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On the super-narrow gamma lines of nuclei in a Bose – Einstein condensate

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***Abstract*. It is shown that due to the quantum coherence of atoms in a Bose – Einstein condensate, the width of gamma lines of the long-lived metastable states of isomeric nuclei in these atoms can be close to the natural radiative width. This opens up the possibility of observing super-narrow gamma lines, which are narrower by a few orders of magnitude than the Mössbauer gamma line.**

Keywords: *Bose – Einstein condensate, narrow nuclear gamma lines.*

It is known that narrow gamma lines with the natural radiative width are observed upon the zero-phonon Mössbauer transitions involving comparatively short-lived metastable states of nuclei with the decay period not exceeding 10 μs (for example, $^{67\mathrm{m}}$Zn) [1]. Exotic exceptions are $^{107\mathrm{m}}$Ag and $^{109\mathrm{m}}$Ag with the decay periods 44 and 40 s, respectively [2]. The assumed possibility of observing narrow nuclear gamma lines in free atoms deeply cooled by laser manipulation methods is even more strictly limited by nanosecond lifetimes [3]. Such a narrowing of gamma lines down to the natural width is achieved in fact by eliminating the action of the thermal motion of atoms.

The width of the Mössbauer line is limited by various perturbations inherent in a condensed state such as fluctuations of the dipole interaction between the magnetic moments of neighbouring nuclei, the interaction of the nuclear moments with the moments of neighbouring paramagnetic atoms, the hyperfine electric quadrupole interaction of a nucleus with an inhomogeneous internal electric field produced by defects in a crystalline lattice, etc. (see, for example, [2]).

In the method of deep cooling of free atoms in gases, which is devoid of the above-mentioned negative properties of a condensed state, the linewidth is limited by the experimentally achievable cooling temperature of an atomic ensemble, which lies in the submicrokelvin range.

These limitations can be eliminated and substantially narrower gamma lines with the natural radiative width can be observed in long-lived nuclear isomers in atoms forming a Bose – Einstein condensate. All the atoms in such a condensate are in the same lower quantum state with the overlapped wave functions (so that the condensate can be conditionally called *a megaatom*) and, hence, it can be expected that the broadening of the gamma line caused by motions of individual atoms will be eliminated or at least strongly suppressed. At the same time, the gamma-line broadening inherent in condensed media can be also almost completely eliminated in a rarefied condensate gas.

Therefore, the quantum coherence of atoms in the condensate can be used for obtaining gamma lines with the natural radiative width. In the ideal case of the complete quantum coherence, all the atoms in the condensate have the same momentum equal to zero. As a result, the inhomogeneous broadening of nuclear gamma lines caused by the random motion of atoms is completely absent and the gamma lines will have the natural radiative width determined by the lifetime of a nuclear state and possible broadening by the finite lifetime of an atom in the condensate. However, it is unlikely that this ideal case can be realised experimentally.

The simplest estimates of the deviation of the condensate from this ideal case can be obtained from the two basic uncertainty relations for (i) the condensate spatial extent and the dispersion of the momentum of atoms and (ii) the lifetime of atoms in the condensate and the dispersion of their energy. The estimates based on the first approach [4] suffer from some intrinsic discrepancy (which was, however, earlier pointed out in [4] assuming that it will not affect qualitatively the result) because, on the one hand, the condensate volume was assumed finite, and on the other, the density of states for atoms in a free space was used (instead of the discrete spectrum of atomic traps). The second approach is briefly discussed below.

The first approach [4] has led to the conclusion that the quantum coherence of atoms is not absolute, and the momentum and energy E_{BEC} of atoms in the condensate in a lower state with the 'zero' momentum have some finite distribution and therefore are not exactly zero, as the energy width ΔE of the lower state. The quantum coherence is characterised by the energy width E_{BEC} of an ensemble of atoms in the condensate. According to [5, 6], this quantity is estimated from the ratio of the concentration n_{sp} of atoms spontaneously filling the lower state in accordance with the equilibrium Bose – Einstein distribution with the zero chemical potential to the concentration n_{BEC} of atoms condensed to the lower state during the phase transition to the condensate:

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$$\Delta E_{\text{BEC}} = \Delta E \, \frac{n_{\text{sp}}}{n_{\text{BEC}} + n_{\text{sp}}} \approx \Delta E \, \frac{n_{\text{sp}}}{n_{\text{BEC}}}. \qquad (1)$$

Approximate equality (1), which is valid for $n_{\text{sp}}/n_{\text{BEC}} \ll 1$, is definitely fulfilled because $(nV)^{1/3} \gg (T/T_0)[1-(T\times T_0^{-1})^{3/2}]^{-1}$, where n is the total concentration of atoms in a gas; T_0 is the critical temperature of the phase transition to the condensate; T is the thermodynamic temperature of an ensemble of atoms; and V is the condensate volume.

According to [4], the energy width of an ensemble of the condensate atoms is

$$\Delta E_{\text{BEC}} \approx \frac{2k_{\text{B}}T}{Vn_{\text{BEC}}} = \frac{2k_{\text{B}}T/Vn}{1-(T/T_0)^{3/2}}. \qquad (2)$$

Here, the estimate of the energy width of the lower state was used:

$$\Delta E = \frac{2^{2/3}\pi^{4/3}\hbar^2}{(2J+1)^{2/3}MV^{2/3}} \approx \frac{3\times10^{-12}}{(2J+1)^{2/3}AV^{2/3}} \qquad (3)$$

where ΔE is measured in microelectronvolts; $n_{\text{sp}} \approx 2k_{\text{B}}T/V\Delta E$; $n_{\text{BEC}} = n[1-(T/T_0)^{3/2}]$; M is the atom mass; J is the angular momentum of the atom; A is the isotopic number; and k_{B} is the Boltzmann constant. In addition, because the energy E of the state has a distribution ΔE, it is assumed equal to $\Delta E/2$. It is also assumed that the difference in the spectra of states of atoms located in a limited volume V and in a free space can only weakly affect the estimate (3).

The energy width ΔE_{BEC} allows the introduction of the effective (not real thermodynamic) temperature of condensate atoms $T_{\text{eff}} = 4T/3Vn_{\text{BEC}}^{-1}$. Note that the estimates of T_{eff} are astonishing: thus, $T_{\text{eff}} \approx 4\times10^{-17}$ and $\Delta E_{\text{BEC}} \approx 3\times10^{-30}$ eV if $T=10^{-8}$ K, $T/T_0 = 0.8$, and $Vn = 10^8$!

The energy width ΔE_{BEC} (2) (and also the effective temperature T_{eff}) of an ensemble of condensate atoms, as a measure of the incompleteness of their quantum coherence, characterises the preserved individual motion of atoms. This motion determines the inhomogeneous broadening of the nuclear gamma line (which has in fact a meaning of the Doppler width) in terms of the effective dispersion Δv of atomic velocities corresponding to the energy width ΔE_{BEC}:

$$\frac{\Delta v}{c} = \frac{\Delta E_{\text{BEC}}}{(Mc^2\Delta E)^{1/2}} = \left(4\,\frac{2J+1}{V^2}\right)^{1/3} \frac{k_{\text{B}}T}{\pi^2 c\hbar n_{\text{BEC}}}$$

$$\approx \left(\frac{2J+1}{V^2}\right)^{1/3} \frac{T}{n_{\text{BEC}}}, \qquad (4)$$

where c is the speed of light and T is measured in kelvins. This gives, for example, the value $\Delta v/c \approx 10^{-18}$ for $n_{\text{BEC}} = 10^{14}$ cm^{-3}, $V = 10^{-6}$ cm^3, and $T = 10^{-8}$ K, i.e., the Doppler width ~ 1 Hz for the transition with the energy 10 keV.

Apart from the Doppler width of the gamma line caused by the energy width ΔE_{BEC} of condensate atoms, other perturbations should be taken into account. Some of them are estimated below.

The above-mentioned mechanisms of line broadening inherent in condensed media are excluded to a great extent in a rarefied condensate. The probability for two neighbouring atoms in a gas with the concentration n to be separated by the distance r is

$$w = \left(\frac{4}{3}\,\pi r^3 n\right)^2. \qquad (5)$$

Then, assuming that the radius r of the mutual effective perturbation of nuclei is, for example, 5×10^{-7} cm, i.e., is twenty times larger than the atom size, we obtain the estimate $w \approx 3\times10^{-7}$ for $n = 10^{15}$ cm^{-3}. Only this small fraction of nuclei can experience an excess broadening, which is manifested as a weak plane pedestal under a narrow unperturbed gamma line, whereas in the case of small condensate volumes $V < (nw)^{-1}$, the mutually perturbed nuclei are absent at all.

The broadening $\Delta\omega_{\text{col}}$ of the gamma line due to a finite transit time Δt_{col} of atoms between two successive collisions in the condensate prevents the observation of the natural linewidth if Δt_{col} is shorter or approximately equal to the metastable-state lifetime. This broadening is estimated, taking into account the gas-dynamic cross section σ_{col} for atomic collisions and the average velocity $\bar{v}$ of the individual motion of atoms, as

$$\frac{\Delta\omega_{\text{col}}}{2\pi} = \frac{1}{\Delta t_{\text{col}}} = \sigma_{\text{col}}\, n^* \bar{v}, \qquad (6)$$

where $n^* = n - n_{\text{BEC}} = n(T/T_0)^{3/2}$ is the concentration of atoms minus the condensate concentration. Assuming that $\sigma_{\text{col}} = 10^{-16}$ cm^2 and the average velocity of deeply cooled atoms is $\bar{v} \sim 0.1$ cm s^{-1}, we obtain for $n = 10^{14}$ cm^{-3} the limiting estimates $\Delta t_{\text{col}} \approx 10^4$ s and $\Delta\omega_{\text{col}}/2\pi \approx 10^{-4}$ Hz.

The inhomogeneous gravitational broadening of the gamma line with the natural width observed in the vertical direction can become noticeable if the product of the metastable-state lifetime τ by the vertical extent ΔL of the nuclear medium is $\tau\Delta L \geqslant 2\pi c^2\hbar/E_{\text{m}}g$ (g is the acceleration of gravity and E_{m} is the metastable-state energy). Thus, this broadening becomes noticeable for $E_{\text{m}} = 10$ keV already for $\tau\Delta L \approx 1$ s cm. This broadening is absent in the perpendicular case, which specifies an exact orientation along the gravitational horizontal.

One can see that, if the experimental conditions are appropriately selected, it is unlikely that the perturbing factors considered above can cause a considerable line broadening. The role of zero-point vibrations of atoms, which can in principle prevent the observation of the natural linewidth for very long-lived isomers, requires a separate analysis.

Of course, the unique properties of narrow gamma lines can be completely manifested only when the observation time is comparable with the metastable-state lifetime. This requires the optimistic assumption that progress in experiments with a Bose–Einstein condensate will provide a substantial increase in its storage time.

Going to the brief discussion of the second approach, it is important to emphasise that the condensate can be considered during its storage as being in the detailed equilibrium state with the rest of the atoms, when the atoms are continuously interchanged between these two phases. Therefore, the real average lifetime Θ of an atom in the condensate until its transition to another phase can substantially differ from the storage time. However, it is this lifetime [which, in particular, can be close to the estimate (6)] that determines the energy width (dispersion) $\Delta E_{\text{BEC}} \approx \hbar/\Theta$ of an ensemble of condensed atoms and, hence, the inhomogeneous broadening of the gamma line. Therefore, the study of the kinetics of atoms in the condensate in order to estimate their average lifetime Θ and a comparison with results [4] is an urgent task.

Note also that the recoil existing during any radiative transition in free nuclei always leads to the mutual shift of the emission and absorption lines by the doubled recoil energy.

An example of a convenient, but probably not optimal nuclide with a boson atom is the ^{81m}Rb isomer with $E_m \approx 86.3$ keV and $\tau \approx 30$ min, for which the value of $T_{eff} \approx 10^{-22}$ K and the gamma-line width of the order of millihertz can be expected for $n_{BEC} = 10^{15}$ cm^{-3} in the volume $V \approx 1$ cm^3.

Therefore, the quantum coherence of atoms in a Bose–Einstein condensate opens up the possibility to eliminate the inhomogeneous broadening of gamma lines of long-lived metastable states of isomeric nuclei of atoms in the condensate and to observe super-narrow gamma lines with the natural width, which are a few orders of magnitude narrower than the Mössbauer gamma line.

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