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Formation of shock waves in inhomogeneous active fibres

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***Abstract*. The formation dynamics of the shock wave of a pulse envelope is considered in fibres with the gain, dispersion, and nonlinearity distributed over the fibre length. It is shown that the wave-front steepness for inhomogeneous optical fibres can be strongly increased even when the local group-velocity dispersion substantially differs from zero if its average value (over the length of the shock-wave formation) is close to zero.**

Keywords: *shock waves, optical fibres, interaction of radiation with matter.*

The formation of the shock wave of a wave-packet envelope in a nonlinear medium is one of the most important nonlinear effects, which has been attracting the attention of researchers for decades [1 – 3]. This effect is caused by the dependence of the group velocity of a pulse on its intensity. The concept of formation of optical shock waves was first illustrated by the example of liquid nonlinear media [1, 2] and was further developed by studying the dynamics of pulses in nonlinear optical fibres [3]. The development of light-guiding systems of a new type (active fibres, fibre lasers, photonic-crystal and hollow-core fibres, etc.) [4] makes urgent the study of the possibility of increasing the wave-front steepness in such systems. In our opinion, of special importance is the investigation of the conditions for formation of shock waves and the possibility to control their dynamics in fibres with distributed material parameters, which have recently become very popular [5 – 10].

In this paper, we study the formation dynamics of the shock wave of a pulse envelope in active nonlinear fibres with the inhomogeneous distributions of the gain, group-velocity dispersion (GVD), and cubic (Kerr) nonlinearity over the fibre length. It is shown that the wave-front steepness of optical pulses in fibres with the local GVD values substantially different from zero can be considerably increased if its average value over the length of shock-wave formation is close to zero.

1. The formation of a shock wave with the amplitude A in a nonlinear amplifying medium with distributed nonlinear and dispersion parameters is analysed based on the equation

$$\frac{\partial A}{\partial z} - \mathrm{i}\,D(z)\,\frac{\partial^2 A}{\partial \tau^2} + \mathrm{i}R(z)|A|^2 A + $$
$$+ \beta_2(z)\,\frac{\partial}{\partial \tau}\left(|A|^2 A\right) = \delta(z)A. \qquad (1)$$

Here,

$$\tau(z) = t - \int_0^z u_{\mathrm{g}}^{-1}(\xi)\mathrm{d}\xi$$

is the time in the running coordinate system, in which the group velocity u_{g} of a low-power pulse in a medium with the distributed refractive index $n(z)$ is described by the relation $u_{\mathrm{g}}^{-1}(z) = n/c + (\omega/c)(\partial n/\partial\omega)$; and R, D, β_2, and δ are the Kerr nonlinearity, GVD, wave-front self-sharpening, and gain in the fibre depending on z, respectively.

Equations of type (1) with variable coefficients have been widely used recently [6 – 10] to analyse the behaviour of radiation in fibres with material parameters (permittivity, GVD, Kerr nonlinearity, etc.) distributed inhomogeneously over the fibre length. In the case of a continuous distribution of these parameters, the necessary condition for Eqn (1) to be correct is a continuous (slow) change in the effective refractive index of the fibre. This condition can be written in the form $|\partial n/\partial z| \ll nk$, where $k = n\omega/c$ is the propagation constant, which also depends in the general case on the coordinate z.

In the case of a sectional structure of the fibre, when the refractive index (and, therefore, all other material parameters) abruptly changes at splices of different sections, the condition of applicability of Eqn (1) is the inequality $L_{\mathrm{f}} \gg 1/k$, where L_{f} is the section length. This inequality can be treated as the condition of the absence of a backward wave caused by the Bragg or Mandelshtam – Brillouin scattering of a forward wave from the interface. In addition, all the characteristic lengths related to the above-mentioned parameters also should greatly exceed the value of $1/k$, which is the condition of applicability of the slowly varying amplitude approximation [2], which we used to drive Eqn (1).

It is convenient to pass to new amplitudes $\Psi(z,\tau)$ in Eqn (1) by using the transformation

$$A(z,\tau) = \Psi(z,\tau)\exp\left(\int_0^z \delta(\xi)\mathrm{d}\xi\right). \qquad (2)$$

New amplitudes satisfy the nonlinear Schrödinger equation with variable coefficients

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$$\frac{\partial \Psi}{\partial z} - \mathrm{i}D(z)\,\frac{\partial^2 \Psi}{\partial \tau^2} + \mathrm{i}\tilde{R}(z)|\Psi|^2\Psi + \tilde{\beta}_2(z)\frac{\partial}{\partial \tau}\left(|\Psi|^2\Psi\right) = 0. \quad (3)$$

Here, the effective nonlinearity and self-sharpening parameters

$$\tilde{R}(z) = R(z)\exp\left(2\int_0^z \delta(\xi)\mathrm{d}\xi\right),$$

$$\tilde{\beta}_2(z) = \beta_2(z)\exp\left(2\int_0^z \delta(\xi)\mathrm{d}\xi\right)$$

are introduced.

Consider a classical situation, when the GVD influence can be neglected, which is correct either for sufficiently long optical pulses (with $\tau_{\mathrm{p}} \gg 10^{-9}$ s) or for media and frequency ranges for which $D(z) \to 0$. In this case, we will seek the solution of Eqn (3) in the form

$$\Psi(z,\tau) = \rho(z,\tau)\exp[\mathrm{i}\phi(z,\tau)], \quad (4)$$

where ρ and ϕ are the real amplitude and phase of a wave packet. By substituting this solution into (3) and separating the real and imaginary parts, we obtain the system of equations

$$\frac{\partial \rho}{\partial z} + 3\tilde{\beta}_2(z)\rho^2\,\frac{\partial \rho}{\partial \tau} = 0,$$
$$(5)$$
$$\frac{\partial \phi}{\partial z} + \tilde{\beta}_2(z)\rho^2\,\frac{\partial \phi}{\partial \tau} + \tilde{R}(z)\rho = 0$$

for the amplitude and phase of the wave packet.

2. Let us analyse the solution of system (5) for a Gaussian pulse, for which the relation

$$A(\tau,0) = \rho_0\exp\left(-\frac{\tau^2}{2\tau_0^2}\right) \quad (6)$$

is valid at the fibre input, where τ_0 is the initial duration of a wave packet coupled to the fibre. In this case, the solution of the equation for the amplitude $\rho(\tau,z)$ determining the pulse shape can be written in the form

$$\rho(\tau,z) = \rho_0\exp\left\{-\left[\tau - 3\rho^2\int_0^z \tilde{\beta}_2(\xi)\mathrm{d}\xi\right]^2\frac{1}{2\tau_0^2}\right\}. \quad (7)$$

Taking into account the definition of the time τ in the running coordinate system, the velocity of the wave-packet maximum is described by the expression

$$u_{\max}(z) = z\left[\int_0^z u_{\mathrm{g}}^{-1}(\xi)\mathrm{d}\xi + 3\rho^2\int_0^z \tilde{\beta}_2(\xi)\mathrm{d}\xi\right]^{-1}, \quad (8)$$

which is in the general case a complicated function of the coordinate z. In the particular case of $\delta = 0$, $\beta_2 = \mathrm{const}$, and, therefore, $u_{\mathrm{g}} = \mathrm{const}$, the expression for the velocity of the envelope maximum takes the known form [2, 3]

$$u_{\max} = \frac{u_{\mathrm{g}}}{1 + 3\beta_2 u_{\mathrm{g}}\rho_0^2}. \quad (9)$$

It is obvious that in the linear approximation (when $\beta_2\rho_0^2 \to 0$), the velocity of the envelope maximum coincides with the group velocity of a pulse of an infinitely low power ($u_{\max} = u_{\mathrm{g}}$).

To construct the pulse shape in a nonlinear amplifying medium, it is convenient to write expression (7) in the form

$$\tau = 3\rho^2\int_0^z \tilde{\beta}_2(\xi)\mathrm{d}\xi \mp \tau_0\left(2\ln\frac{\rho_0}{\rho}\right)^{1/2}, \quad (10)$$

where the minus sign corresponds to the leading edge of the pulse and the plus sign to its trailing edge. An increase in the pulse steepness leads finally to the discontinuity over some length L_0 at which $|\partial\rho/\partial\tau| \to \infty$ (or $|\partial\tau/\partial\rho| = 0$), i.e., the envelope shock wave is formed.

We can obtain from (10) the following implicit relation between the formation length L_0 of the envelope shock wave and fibre and pulse parameters:

$$\int_0^{L_0}\left\{\beta_2(z)\exp\left[2\int_0^z \delta(\xi)\mathrm{d}\xi\right]\right\}\mathrm{d}z$$
$$= \mathrm{sign}\,\beta_2\,\frac{\tau_0(\mathrm{e}/2)^{1/2}}{3\rho_0^2}. \quad (11)$$

Thus, for $\beta_2 = \mathrm{const}$ and $\delta = \mathrm{const}$, we have

$$L_0 = \frac{1}{2\delta}\ln\left[\frac{2\delta\tau_0(\mathrm{e}/2)^{1/2}}{3|\beta_2|\rho_0^2} + 1\right]. \quad (12)$$

For the case $\delta = 0$, expression (12) takes a standard well-known form [2, 3]

$$L_0 = \frac{\tau_0(\mathrm{e}/2)^{1/2}}{3|\beta_2|\rho_0^2}. \quad (13)$$

If $\beta_2 > 0$, the shock wave is formed on the trailing edge of the pulse, while for $\beta_2 < 0$, it is formed on the leading edge. For $\beta_2 > 0$, the pulse maximum propagates at a velocity that is lower than the group velocity of the wave packet in the medium, which means that the pulse maximum is displaced to the trailing edge of the wave packet and the steepness of the trailing edge increases. The situation corresponding to $\beta_2 < 0$ should lead to an increase in the steepness of the leading edge of the wave front. It can be realised, for example, in a two-level medium, where this process occurs due to the predominant amplification at the leading edge of the pulse [11]. The effective medium with the negative value of the self-sharpening parameter can be also produced by preparing a two-mode wave packet consisting of two strongly interacting co-propagating pulses [12, 13].

When $\beta_2 < 0$, the pulse maximum propagates at a greater velocity than its wings, and if some additional conditions are satisfied [12], its velocity proves to be even greater than the speed of light in vacuum. This circumstance is important, because, as a rule, it is assumed that superluminal pulses can be observed in an active (amplifying) medium. However, it was pointed out in [12] that pulses with the superluminal velocity of the envelope can be obtained in a non-amplifying nonlinear medium in the case of a strong interaction between the waves (taking into account the influence of nonlinearity dispersion and the coupling between the waves). This means that the region of existence of superluminal optical pulses can be extended to the systems in which the transformation of the wave packet is performed without its amplification. In this case, the mechanism of formation of superluminal pulses, as in the classical case with their amplification, is related to the transformation of the pulse profile [14] and does not contradict to special relativity.

It is known that upon GVD-induced phase modulation of a wave packet, the increase in the wave-front steepness slows down [2, 3]. Because of this, to obtain the maximum steepness, media with a small GVD or long pulses are used, as a rule, for which the formation length of a shock wave is much shorter than the dispersion spread length ($L_0 \ll L_d \equiv \tau_0^2/2|D|$) and, therefore, the influence of GVD over the formation length L_0 of the shock wave can be neglected. On the other hand, the envelope shock wave can be obtained in fibres with nonzero local GVD ($|D(z)| \geqslant 10^{-27}$ s^2 m^{-1}) if the average GVD value is zero over the length L_0:

$$\int_0^{L_0} D(z)\mathrm{d}z \simeq 0. \qquad (14)$$

If this condition is fulfilled, the total GVD-induced phase modulation of the pulse over the length L_0 also vanishes, irrespective of the local values of the parameter $D(z)$ [15]. This condition can be readily realised by using any of the two technological methods available at present. The first method, which is now widely employed in fibreoptic communication links [5–9], is based on the use of spliced fibres with GVDs of opposites signs. The second efficient method, which allows one to realise condition (14), uses a change in the radius of the fibre core. Thus, researchers at the Fiber Optics Research Center, A.M. Prokhorov General Physics Institute, RAS have developed the technology for manufacturing optical fibres with the GVD profile varying over the fibre length due to a change in the fibre cross section [16]. Therefore, it is possible now to obtain virtually any GVD profile, in particular, a profile providing the fulfilment of condition (14) over the length L_0 required for the shock-wave formation.

The simultaneous fulfilment of conditions (11) and (14) allows one to obtain envelope shock waves with large wave-front steepness in active inhomogeneous fibres. In our opinion, the generation of such shock waves is of considerable practical interest. Thus, it was proposed in one of the first methods for compression of laser pulses [11, 14] to use standard optical amplifiers as compressors. However, if a pulse has a flat leading edge, the amplification of the entire front part of the pulse in the amplifier will not only result in its compression but, on the contrary, can considerably broaden the pulse. It is under such conditions that pulses with the superluminal velocity of the envelope maximum were first observed experimentally [17]. This broadening is explained by the fact that pulses generated by real lasers always have a long flat leading edge. For this reason, to perform pulse compression, a device (for example, a Kerr or Pockels cell) is placed in front of the amplifier to cut off the leading edge of the pulse. The method for obtaining shock waves at the leading edge of a pulse considered in the paper allows one to perform pulse compression in an active medium without using any additional cut off devices.

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