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On the induced release of the energy of isomeric nuclei in a Bose – Einstein condensate

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Abstract. **It is shown that the asymptotic temporal behaviour of the current value of the stimulated emission cross section leads to a complex dynamics of the radiative release of the nuclear energy of long-lived metastable states induced by the external flux of resonance gamma quanta. The stimulated emission cross section of isomeric nuclei in atoms of a Bose – Einstein condensate becomes sufficient to achieve a comparatively high efficiency of this process, which is manifested in a considerable increase in the decay rate of metastable states.**

Keywords: *isomeric nuclei, Bose – Einstein condensate.*

1. The possibility of releasing the energy of nuclear isomers in stimulated transitions from long-lived metastable states with strongly forbidden spontaneous decay is based on the radical suppression of the inhomogeneous broadening of emission lines of isomeric nuclei in the atoms of a Bose – Einstein condensate (BEC) [1, 2]. Such an atomic ensemble with the overlapped wave functions of individual boson atoms can be conditionally called a *megaatom*, in which the differences between the states of atoms become minimal because of their mutual quantum coherence. This eliminates the main factors producing the inhomogeneous (in particular, Doppler) line broadening. Note that the narrowing of lines down to their natural width is accompanied by the increase both in the stimulated emission cross section upon downward transitions and the absorption cross section upon upward transitions up to the maximum value equal approximately to the square of the emission wavelength and independent of the transition matrix element and the degree of transition forbiddeness.

The potential possibility of the induced release of the energy of isomers during stimulated emission or absorption of gamma quanta in transitions from metastable states requires the analysis of the development of this process in time, which proves to be significant because of a long lifetime of the states and a small natural radiative linewidth of transitions. In this case, the temporal behaviour is determined, on the one hand, by the kinetics of the transition cross section and, on the other, by the asymptotic establishment of the spectrum of the acting electromagnetic wave from the infinite width and zero amplitude at the zero instant up to a finite stationary value at infinity [2]. The latter dependence follows in fact from the classical Fourier uncertainty relation (see, for example, [3]). Such analysis was performed, in particular, in [4] for the problem of the temporal development of the amplification of gamma rays in Mössbauer nuclei in a crystal, which substantially differs from the problem of induced release of the nuclear energy in a BEC.

If the induced release of the energy of isomeric nuclei is treated as the stimulated decay of a great part of metastable states during the time that is much shorter than their spontaneous decay time τ_γ, it is obvious that this energy release can be achieved when the stimulating flux of gamma quanta has a sufficiently high density $\Phi > 8\pi/(\lambda^2\tau_\gamma)$, which leads to rather high numerical estimates due to a small wavelength λ of gamma rays. This is a direct consequence of the proportionality between the required density of the stimulating gamma-ray flux Φ and the density of radiation oscillators of a free space. The requirements to Φ can be alleviated to some extent by using high-Q resonators; however, this is not simple in the gamma ray region.

2. Consider a condensate of metastable isomers at the concentration $n^*_{\rm BEC}$ irradiated by gamma quanta with the flux density Φ, the energy $\hbar\omega$, and the wavelength $\lambda = 2\pi c/\omega$. The variation of Φ during the propagation of gamma quanta in the condensate along the coordinate z is described by a standard equation

$$\frac{{\rm d}\Phi}{{\rm d}z} = [\sigma_{\rm st}(z,t)n^*_{\rm BEC}(z,t) - \chi n]\Phi, \quad (1)$$

where $\sigma_{\rm st}$ is the current value of the stimulated emission cross section depending on the coordinate z and time t; n is the total concentration of atoms; and χ is the total cross section of photon losses of all types. Because of the existence of a hidden population inversion in the rarefied condensate, Eqn (1) does not contain the term of resonance absorption of gamma quanta by nuclei, and the quantities Φ, $n^*_{\rm BEC}$ and $\sigma_{\rm st}$ depend both on the longitudinal coordinate z and time t.

In the first-order perturbation theory, the amplitude of downward transition from a metastable state stimulated by the wave field is [5]

$$a = -\frac{\rm i}{\hbar}\int_0^t \mu A \exp[-{\rm i}(\omega - \omega_0)t]{\rm d}t, \quad (2)$$

where μA is the perturbation matrix element; $A = A(t)$ is the time-dependent amplitude of the vector potential of the

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electromagnetic wave; $\mu = \text{const}$ is the matrix element of the transition from the metastable state corresponding to the unit amplitude A; $\hbar\omega_0 = E_{\text{tr}} - E_{\text{rec}}$ is the transition energy E_{tr} minus the recoil energy of a nucleus accompanying the emission transition,

$$E_{\text{rec}} = \frac{E_{\text{tr}}^2}{2Mc^2}; \qquad (3)$$

and M is the nucleus mass. Integration in (2) is performed from the instant $t = 0$ of incidence of the gamma-ray flux on a one-dimensional nuclear medium at the point $z = 0$.

According to the principle of detailed balancing, the constant μ is uniquely related to the natural radiative lifetime τ_γ of the metastable state:

$$|\mu|^2 = \frac{\hbar\lambda}{8\pi\tau_\gamma(2J_0 + 1)}, \qquad (4)$$

where J_0 is the angular momentum of the lower state of the transition. Therefore, taking into account the dependence of Φ on A

$$\Phi = \frac{2\pi c}{\lambda^2}\frac{A^2}{\hbar\omega} = \frac{A^2}{\hbar\lambda} \qquad (5)$$

we can rewrite (2) in the form

$$a(t) = -\frac{\mathrm{i}\lambda}{[8\pi\tau_\gamma(2J_0 + 1)]^{1/2}}\int_0^t \Phi^{1/2}\exp[-(\omega - \omega_0)t]\mathrm{d}t. \qquad (6)$$

In the case of a sufficiently exact resonance and (or) a relatively short time of the process, when

$$|\omega - \omega_0|t \ll 1 \text{ and (or) } t \ll \tau_\gamma, \qquad (7)$$

the exponential in (6) turns to unity, i.e.,

$$a(t) \approx -\frac{\mathrm{i}\lambda}{[8\pi\tau_\gamma(2J_0 + 1)]^{1/2}}\int_0^t \Phi^{1/2}\mathrm{d}t. \qquad (8)$$

Condition (7) means that the incident radiation is highly monochromatic and (or) its duration is limited, although the absolute value of the latter can be not small because of a long lifetime of metastable states. The limited time of the interaction process means in fact that approximation (8) can be applied only to the initial stage of stimulated emission. It should be emphasised, however, that because we consider the release of the energy of metastable states of isomers in the form of stimulated gamma rays for the time that is considerably shorter than τ_γ, it is this initial stage of the process that is of special interest.

3. The stimulated emission cross section is, by definition,

$$\sigma_{\text{st}} = \Phi^{-1}\frac{\mathrm{d}}{\mathrm{d}t}|a(t)|^2, \qquad (9)$$

i.e., taking (8) into account, we have

$$\sigma_{\text{st}} \approx \frac{\lambda^2}{8\pi\tau_\gamma\Phi}\frac{\mathrm{d}}{\mathrm{d}t}\left|\int_0^t \Phi^{1/2}\mathrm{d}t\right|^2 = \frac{B}{\Phi^{1/2}}\int_0^t \Phi^{1/2}\mathrm{d}t, \qquad (10)$$

where

$$B \equiv \lambda^2/(4\pi\tau_\gamma) \qquad (11)$$

and the factor $2J_0 + 1$ is absent because σ_{st} is proportional to the statistical weight of the final lower state.

In addition, we assume for simplicity that the line is not subjected to any broadening exceeding the natural radiative linewidth, i.e., that the ratio β of the natural linewidth τ_γ^{-1} to the total linewidth $\Delta\omega_{\text{tot}}$ (including the inhomogeneous linewidth) is unity,

$$\beta = \frac{2\pi}{\tau_\gamma\Delta\omega_{\text{tot}}(1 + \alpha)} = 1, \qquad (12)$$

and the coefficient of internal electron conversion is $\alpha \ll 1$. In the case of long-lived metastable states, this assumption requires, according to [2], the formation of a condensate with a large total number of atoms $n_{\text{BEC}}V$ in a large volume V,

$$n_{\text{BEC}}V \geqslant (2J + 1)^{1/3}(4V)^{1/3}\frac{k_{\text{B}}T}{\pi^2\hbar\lambda}\tau_\gamma, \qquad (13)$$

where J is the angular momentum of an atom; T is the absolute temperature; and k_{B} is the Boltzmann constant. Thus, $n_{\text{BEC}}V \approx 1.3\times10^{16}(2J + 1)^{1/3}$ for $\lambda = 1.5\times10^{-9}$ cm, $T = 10^{-6}$ K, $\tau_\gamma = 3600$ s and $V = 13$ cm^3. In addition, the storage time of the condensate should be at least close to τ_γ.

The required values of $n_{\text{BEC}}V$ and the storage time estimated above differ by many orders of magnitude from real experimental values achieved at present. Moreover, no theoretical predictions exist about the possibility of any experimental progress in this direction. Therefore, it should be emphasised that our consideration is based, on the one hand, on optimism to a great extent and, on the other, on the hope that it will stimulate the relevant theoretical and experimental investigations of the BEC.

In a simple case, when the gamma-ray flux (5) has the form of a pulse with the leading edge raising as

$$\Phi = \Phi_0\left(\frac{t}{t_0}\right)^{2m} \qquad (14)$$

($\Phi_0 = \text{const}$, $t_0 = \text{const}$, $m = \text{const}$), the stimulated emission cross section increases as

$$\sigma_{\text{st}} = \frac{Bt}{m + 1}. \qquad (15)$$

Therefore, at the initial stage of the process, the cross section (15) increases linearly with time, beginning with $\sigma_{\text{st}} = 0$ for $t = 0$, irrespective of the values of parameters t_0 and Φ_0; the faster, the smaller m. Therefore, we will consider below mainly the case $m = 0$.

As σ_{st} increases, the concentration $n^*_{\text{BEC}}(t)$ of isomeric nuclei decreases according to the equation

$$\frac{\mathrm{d}n^*_{\text{BEC}}}{\mathrm{d}t} = -n^*_{\text{BEC}}\left[\frac{1}{\tau_\gamma} + \sigma_{\text{st}}(t)\Phi(t)\right]$$
$$= -n^*_{\text{BEC}}\left(\frac{1}{\tau_\gamma} + \frac{B\Phi_0}{m + 1}\frac{t^{2m+1}}{t_0^{2m}}\right), \qquad (16)$$

which has the solution

$$n^*_{\text{BEC}}(t) = n^*_{\text{BEC}}(t = 0)\times$$

$$\times \exp\left[-\left(\frac{1}{\tau_\gamma}+\frac{B\Phi_0}{2(m+1)^2}\frac{t^{2m+1}}{t_0^{2m}}\right)t\right], \quad (17)$$

which is caused both by the spontaneous decay of metastable states and the action of stimulating gamma quanta with the flux density $\Phi(t)$ (14).

4. Because the quantities Φ, $n^*_{\rm BEC}$ and $\sigma_{\rm st}$ depend on the longitudinal coordinate z and time t, it is useful first to analyse Eqn (1) for the fixed value $z=0$, i.e., at the input of the external gamma-ray flux to a nuclear medium, when its density Φ is equal to the input value $\Phi_{\rm in}$.

Due to a gradual increase of the cross section $\sigma_{\rm st}$, beginning with $\sigma_{\rm st}=0$ for $t=0$, the right-hand side of (1) and the derivative $d\Phi/dz$ prove to be first negative up to the start instant $t=t_{\rm del}$, when

$$\sigma_{\rm st}(t_{\rm del}) \geqslant \chi\frac{n}{n^*_{\rm BEC}(t_{\rm del})} \quad (z=0), \quad (18)$$

where $n^*_{\rm BEC}(t_{\rm del})$ is given by solution (17). Taking into account that the expected delay time $t_{\rm del} \ll \tau_\gamma$, considering inequality (7), and also that the exponent in (17) is substantially lower than unity for moderate gamma-ray fluxes Φ, we can rewrite (17) as

$$n^*_{\rm BEC}(t) \approx n^*_{\rm BEC}(t=0)$$
$$\times\left[1-\left(\frac{1}{\tau_\gamma}+\frac{B\Phi_0}{2(m+1)^2}\frac{t^{2m+1}}{t_0^{2m}}\right)t\right]. \quad (19)$$

In this case, the delay time $t_{\rm del}$ of the amplification onset is determined from equality (18) by the condition

$$t_{\rm del}^{2m+2} - 2\frac{n^*_{\rm BEC}(t=0)}{n}\frac{t_0^{2m}}{\chi\Phi_0}(m+1)^2 t_{\rm del}$$
$$\times\left[1-4\pi\frac{\chi}{\lambda^2}\frac{n}{n^*_{\rm BEC}(t=0)}\right] + 2\frac{t_0^{2m}}{B\Phi_0}(m+1)^2 = 0, \quad (20)$$

where the second term in brackets in the second term of (20), which is considerably smaller than unity, can be omitted. In a simple case of $m=0$, this gives

$$\frac{t_{\rm del}}{\tau_\gamma} \approx 4\pi\frac{\chi}{\lambda^2}\frac{n}{n^*_{\rm BEC}(t=0)} \ll 1, \quad (21)$$

which can be directly obtained from (15) and (18).

By neglecting only the stimulated component of the decay of metastable states and retaining the influence of spontaneous decay in (19), we obtain from (17) the equation for determining the delay time

$$t_{\rm del}\exp\left(-\frac{t_{\rm del}}{\tau_\gamma}\right) \approx t_{\rm del}\left(1-\frac{t_{\rm del}}{\tau_\gamma}\right) \geqslant \frac{\chi n(m+1)}{Bn^*_{\rm BEC}(t=0)}, \quad (22)$$

where the approximate equality corresponds to condition (7). The approximate left-hand side of (22) increases from zero, achieving the maximum $\tau_\gamma/4$ for $t_{\rm del}=\tau_\gamma/2$. Therefore, inequality (22) can be satisfied only when

$$\frac{n^*_{\rm BEC}(t=0)}{n} \geqslant 16\pi\frac{\chi}{\lambda^2}(m+1). \quad (23)$$

Thus, for $\chi/\lambda^2 = 2\times10^{-3}$, inequality (23) is satisfied if $n^*_{\rm BEC}(t=0)/n \geqslant 0.1(m+1)$. The delay time $t_{\rm del}<\tau_\gamma$ can be calculated from (22) for the specified values of the right-hand side.

A critical situation appears if inequality (23) has the opposite sign, when the positive derivative $d\Phi/dz$ (1) and amplification cannot be realised at all; thus, equality (23) determines the critical value of the ratio $[n^*_{\rm BEC}(t=0)/n]_{\rm crit}$. Because the maximum (although really inaccessible) concentration ratio is $[n^*_{\rm BEC}(t=0)/n]_{\rm crit}=1$, the condition

$$\left(\frac{\lambda^2}{\chi}\right)_{\rm abs} \geqslant 50(m+1) \quad (24)$$

is the absolute criterion for the possibility of achieving amplification. It also follows from (22) the equivalent dimensionless critical condition

$$[\varepsilon_0(1-\theta^{3/2})]_{\rm crit} \geqslant 16\pi\frac{\chi}{\lambda^2}(m+1), \quad (25)$$

where $\varepsilon_0 = n^*_{\rm BEC}(t=0)/n_{\rm BEC} \leqslant 1$ is the initial ratio of the concentration of metastable nuclei in the condensate to the total condensate concentration; $\theta = T/T_0 < 1$ is the ratio of the thermodynamic temperature T of atoms to the critical temperature of their phase transition to the condensate:

$$T_0 = 3.3\frac{\hbar^2}{k_{\rm B}M}\left(\frac{n}{2J+1}\right)^{2/3}$$
$$\approx \frac{1.6\times10^{-14}}{A_{\rm i}}\left(\frac{n}{2J+1}\right)^{2/3} \quad (26)$$

(here, $A_{\rm i}$ is the isotopic number).

5. The amplification of the gamma-ray flux, described in (1) by the first term in the right-hand side of the equation, changes due to the increase in the cross section $\sigma_{\rm st}(t)$ and the decrease in the concentration $n^*_{\rm BEC}(z,t)$ of metastable nuclei because of their spontaneous decay and stimulated transition to the lower state. The competition of these processes determines the development of amplification after the stimulated emission cross section has achieved the start value (18) for $t=t_{\rm del}$.

A change in the concentration $n^*_{\rm BEC}$ near $z=0$ during stimulated transitions is described by the integral of Eqn (16) in the range from $t=t_{\rm del}$ to $t>t_{\rm del}$ with the initial condition $n^*_{\rm BEC}=n^*_{\rm BEC}(t=t_{\rm del})$:

$$n^*_{\rm BEC}(t) = n^*_{\rm BEC}(t_{\rm del})\exp\left[-\left(\frac{t-t_{\rm del}}{\tau_\gamma}+\frac{B\Phi_0}{2(m+1)^2}\right.\right.$$
$$\left.\left.\times\frac{t^{2m+2}-t_{\rm del}^{2m+2}}{t_0^{2m}}\right)\right], \quad (27)$$

where $t_{\rm del}$ is determined from (20) and $t-t_{\rm del} \ll \tau_\gamma$.

Then, taking (27) into account, Eqn (1) near $z=0$ can be rewritten as

$$\frac{1}{\Phi}\frac{d\Phi}{dz} = n\chi\left\{\frac{t}{t_{\rm del}}\exp\left[-\frac{t-t_{\rm del}}{\tau_\gamma}-\frac{B\Phi_0}{2(m+1)^2 t_0^{2m}}\right.\right.$$
$$\left.\left.\times\left(t^{2m+2}-t_{\rm del}^{2m+2}\right)\right]-1\right\}, \quad (28)$$

where the value of $n\chi$ is determined by inequalities (15) and (18). The right-hand side of (28) is zero at $t=t_{\rm del}$ and increases for $t > t_{\rm del}$ (when the amplification of the gamma-ray flux begins) only if

$$\Phi_0 < \frac{m+1}{B}\frac{t_0^{2m}}{t_{\rm del}^{2m+2}}\left(1-\frac{t_{\rm del}}{\tau_\gamma}\right) \approx \frac{m+1}{B}\frac{t_0^{2m}}{t_{\rm del}^{2m+2}}, \quad (29)$$

achieving the maximum

$$\frac{1}{\Phi}\frac{{\rm d}\Phi}{{\rm d}z}\bigg|_{\max} = n\chi\left\{\frac{t_1}{t_{\rm del}}\exp\left[-\frac{t_1}{\tau_\gamma}\left(1-\frac{t_{\rm del}}{t_1}\right)\right.\right.$$
$$\left.\left. -\frac{B\Phi_0 t_1^{2m+2}}{2(m+1)^2 t_0^{2m}}\left(1-\left(\frac{t_{\rm del}}{t_1}\right)^{2m+2}\right)\right]-1\right\}$$
$$= n\chi\left\{\frac{t_1}{t_{\rm del}}\exp\left[-\frac{t_1}{\tau_\gamma}\left(1-\frac{t_{\rm del}}{t_1}\right)-\frac{1}{2(m+1)}\right.\right.$$
$$\left.\left.\times\left(1-\frac{t_1}{\tau_\gamma}\right)\left(1-\left(\frac{t_{\rm del}}{t_1}\right)^{2m+2}\right)\right]-1\right\} \quad (30)$$

at $t = t_1$, which is determined from the equation

$$t_1^{2m+2} - \frac{m+1}{B\Phi_0}t_0^{2m}\left(1-\frac{t_1}{\tau_\gamma}\right) = 0. \quad (31)$$

For $m = 0$, these expressions are simplified and take the form

$$\frac{1}{\Phi}\frac{{\rm d}\Phi}{{\rm d}z}\bigg|_{\max} = n\chi\left\{\frac{t_1}{t_{\rm del}}\exp\left[-\frac{t_1}{\tau_\gamma}\left(1-\frac{t_{\rm del}}{t_1}\right)\right.\right.$$
$$\left.\left.-\frac{1}{2}\left(1-\frac{t_1}{\tau_\gamma}\right)\left(1-\left(\frac{t_{\rm del}}{t_1}\right)^2\right)\right]-1\right\}, \quad (32)$$

$$t_1 = (B\Phi_0)^{-1/2}\{[1+1/(4B\Phi_0\tau_\gamma^2)]^{1/2} - 1/[2(B\Phi_0)^{1/2}\tau_\gamma]\}. \quad (33)$$

The amplification ceases when the right-hand side of (28) again vanishes for $t = t_2 > t_1 > t_{\rm del}$. In this case,

$$\exp\left\{\frac{t_2}{\tau_\gamma}\left(1-\frac{t_{\rm del}}{t_2}\right)+\frac{B\Phi_0 t_2^{2m+2}}{2(m+1)^2 t_0^{2m}}\left[1-\left(\frac{t_{\rm del}}{t_2}\right)^{2m+2}\right]\right\}$$
$$= \exp\left\{\frac{t_2}{\tau_\gamma}\left(1-\frac{t_{\rm del}}{t_2}\right)+\frac{(t_2/t_1)^{2m+2}}{2(m+1)}\left(1-\frac{t_1}{\tau_\gamma}\right)\right.$$
$$\left.\times\left[1-\left(\frac{t_{\rm del}}{t_2}\right)^{2m+2}\right]\right\} = \frac{t_2}{t_{\rm del}}. \quad (34)$$

Let us present some numerical estimates. Let $\lambda = 1.5\times10^{-9}$ cm (energy 83 keV), $\chi/\lambda^2 = 10^{-3}$, $\tau_\gamma = 3600$ s, and $B \approx 5\times10^{-23}$ cm^2 s^{-1}. Then, for $m = 0$, $\varepsilon_0 \equiv n^*_{\rm BEC}(t=0)n^{-1} = 0.5$ and $\Phi_0 = 5\times10^{17}$ cm^{-2} s^{-1}, we have, according to (20), $t_{\rm del} \approx 0.032\tau_\gamma = 115$ s, and according to (29), $\Phi_0 = 5\times10^{17} < 1.5\times10^{18}$ cm^{-2} s^{-1}. It follows from equality (31) that $t_1 \approx 0.054\tau_\gamma \approx 195$ s, according to (32), the gain in the maximum is $\Phi^{-1}({\rm d}\Phi/{\rm d}z)_{\max} \approx 1.2n\chi$, and according to (34), we have $t_2 \approx 0.075\tau_\gamma \approx 270$ s and $\Delta t = t_2 - t_{\rm del} \approx 0.042\tau_\gamma \approx 155$ s.

These estimates show that the absolute value of the gain $\Phi^{-1}|{\rm d}\Phi/{\rm d}z|$ is small during the entire time of the action of the stimulating gamma-ray flux, beginning with $|-n\chi|$ for $t = 0$. Near $z = 0$, this parameter is $\sim n\chi$ in the interval $0 < t < t_2$, i.e., hardly exceeds 10^{-6} cm^{-1}. Therefore, when the start condition of the type (18) is satisfied, the stimulated emission and amplification of the gamma-ray flux in the nuclear medium for $z > 0$ proceed virtually in the same way as on the interval near $z = 0$ considered above. The coordinate $z > 0$ for which this condition is fulfilled propagates in the medium at the speed of light. This quantitative similarity is explained by a low gain estimated above. As a result, the gamma-ray flux density $\Phi(z, t)$ in the nuclear medium only slightly differs from the input density $\Phi_{\rm in}$, while the output flux density $\Phi_{\rm out}$ for $z = L$ virtually coincides with the initial density. Therefore, the output gamma-ray flux can be again used to stimulate the energy release in a new fragment of the isomeric medium.

6. Within the time t_2 after the initial instant $t = 0$, the concentration of metastable nuclei near $z = 0$ decreases according to (17) due to spontaneous emission and under the action of the stimulating gamma quanta. In the absence of the latter, the decrease in the concentration is described only by the first term in (17). Therefore, the efficiency of stimulated emission achieved to the instant $t = t_2$ is characterised by the ratio of the concentration decrease in the presence of stimulating gamma quanta and without them:

$$G = \left\{1-\exp\left[-\left(\frac{1}{\tau_\gamma}+\frac{B\Phi_0}{2(m+1)^2}\frac{t_2^{2m+1}}{t_0^{2m}}\right)t_2\right]\right\}$$
$$\times\left[1-\exp\left(-\frac{t_2}{\tau_\gamma}\right)\right]^{-1}. \quad (35)$$

The acceleration of the decay of metastable states can be characterised by the ratio of the effective time $\tau_{\rm eff}$ of stimulated emission to τ_γ:

$$\frac{\tau_{\rm eff}}{\tau_\gamma} = \left[1+\frac{B\Phi_0}{2(m+1)^2}\frac{\tau_\gamma t_2^{2m+1}}{t_0^{2m}}\right]^{-1}. \quad (36)$$

For numerical estimates presented after expression (34), this gives $G \approx 8.4$ and $\tau_{\rm eff}/\tau_\gamma \approx 0.075$ for the time $t_2 = 0.075 \times\tau_\gamma = 270$ s. There is no sense to introduce the energy gain factor because, as mentioned above, the process occurs in fact without any consumption of stimulating gamma quanta. A quite noticeable decrease in the concentration of metastable nuclei, which occurs much faster than in the case of the spontaneous decay, demonstrates the acceptable efficiency of the isomer-energy release due to stimulated emission. This process also continues after the time $t = t_2$; however, because conditions (7) are violated, it cannot be estimated by the method developed above.

The noticeable values of G and $\tau_{\rm eff}/\tau_\gamma$ are achieved, despite very low gain $\Phi^{-1}({\rm d}\Phi/{\rm d}z)$ of the gamma-ray flux, due to prolonged stimulating action of gamma quanta on the population of metastable states during the time t_2, which is many orders of magnitude longer than the transit time of gamma quanta through a nuclear medium of length L. The ratio of these times is $t_2/(L/c)$ and for $L = 1$ cm and the above numerical estimates it amounts to $\sim10^{12}$.

Because the gamma-ray flux density in the medium is invariable, $\Phi(z) \approx \Phi_{\rm in} \approx \Phi_{\rm out}$, the results concerning the relative decrease in the concentration of metastable states G (35) and the decrease in the decay time of metastable states (36) near $z = 0$ are also valid for the entire nuclear medium of the unit cross section in the interval $0 \leqslant z \leqslant L$, i.e., for a total number of emitting isomeric nuclei $\Delta N^*_{\rm BEC} =$

$n^*_{\rm BEC}(t=0)[1-n^*_{\rm BEC}(t=t_2)]L$. Thus, for $n^*_{\rm BEC}(t=0)=10^{14}$ cm^{-3} and numerical estimates presented above, we have $\Delta N^*_{\rm BEC}/L \approx 0.63\times10^{14}$ cm^{-3}, and the specific energy released in the form of gamma quanta is ~ 0.83 J cm^{-3}.

It is important to note that the stimulating gamma-ray flux should have a comparatively high brightness (for the same estimates, $\Phi_{\rm in}\tau_\gamma \approx 1.8\times10^{21}$ phot cm^{-2} or ~ 25 phot cm^{-2} in the frequency band equal to 0.1 % of the gamma quantum frequency.

An example (by no means, optimal) of a nuclide suitable for using in the method of induced release of the energy of isomeric nuclei is $^{135}_{55}$Cs with the energy and lifetime of the metastable state $E_{\rm m}=846$ keV and $\tau_\gamma=53$ min, respectively. The atomic properties of this boson isotope of cesium are convenient for formation of a BEC with the help of an optical laser.

7. Because of adopted considerable simplifying assumptions, the analysis performed above describes the amplification dynamics of gamma quanta in isomeric nuclei in the BEC only in general terms. In particular, the conclusion made in section 2 and expression (9) for the stimulated cross section are valid only in the dipole approximation, which obviously cannot be applied to high-multiplicity transitions. However, the analysis has revealed substantial features of the process such as the delay of the initial amplification, the presence of the start and critical conditions for the appearance of amplification, a virtual invariability of the stimulating gamma-ray flux density in a nuclear medium, i.e., almost complete transparency of the medium, as well as a rather high efficiency of the release of the energy of metastable states, the increase in their decay rate, etc.

It should be emphasised once more that all the problems of radiative gamma-ray transitions in long-lived isomeric nuclei considered in the paper can acquire a realistic nature only if the storage time of a Bose–Einstein condensate is long enough to be comparable with the lifetime of a metastable isomer state, which is far from the experimental data obtained at present. Unfortunately, the BEC storage time is many orders of magnitude shorter than the required time. In addition, the question of the BEC degradation rate caused by radiative processes in a nuclear medium remains open. Therefore, both theoretical and especially experimental studies of the methods for increasing the BEC storage time are of great importance.

No less important (along with further investigations of dynamics) is the study of some other features of the BEC behaviour under the conditions of intense radiative transitions in BEC nuclei, in particular, collective BEC excitations and the propagation of waves in the BEC [6–8], which probably can improve the estimates [2] of the inhomogeneous broadening of gamma lines.

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