, $\eta = x/r_0$ and $\zeta = y/r_0$ are the transverse coordinates normalised to the characteristic width $r_0$ of the input beam; $\xi$ is the longitudinal coordinate normalised to the diffraction length; $L_{\mathrm{dif}} = kr_0^2$; $k$ is the wave number; the parameter $p = L_{\mathrm{dif}}/L_{\mathrm{ref}}$ is proportional to the modulation depth of the refractive index $\delta n$; $L_{\mathrm{ref}} = c/(\delta n\omega)$ is the refraction length; $R(\eta) = \cos(2\pi\eta/T)$ is the function describing the refractive-index profile along the $\eta$ axis; and $T$ is the modulation period of the refractive index. We assume that the nonlinear addition to the refractive index is comparable with its modulation depth.

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Note that evolution equation (1) admits the preservation of a number of integral characteristics of the beam during its propagation, among which the most important are the energy flux (hereafter, simply energy) $U$ and the Hamiltonian $H$:

$$U=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|q|^2\mathrm{d}\eta\mathrm{d}\zeta,$$
$$H=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{2}|\nabla q|^2-\frac{1}{2}|q|^4-pR(\eta)|q|^2\right)\mathrm{d}\eta\mathrm{d}\zeta. \quad (2)$$

Let us represent the simplest stationary solutions of initial equation (1) in the form $q(\eta,\zeta,\xi)=w(\eta,\zeta)\exp(\mathrm{i}b\xi)$, where $w(\eta,\zeta)$ is the real function describing the soliton profile; $b$ is the real propagation constant describing the nonlinear phase shift. By substituting this expression for $q(\eta,\zeta,\xi)$ into (1), we obtain the equation

$$\frac{1}{2}\left(\frac{\partial^2 w}{\partial\eta^2}+\frac{\partial^2 w}{\partial\zeta^2}\right)+pR(\eta)w-bw+w^3=0. \quad (3)$$

The families of soliton solutions of (3) are determined by the values of the propagation constant $b$, modulation period $T$, and parameter $p$. Because different soliton families can be obtained from one known family by using the transformation $q(\eta,\zeta,\xi,p)\to\chi q(\chi\eta,\chi\zeta,\chi^2\xi,\chi^2 p)$ with an arbitrary coefficient $\chi$, we will fix the modulation period ($T=\pi/2$) and will vary $b$ from 0 to 6 and $p$ from 0 to 12.

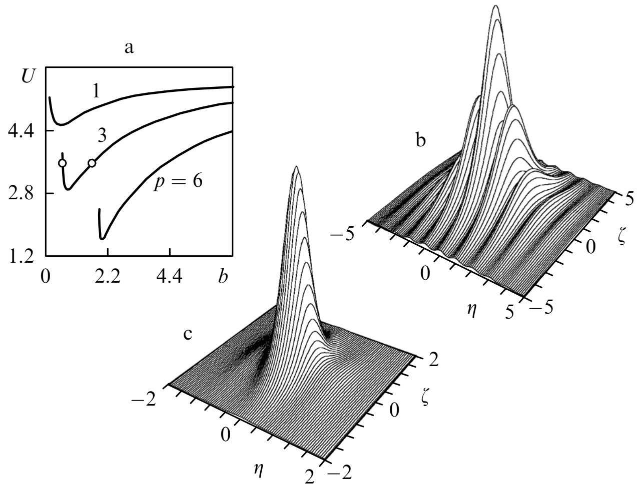


**Figure 1.** Soliton energy as a function of the propagation constant for different modulation depths $p$ of the refractive index (a) and the soliton profiles [the function $w(\eta,\zeta)$] corresponding to points indicated by circles in Fig. 1a (b, c). The modulation period of the refractive index is $T=\pi/2$, as in other figures.

The exact solutions of Eqn (3) can be found numerically by the relaxation method. As the initial conditions for the function $w(\eta,\zeta)$ describing the soliton profile, we used the Gaussian $w(\eta,\zeta)=A\exp[-(\eta^2+\zeta^2)/R_0^2]$,where $A$ and $R_0$ are the initial soliton amplitude and width, respectively. The properties of these solutions are demonstrated in Fig. 1. The soliton energy $U$ is a nonmonotonic function of the propagation constant $b$ (Fig. 1a). In addition, there exists the lower cut-off value $b_{\mathrm{co}}$ of the propagation constant, below which ($b<b_{\mathrm{co}}$) we did not find spatially localised stationary solutions of Eqn (3). For $b\to b_{\mathrm{co}}$, the soliton energy increases infinitely, its width also increases significantly, and a deep modulation of the refractive index appears along the $\eta$ axis (Fig. 1b). As $b$ increases, the soliton width decreases (Fig. 1c) and its energy asymptotically approaches the limiting value $U\approx 5.85$ for $b\to\infty$ irrespective of the value of $p$. In this case, the soliton itself becomes virtually axially symmetric. Note that, when the soliton energy $U$ is fixed, the soliton-profile modulation becomes more noticeable with increasing the modulation depth of the refractive index, while the peak amplitude of the soliton decreases. Apart from simplest fundamental solitons with the amplitude maximum at the point $\eta=\zeta=0$ considered here, the so-called even and twisted solitons exist in a medium with the modulated refractive index. However, all of them are unstable within the framework of the model under study.

By using the Vakhitov–Kolokolov criterion [15], we can assert that the soliton stability is absent in a homogeneous medium ($p=0$) where the soliton energy is $U\approx 5.85$ and is independent of $b$. Here, the propagation of light beams is accompanied by their spreading or collapse. In a medium with the modulated refractive index (Fig. 1a), a stable propagation of solitons is possible in the region $b_{\min}<b<\infty$, where $\mathrm{d}U/\mathrm{d}b>0$. The lower boundary $b_{\min}$ of the stability region, which is determined from the condition $\mathrm{d}U/\mathrm{d}b=0$, depends nonmonotonically on $p$ (Fig. 2a). In this case, $b_{\min}\to\infty$ for $p\to 0$ and $p\to\infty$. The energy $U_{\min}$ [$U_{\min}=U(b_{\min})$] monotonically decreases with increasing the modulation depth of the refractive index (Fig. 2b).

To confirm the preliminary results of analysis of the soliton stability based on the Vakhitov–Kolokolov criterion, we performed the numerical study of the stability. The solutions of Eqn (1) were represented in the form

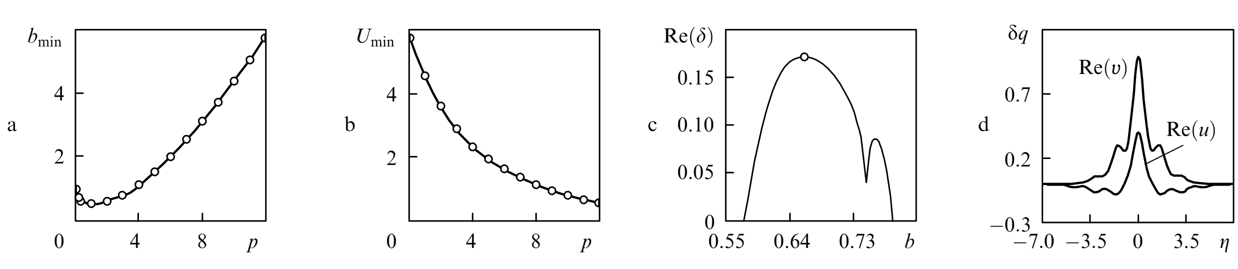


**Figure 2.** Propagation constant corresponding to the minimum of the dependence $U(b)$ as a function of the modulation depth of the refractive index (a), the minimal soliton energy as a function of the modulation depth of the refractive index (b), the real part of the increment of growth of a small perturbation as a function of the propagation constant for $p=3$ (c), and the distribution of the perturbation field for $\zeta=0$ (d). The perturbation corresponds to the point indicated by a circle in Fig. 2c.

$q(\eta,\zeta,\xi) = [w(\eta,\zeta) + u(\eta,\zeta,\xi) + \mathrm{i}v(\eta,\zeta,\xi)]\exp(\mathrm{i}b\xi)$, where the components $u$ and $v$ of a small perturbation can increase with the complex argument $\delta$ during soliton propagation. By substituting the wave field in this form into (1) and performing linearisation, we obtain the system of equations for the perturbation components

$$\frac{\partial v}{\partial \xi} = \frac{1}{2}\left(\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \zeta^2}\right) - bu + pRu + 3uw^2,$$
$$-\frac{\partial u}{\partial \xi} = \frac{1}{2}\left(\frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 v}{\partial \zeta^2}\right) - bv + pRv + vw^2. \qquad (4)$$

System of equations (4) was solved numerically by the method of splitting over physical factors. The numerical analysis of the soliton stability confirms the validity of the Vakhitov–Kolokolov criterion for the problem considered. The real part $\mathrm{Re}(\delta)$ of the increment of growth vanishes for $b = b_{\min}$ (Fig. 2c), where the sign of the derivative $\partial U/\partial b$ changes. A greater part of the instability region shown in Fig. 2c corresponds to the exponential growth of the perturbation. However, for $b \to b_{\min}$, when a hole appears in the dependence of $\mathrm{Re}(\delta)$ on $b$, the exponential instability changes to the oscillatory one. For $b > b_{\min}$, we found no perturbations with $\mathrm{Re}(\delta) > 0$. A typical distribution of the field of exponentially growing perturbation for $\zeta = 0$ is shown in Fig. 2d.

Finally, we considered the propagation dynamics of solitons in the developed instability regime. For this purpose, initial equation (1) was solved numerically by the method of splitting over physical factors with the initial conditions $q(\eta,\zeta,\xi=0) = w(\eta,\zeta)[1+\rho(\eta,\zeta)]$, where $w(\eta,\zeta)$ is the stationary solution of Eqn (3), $\rho(\eta,\zeta)$ is the function describing either white noise with the Gaussian distribution and dispersion $\sigma^2_{\text{noise}}$ or the regular perturbation of the input profile. We found that noise perturbations with a small dispersion $\sigma^2_{\text{noise}}$, as a rule, cause the spreading of unstable solitons. In the presence of a regular perturbation, which somewhat reduces the amplitude and energy of the unstable soliton, the soliton also spreads (Figs 3a, b). However, when the initial amplitude increases weakly, a broad soliton of the unstable branch can transform to a rather narrow soliton of the stable branch, which has almost the same energy (Figs 3c, d). In the presence of noise, stable solitons propagate without distortions over the distances exceeding the lengths of available crystals by a few orders of magnitude (Figs 3e, f).

Therefore, a periodic modulation of the refractive index in one of the transverse directions in a nonlinear cubic medium enhances linear refraction of radiation in this direction, resulting in the weakening of nonlinear refraction in both mutually orthogonal directions. Under such conditions, the soliton collapse can be suppressed and soliton beams can be stabilised within the entire region of their existence. The stability of fundamental solitons supported by a cubic medium with the transversely modulated refractive index can be analysed using the Vakhitov–Kolokolov stability criterion.

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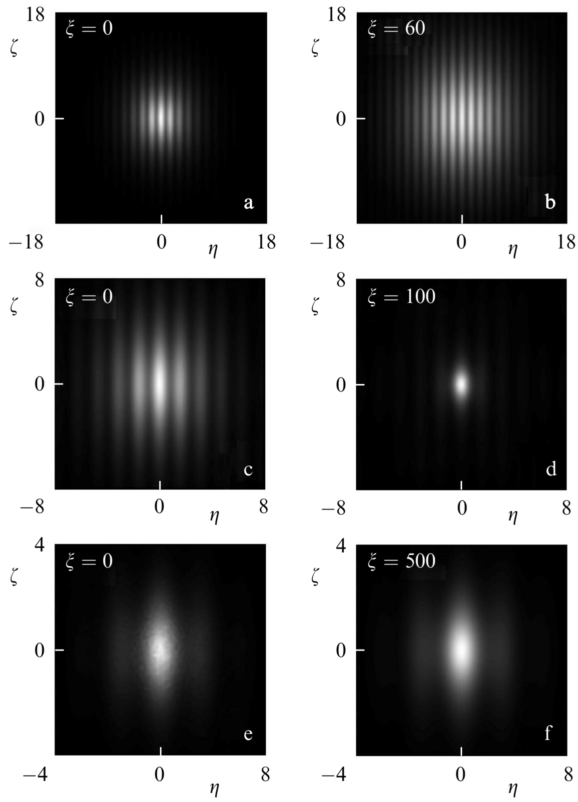


**Figure 3.** Propagation of the soliton shown in Fig. 1b with the somewhat decreased (a, b) and increased (c, d) amplitudes. Figures 3e, f show the propagation of a stable soliton for $b = 0.9$ and $p = 3$.

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