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Absorption of an electromagnetic wave by an inhomogeneous cylindrical particle

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Abstract. **The absorption cross section is calculated for an electromagnetic wave whose field is directed along the symmetry axis of an inhomogeneous cylindrical particle. The general case of an arbitrary ratio of radii of a dielectric nucleus and a particle is considered. The condition of diffuse reflection of electrons from the internal and external surfaces of the metal particle layer is used as the boundary condition. The limiting cases are also analysed and the results are discussed.**

Keywords: *metal particle, electromagnetic radiation, absorption cross section.*

1. Introduction

The electromagnetic properties of small metal particles can substantially differ from those of bulky metal samples [1]. If the linear transverse size R of a metal sample is approximately equal to or smaller than the electron mean free path Λ ($R < \Lambda$), the interaction of electrons with the metal sample surface begins to affect noticeably their response to an external electromagnetic field. As a result, a metal particle acquires special optical properties. Therefore, when the condition $R < \Lambda$ is fulfilled, one of the basic optical characteristics, the absorption cross section, reveals a nontrivial dependence on the ratio R/Λ. The electron mean free path in good metal conductors (aluminium, copper, silver, etc.) at room temperature lies within 10 – 100 nm. The size of metal particles studied in experiments amounts to a few nanometres, i.e., the condition $R < \Lambda$ is valid.

We will describe the electron response to the external electromagnetic field, taking into account the interaction of electrons with a sample surface, by using the standard kinetic theory of conduction electrons in a metal [2]. In this case, no restrictions are imposed on the relation between the electron mean free path and a sample size.

The equations of macroscopic dynamics are valid only for 'bulky' samples, when $R \gg \Lambda$. Therefore, the known Mie theorem, which describes the interaction of an electromagnetic wave with metal bodies within the framework of microscopic electrodynamics, cannot be used to describe this size effect.

The theory of the interaction of electromagnetic radiation with a spherical particle was developed in papers [3, 4]. Earlier [5, 6], the result was obtained in the limiting case $R \ll \Lambda$ for low frequencies (far-IR range), which coincides with that presented in [3]. The authors of the above papers used the approach based on the solution of the kinetic Boltzmann equation for conduction electrons in a metal. The alternative approach to this problem was proposed and developed in papers [7, 8].

Recently, interest was aroused in the problem of interaction of electromagnetic radiation with nonspherical particles [9]. A number of papers [10 – 13] was devoted to the description of the interaction of electromagnetic radiation with a cylindrical particle. Note also papers in which an attempt was made to take quantum-mechanical effects into account in this problem, which is especially important at low temperatures [14, 15]. In [3 – 6] and [10 – 13], only magnetic dipole absorption of radiation by small metal particles was described. All the papers mentioned above considered only homogeneous particles, i.e., the internal structure of absorbing particles was not discussed.

However, recently the experimental studies of particles with a complex internal structure were reported [16, 17]. Such particles consist of a dielectric (or metal) nucleus surrounded by a metal shell, which naturally affects their optical properties. The importance of studying particles with a complex internal structure is pointed out, in particular, in paper [18]. Note that the problem of absorption of an electromagnetic wave by small metal particles is of current interest, for example, for the description of the interaction of laser radiation with matter.

In this paper, we calculated by the kinetic method the distribution function describing a linear response of conduction electrons in an inhomogeneous cylindrical particle (metal particle with a dielectric nucleus) to the alternating electric field of a plane electromagnetic wave. This distribution function allows us to calculate the dependence of the absorption cross section on the particle radius and frequency, as well as on the ratio of radii of the nucleus and particle. The important case of a low-frequency external field and low-frequency collisions between electrons in a metal layer is separately discussed.

2. Mathematical model

Consider a cylindrical particle of length L consisting of a dielectric nucleus of radius R_1 surrounded by a non-

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magnetic metal shell of radius R_2 (we assume that $L \gg R_2$), which is placed in the field of a plane electromagnetic wave of frequency ω, which is much lower than the plasma resonance frequency ω_p in metals ($\omega_p \sim 10^{16}$ s^{-1}). We assume that the electric-field direction in the electromagnetic wave coincides with the axis of the inhomogeneous cylinder. The particle is assumed small, which means that the condition $R_2 \ll 2\pi c/\omega$ (c is the speed of light in vacuum) is valid. The inhomogeneity of the external field of the wave and the skin effect are neglected (it is assumed that R_2 is smaller than the skin layer depth). In the frequency range under study and the given orientation of the electric field, the contribution of currents of the dipole electric polarisation dominates over the contribution from eddy currents, which are induced by the external magnetic field of the wave [3]. Therefore, the action of the external magnetic field of the wave is neglected.

For a cylinder long enough, the electric field of the wave in a greater part of the cylinder volume remains unscreened. To estimate parameters at which such a regime takes place, we consider the known solution for a prolate ellipsoid in an electric field [19], assuming that a sufficiently long cylinder can be approximated by a prolate ellipsoid. We will use the results obtained in paper [19], where the electric field strength was calculated inside a prolate ellipsoid of revolution (in fact, an infinite cylinder) with semiaxes a, b, and d ($a > b = d$) placed in an external homogeneous electric field directed along the symmetry axis of the ellipsoid:

$$E_{\text{int}} = \frac{E_{\text{ext}}}{1 + (\varepsilon_{\text{int}} - 1)s(\beta)}, \quad s(\beta) = \frac{1-\beta^2}{2\beta^3}\left[\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta\right],$$

where E_{ext} is the external electric field strength; E_{int} is the electric field strength in the ellipsoid; ε_{int} is the permittivity of the ellipsoid; $s(\beta)$ is the coefficient depending on the ellipsoid eccentricity β [$\beta = (1 - b^2/a^2)^{1/2}$]. If screening is absent, then $E_{\text{int}} \approx E_{\text{ext}}$ and therefore $1 + (\varepsilon_{\text{int}} - 1)s(\beta) \approx 1$, which is possible for $|\varepsilon_{\text{int}} s(\beta)| \ll 1$ (unity in the parentheses can be neglected because the permittivity of metals is very large).

By using the Drude formulas for the frequency dependence of the permittivity $\varepsilon(\omega)$ and conductivity $\Sigma(\omega)$ of a metal [20] (we assume that the external field frequency is low compared to the collision frequency between electrons inside a particle, i.e., $\omega\tau \ll 1$)

$$\varepsilon(\omega) = 1 + \mathrm{i}\left[\frac{4\pi\Sigma(\omega)}{\omega}\right], \quad \Sigma(\omega) = \frac{\Sigma(0)}{1 - \mathrm{i}\omega\tau}, \quad \Sigma(0) = e^2 n \frac{\tau}{m}$$

(where e and m are the electron charge and effective mass in a metal, n is the concentration of conduction electrons, and τ is the electron relaxation time) and the definition of the eccentricity (the semiaxes b and a of a prolate ellipsoid approximating an infinite cylinder are treated as the radius and half-length of the cylinder: $b = R$, $a = L/2$; for a prolate cylinder, $\beta \to 1$), we obtain by the method of successive approximations the required limiting relation between the radius and length of the particle ($\Gamma = R/L$):

$$\Gamma \ll \left[\frac{\omega}{2\pi\Sigma(0)}\right]^{1/2} \Big/ \left\{\ln\left[\frac{4\pi\Sigma(0)}{\omega}\right]\right\}^{1/2}.$$

An estimate by this expression for an external electric field with the frequency, for example, 10^9 s^{-1} shows that in this case the particle length should exceed its radius approximately by a factor of four (in the case of high-frequency external fields, the screening is absent in fact).

We also used the accepted physical assumptions according to which the conduction electrons in a metal shell are treated as a degenerate Fermi gas and their response to an external alternate field is described by the Boltzmann equation. The boundary conditions take into account that electrons experience diffusion reflection from the internal surface of the metal shell and the nucleus surface.

Absorption of the electromagnetic wave energy by an inhomogeneous cylindrical particle can be described as follows. The homogeneous time-periodic electric field of the wave

$$\boldsymbol{E} = \boldsymbol{E}_0 \exp(-\mathrm{i}\omega t) \tag{1}$$

acts on the conduction electrons in the particle and causes the deviation f_1 of their distribution function f from the equilibrium Fermi function f_0:

$$f(\boldsymbol{r}, \boldsymbol{v}) = f_0(\mathscr{E}) + f_1(\boldsymbol{r}, \boldsymbol{v}), \quad \mathscr{E} = \frac{m\boldsymbol{v}^2}{2},$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are the electron radius vector (the coordinate origin is located on the particle axis) and velocity, respectively. This results in the appearance of a high-frequency current with the density

$$\boldsymbol{j} = e\int \boldsymbol{v} f \frac{2d^3(m v)}{h^3} = 2e\left(\frac{m}{h}\right)^3 \int \boldsymbol{v} f_1 d^3 v \tag{2}$$

(where h is Planck's constant) and in the energy dissipation in the particle volume. The energy dissipated per unit time is [19]

$$\bar{Q} = \int \overline{\mathrm{Re}\,\boldsymbol{E}\,\mathrm{Re}\,\boldsymbol{j}}\, d^3 r = \frac{1}{2}\mathrm{Re}\int \boldsymbol{j}\boldsymbol{E}^* d^3 r. \tag{3}$$

Here, the bar denotes time averaging and the asterisk means complex conjugation.

Expression (2) uses a standard normalisation of the distribution function f at which the density of electron states is $2/h^3$. The equilibrium function $f_0(\mathscr{E})$ is approximated by the step function [20]

$$f_0(\mathscr{E}) = \theta(\mathscr{E}_{\mathrm{F}} - \mathscr{E}) = \begin{cases} 1, & 0 \leqslant \mathscr{E} \leqslant \mathscr{E}_{\mathrm{F}}, \\ 0, & \mathscr{E}_{\mathrm{F}} < \mathscr{E}, \end{cases}$$

where $\mathscr{E}_{\mathrm{F}} = m v_{\mathrm{F}}^2/2$ is the Fermi energy (v_{F} is the Fermi velocity). It is assumed that the Fermi surface is spherical.

We assume that the energy spectrum of an electron is continuous (quasi-classical approximation). This assumption is valid when the characteristic size of a conductor exceeds 3–4 nm because the de Broglie wavelength of the electron on the Fermi surface in the particle shell should be many times smaller than the corresponding linear size of the metal particle. Thus, we will assume that the particle shell thickness exceeds 3–4 nm.

The problem is reduced to the determination of the deviation f_1 of the electron distribution function from the equilibrium function f_0 produced by the field (1). In the linear approximation in the external field, the function f_1 satisfies the kinetic equation [2, 20]

$$-\mathrm{i}\omega f_1 + \boldsymbol{v}\frac{\partial f_1}{\partial \boldsymbol{r}} + e(\boldsymbol{vE})\frac{\partial f_0}{\partial \mathscr{E}} = -\frac{f_1}{\tau}, \tag{4}$$

where the stationary time dependence is assumed and the collision integral is taken in the time relaxation approximation:

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = -\frac{f_1}{\tau}.$$

By solving Eqn (4) by the method of characteristics [21], we obtain

$$f_1 = \frac{A[\exp(-\nu t') - 1]}{\nu}, \quad t' \geqslant 0, \tag{5}$$

where

$$\nu = \frac{1}{\tau} - \mathrm{i}\omega, \quad A = e(\boldsymbol{vE})\frac{\partial f_0}{\partial \mathscr{E}}, \tag{6}$$

the values of ν and A being constant along the trajectory (i.e., along the characteristic). The parameter t' in expression (5) is the time of the electron motion at the velocity $\boldsymbol{v}$ along the trajectory from the boundary where reflection occurs to the point $\boldsymbol{r}$.

To define the function f_1 uniquely, it is necessary to specify the boundary conditions for it on the cylindrical surfaces of the metal shell and dielectric nucleus of the particle. As the boundary conditions, we will use the conditions of diffuse reflection of electrons from these surfaces [2]. Because electrons can reflect from the internal (R_1) and external (R_2) boundaries of the metal layer, it is necessary to write the two boundary conditions

$$f_{11}(\boldsymbol{r}, \boldsymbol{v}) = 0 \quad \text{for} \quad |\boldsymbol{r}_\perp| = R_1, \ \boldsymbol{r}_\perp\boldsymbol{v}_\perp > 0, \tag{7}$$

$$f_{12}(\boldsymbol{r}, \boldsymbol{v}) = 0 \quad \text{for} \quad |\boldsymbol{r}_\perp| = R_2, \ \boldsymbol{r}_\perp\boldsymbol{v}_\perp < 0, \tag{8}$$

where $\boldsymbol{r}_\perp$ and $\boldsymbol{v}_\perp$ are the components of the radius vector of the electron and its velocity $\boldsymbol{v}$ in a plane perpendicular to the inhomogeneous cylinder axis. The case $\boldsymbol{r}_\perp\boldsymbol{v}_\perp > 0$ ($\boldsymbol{r}_\perp\boldsymbol{v}_\perp < 0$) corresponds to the motion of electrons between nuclei.

Upon reflection of an electron from the internal boundary (R_1), the parameter t' in (5) is determined by the expression

$$t' = t_1 = \frac{\boldsymbol{r}_\perp\boldsymbol{v}_\perp - \left[(\boldsymbol{r}_\perp\boldsymbol{v}_\perp)^2 + (R_1^2 - r_\perp^2)v_\perp^2\right]^{1/2}}{v_\perp^2}, \tag{9}$$

and upon reflection from the external boundary (R_2) – by the expression

$$t' = t_2 = \frac{\boldsymbol{r}_\perp\boldsymbol{v}_\perp + \left[(\boldsymbol{r}_\perp\boldsymbol{v}_\perp)^2 + (R_2^2 - r_\perp^2)v_\perp^2\right]^{1/2}}{v_\perp^2}. \tag{10}$$

This is clear from the following geometrical considerations. By using the obvious vector equality $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t'$, where $\boldsymbol{r}_0$ is the radius vector of the electron at the instant of its reflection from any of the boundaries of the metal layer, and projecting this equality on a plane perpendicular to the cylinder axis, we obtain $\boldsymbol{r}_\perp = \boldsymbol{r}_{0\perp} + \boldsymbol{v}_\perp t'$, where the vectors $\boldsymbol{r}_\perp$, $\boldsymbol{r}_{0\perp}$ and $\boldsymbol{v}_\perp$ are the components of the initial vectors in the projection plane. By squaring both parts of the latter equality and solving the obtained equation for t_1 and t_2, we can obtain expressions (9) or (10).

Therefore, Eqn (4) has two different solutions depending on the site of reflection of a conduction electron inside the metal layer of the particle. Relations (5), (6), (9), and (10) completely determine the solutions f_{11} and f_{12} of Eqn (4) with boundary conditions (7) and (8), which allows one to calculate the current density (2) and dissipation power (3).

It is convenient to calculate integrals (2) and (3) in the cylindrical coordinates, both in the coordinate space ($r_\perp$, φ, z; the vector $\boldsymbol{E}_0$ is parallel to the z axis) and in the velocity space ($v_\perp$, α, v_z; v_z is the polar axis). The cylinder axis coincides with the z axis. Field (1) in cylindrical coordinates has only one z component

$$\boldsymbol{E} = E_z\boldsymbol{e}_z, \quad E_z = E_0\exp(-\mathrm{i}\omega t). \tag{11}$$

Therefore, the current density (2) also has only z component (current lines are straight lines parallel to the z axis):

$$j_z = \frac{3ne^2}{4\pi v_\mathrm{F}^3}\frac{E_z}{\nu}\int v_z^2\delta(\mathscr{E} - \mathscr{E}_\mathrm{F})[1 - \exp(-\nu t')]d^3v. \tag{12}$$

We took into account here that the concentration of conduction electrons in metals is determined by the expression

$$n = 2\left(\frac{m}{h}\right)^3\int f_0 d^3v = 2\left(\frac{m}{h}\right)^3\frac{4}{3}\pi v_\mathrm{F}^3.$$

By integrating relation (12), one should bear in mind that the site of reflection of electrons inside the particle in the velocity space is determined by the angle α ($0 < \alpha \leqslant 2\pi$):

(i) If the inequality $\alpha_0 \leqslant \alpha \leqslant \pi - \alpha_0$ is fulfilled, where the angle α_0 is defined by the expression

$$\alpha_0 = \arccos\left[\frac{(r_\perp^2 - R_1^2)^{1/2}}{r_\perp}\right], \tag{13}$$

the electron trajectory is not intersected with the nucleus and the electron undergoes reflection from the external boundary of the metal layer of the particle. In this case, the function f_1 is $f_{12}(\boldsymbol{r}, \boldsymbol{v})$ ($t' = t_2$).

(ii) If $\pi - \alpha_0 < \alpha \leqslant \pi$, then electrons move to the particle nucleus, and the function f_1 is again $f_{12}(\boldsymbol{r}, \boldsymbol{v})$ ($t' = t_2$).

(iii) Finally, if $0 < \alpha \leqslant \alpha_0$, electrons move from the particle nucleus, and the function f_1 is $f_{11}(\boldsymbol{r}, \boldsymbol{v})$ ($t' = t_1$).

One can easily see that in the first two cases the integrals can be united.

The absorption cross section σ for electromagnetic radiation can be found by dividing the average dissipated power $\bar{Q}$ (3) by the average energy flux $cE_0^2/(8\pi)$ in the wave

$$\sigma = \frac{1}{2}\frac{8\pi}{cE_0^2}\mathrm{Re}\left(\int j_z E_z^* d^3r\right)$$

or, by using (12), we obtain

$$\sigma = \frac{1}{2}\frac{8\pi}{cE_0^2}\mathrm{Re}\left\{\int \frac{3ne^2E_z}{4\pi v_F^3 v}\left[\int v_z^2\delta(\mathscr{E}-\mathscr{E}_F)[1-\exp(-vt')]d^3v\right]\right.$$
$$\left.\times E_z^* d^3r\right\}.$$

By using the properties of the delta function, we have

$$\delta(\mathscr{E}-\mathscr{E}_F) = \frac{2}{m}\delta\left(v_z^2+v_\perp^2-v_F^2\right) = \frac{2}{m}\delta\left[v_z^2-\left(v_F^2-v_\perp^2\right)\right]$$
$$= \frac{2}{m}\delta\left\{\left[v_z-\left(v_F^2-v_\perp 2\right)^{1/2}\right]\left[v_z+\left(v_F^2-v_\perp^2\right)^{1/2}\right]\right\}$$
$$= \frac{1}{m(v_F^2-v_\perp^2)^{1/2}}\left\{\delta\left[v_z-\left(v_F^2-v_\perp^2\right)^{1/2}\right]\right.$$
$$\left.+\delta\left[v_z+\left(v_F^2-v_\perp^2\right)^{1/2}\right]\right\}.$$

Due to the symmetry of the problem, the integration over the entire range of velocities v_z can be replaced by integration over the positive range with the subsequent doubling of the result. Thus, we have

$$\sigma = \frac{1}{2}\frac{8\pi}{cE_0^2}\mathrm{Re}\left\{\int\frac{3ne^2E_z}{4\pi v_F^3 v}\left[\frac{2}{m}\int\frac{v_z^2\delta\left[v_z-(v_F^2-v_\perp^2)^{1/2}\right]}{(v_F^2-v_\perp^2)^{1/2}}\right.\right.$$
$$\left.\left.\times[1-\exp(-vt')]d^3v\right]E_z^*d^3r\right\}.$$

Then, by using (11), we obtain

$$\sigma = \mathrm{Re}\left\{\frac{12\pi ne^2L}{mcv_F^3v}\int_{R_1}^{R_2}r_\perp \mathrm{d}r_\perp\int_0^{v_F}\int_0^{2\pi}v_\perp(v_F^2-v_\perp^2)^{1/2}\right.$$
$$\left.\times[1-\exp(-vt')]\mathrm{d}v_\perp\mathrm{d}\alpha\right\}. \tag{14}$$

For further calculations and analysis of the results, expression (14) can be represented in the form

$$\sigma = \sigma_1+\sigma_2, \tag{15}$$

where

$$\sigma_1 = \mathrm{Re}\left\{\frac{24\pi ne^2L}{mcv_F^3v}\int_{R_1}^{R_2}r_\perp\mathrm{d}r_\perp\int_0^{v_F}\int_{\alpha_0}^{\pi}v_\perp(v_F^2-v_\perp^2)^{1/2}\right.$$
$$\left.\times[1-\exp(-vt_2)]\mathrm{d}v_\perp\mathrm{d}\alpha\right\}, \tag{16}$$

$$\sigma_2 = \mathrm{Re}\left\{\frac{24\pi ne^2L}{mcv_F^3v}\int_{R_1}^{R_2}r_\perp\mathrm{d}r_\perp\int_0^{v_F}\int_0^{\alpha_0}v_\perp(v_F^2-v_\perp^2)^{1/2}\right.$$
$$\left.\times[1-\exp(-vt_1)]\mathrm{d}v_\perp\mathrm{d}\alpha\right\}. \tag{17}$$

(Because the motion of electrons is symmetrical with respect to any diametrical plane in which a point of their location on the trajectory lies, we can assume that the angle α in the velocity space varied from 0 to π and double the result of integration over this variable.)

Let us introduce new variables

$$\xi = \frac{r_\perp}{R_2},\ \rho = \frac{v_\perp}{v_F},\ z = v\frac{R_2}{v_F} = \left(\frac{1}{\tau}-\mathrm{i}\omega\right)\frac{R_2}{v_F} = x-\mathrm{i}y,\ K = \frac{R_1}{R_2}$$

and transform expressions (9), (10), and (13):

$$t_1 = \frac{R_2}{v_\perp}\psi,\quad \psi = \left[\xi\cos\alpha-(K^2-\xi^2\sin^2\alpha)^{1/2}\right],$$

$$t_2 = \frac{R_2}{v_\perp}\eta,\quad \eta = \left[\xi\cos\alpha+(1-\xi^2\sin^2\alpha)^{1/2}\right],$$

$$\alpha_0 = \arccos\left(1-\frac{K^2}{\xi^2}\right)^{1/2}.$$

We took into account here that $\boldsymbol{r}_\perp\boldsymbol{v}_\perp = \boldsymbol{r}_\perp v_\perp\cos\alpha$ (all the electrons on the Fermi surface inside the metal layer of the particle move at the velocity v_F). Then, expressions (16) and (17) take the form

$$\sigma_1 = \mathrm{Re}\left[\frac{24\pi ne^2R_2^3L}{mcv_F}\int_K^1\xi\mathrm{d}\xi\int_0^1\int_{\alpha_0}^{\pi}\rho(1-\rho^2)^{1/2}\right.$$
$$\left.\times\frac{1-\exp(-z\eta/\rho)}{z}\mathrm{d}\rho\mathrm{d}\alpha\right],$$

$$\sigma_2 = \mathrm{Re}\left[\frac{24\pi ne^2R_2^3L}{mcv_F}\int_K^1\xi\mathrm{d}\xi\int_0^1\int_0^{\alpha_0}\rho(1-\rho^2)^{1/2}\right.$$
$$\left.\times\frac{1-\exp(-z\psi/\rho)}{z}\mathrm{d}\rho\mathrm{d}\alpha\right].$$

Let us represent the absorption cross section (15) in the form

$$\sigma = \sigma_0(F_1+F_2),$$

where

$$\sigma_0 = \frac{24\pi ne^2R_2^3L}{mcv_F};$$

$$F_1 = \mathrm{Re}\left[\int_K^1\xi\mathrm{d}\xi\int_0^1\int_{\alpha_0}^{\pi}\rho(1-\rho^2)^{1/2}\frac{1-\exp(-z\eta/\rho)}{z}\mathrm{d}\rho\mathrm{d}\alpha\right]; \tag{18}$$

$$F_2 = \mathrm{Re}\left[\int_K^1\xi\mathrm{d}\xi\int_0^1\int_0^{\alpha_0}\rho(1-\rho^2)^{1/2}\frac{1-\exp(-z\psi/\rho)}{z}\mathrm{d}\rho\mathrm{d}\alpha\right]. \tag{19}$$

Expressions (18) and (19) allow one to calculate the dimensionless absorption cross section for an inhomogeneous cylindrical particle

$$F(x,y,K) = F_1(x,y,K)+F_2(x,y,K) \tag{20}$$

and the absorption cross section for electromagnetic radiation

$$\sigma = \sigma_0F(x,y,K). \tag{21}$$

When $K \to 0$ ($\alpha_0 \to 0$), it follows from (2) that

$$F(x,y) = \mathrm{Re}\left[\int_0^1 \xi \mathrm{d}\xi \int_0^1 \int_0^{\pi} \rho (1-\rho^2)^{1/2} \frac{1-\exp(-z\eta/\rho)}{z} \mathrm{d}\rho \mathrm{d}\alpha\right].$$

This expression coincides with the expression for absorption of an electromagnetic wave by a homogeneous prolate cylindrical metal particle. The results of numerical calculation of $F(x, y, K)$ are presented in Figs 1 and 2.

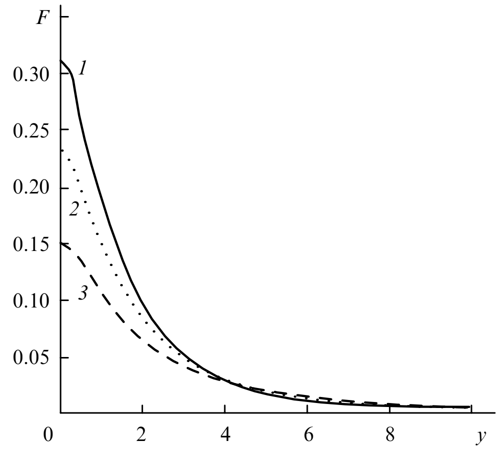

Figure 1. Dependences of the dimensionless absorption cross section F on the dimensionless frequency $y = R_2\omega/v_F$ for $x = 0.3$, $K = 0.5$ (*1*), 0.6 (*2*), and 0.7 (*3*).

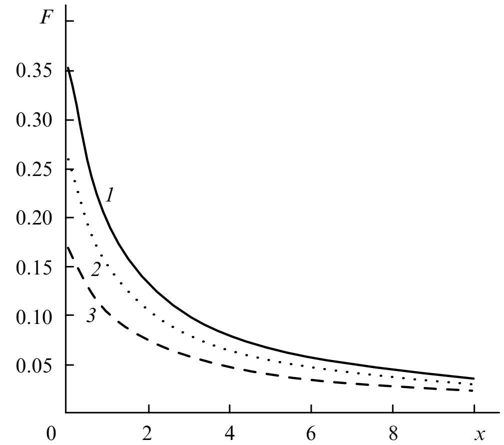

Figure 2. Dependences of the dimensionless absorption cross section F on the dimensionless inverse mean free path of electrons $x = R_2/(\tau v_F)$ for $y = 0.3$, $K = 0.5$ (*1*), 0.6 (*2*), and 0.7 (*3*).

3. Absorption in low- and high-frequency regimes

Consider in detail the case when the external field frequency ω and the electron collision frequency $(1/\tau)$ inside a metal are low compared to the collision frequency of electrons with the surfaces of the cylindrical metal layer of the particle. In other words, consider the case of $|z| \ll 1$. Then, the exponential in expressions (18) and (19) can be expanded in a Taylor series retaining only the first two terms in the expansion. As a result, we obtain

$$F_1 = \int_K^1 \xi \mathrm{d}\xi \int_0^1 \int_{\alpha_0}^{\pi} (1-\rho^2)^{1/2} \left[\xi \cos\alpha + (1-\xi^2 \sin 2\alpha)^{1/2}\right] \mathrm{d}\rho \mathrm{d}\alpha,$$

$$F_2 = \int_K^1 \xi \mathrm{d}\xi \int_0^1 \int_0^{\alpha_0} (1-\rho^2)^{1/2} \left[\xi \cos\alpha - (K^2 - \xi^2 \sin 2\alpha)^{1/2}\right] \mathrm{d}\rho \mathrm{d}\alpha.$$

By integrating over the variable ρ, we have

$$F_1 = \frac{\pi}{4} \int_K^1 \xi \mathrm{d}\xi \int_{\alpha_0}^{\pi} \left[\xi \cos\alpha + (1-\xi^2 \sin^2\alpha)^{1/2}\right] \mathrm{d}\alpha, \qquad (22)$$

$$F_2 = \frac{\pi}{4} \int_K^1 \xi \mathrm{d}\xi \int_0^{\alpha_0} \left[\xi \cos\alpha - (K^2 - \xi^2 \sin^2\alpha)^{1/2}\right] \mathrm{d}\alpha. \qquad (23).$$

Expressions (22) and (23) can be partially calculated analytically. The final results is

$$F_1 = \frac{\pi}{4} \left\{ \frac{4}{3} - \frac{3}{2}K + \frac{1}{6}K^3 - \frac{1}{2} \int_{1-K}^{(1-K^2)^{1/2}} \left[2K^2 - K^4 - 1 - \eta^4 + 2\eta^2(1+K^2)\right]^{1/2} \mathrm{d}\eta \right\},$$

$$F_2 = \frac{\pi}{4} \left\{ \frac{1}{2}K(1-K^2) - \frac{1}{2} \int_{1-K}^{(1-K^2)^{1/2}} \left[2K^2 - K^4 - 1 - \psi^4 + 2\psi^2(1+K^2)\right]^{1/2} \mathrm{d}\psi \right\}.$$

Then, the absorption cross section is described by the expression

$$\sigma = \sigma_0 \frac{\pi}{4} \left\{ \frac{4}{3} - K - \frac{1}{3}K^3 - \int_{1-K}^{(1-K^2)^{1/2}} \left[2K^2 - K^4 - 1 - \eta^4 + 2\eta^2(1+K^2)\right]^{1/2} \mathrm{d}\eta \right\}. \quad (24)$$

Consider the possible limiting cases.

If the particle has a dielectric nucleus whose radius is many times greater than the particle radius, i.e., $K \ll 1$, we can find the correction to absorption by omitting in (24) the terms proportional to K^3 (the contribution from the integral is negligible):

$$\sigma \approx \sigma_0 \frac{\pi}{3} \left(1 - \frac{3}{4}K\right). \qquad (25)$$

For a metal particle without a nucleus ($K \to 0$), it follows from (25) that

$$\sigma = \frac{8\pi^2 n e^2 R_2^3 L}{m c v_F}.$$

This expression coincides with the result for low-frequency electric absorption by a homogeneous prolate cylindrical metal particle.

To find the correction to absorption by expression (24) in the case of a thin metal shell, when $K \to 1$, it is necessary to perform the expansion in a series in the parameter $(1-K) \ll 1$ and to use approximate expressions for calculations. In this case (the contribution of the integral dominates), the absorption cross section is

$$\sigma \approx \sigma_0 \frac{\pi}{2}\left[\frac{3}{2}(1-K)\right]^{1/2}.$$

If $|z| \gg 1$, expression (20) has the asymptotics. By neglecting the terms with exponentials due to their fast decay and performing algebraic transformations, we obtain the expression for the dimensionless absorption cross section $F(z)$

$$F(z) = \mathrm{Re}\left[\frac{1}{z}\int_K^1 \xi \mathrm{d}\xi \int_0^1 \int_0^{\pi} \rho(1-\rho^2)^{1/2} \mathrm{d}\rho \mathrm{d}\alpha\right].$$

It can be easily integrated to obtain

$$F(z) = \mathrm{Re}\left[\frac{1}{z(x,y)}\frac{\pi}{6}\left(1-K^2\right)\right]. \quad (26)$$

As a result, we obtain the absorption cross section (21) in the form

$$\sigma(z) = \sigma_0 \mathrm{Re}\left[\frac{1}{z(x,y)}\frac{\pi}{6}\left(1-K^2\right)\right]$$

$$= \sigma_0 \frac{\pi}{6}(1-K^2)\frac{x}{x^2+y^2}. \quad (27)$$

For a metal particle without a nucleus ($K \to 0$), this expression corresponds to the classical result (the Drude formula) [20] for a homogeneous cylindrical metal particle

$$\sigma(z) = \sigma_0 \frac{\pi}{6}\frac{z}{x^2+y^2}.$$

In the case of a thin metal shell, when $K \to 1$, it is convenient to find the correction to absorption by expression (26) by making the substitution $K = 1 - \delta$, where δ is a small quantity ($\delta \to 0$), and to use expressions for approximate calculations. Indeed, because $1 - K^2 = 1 - (1-\delta)^2 \approx 1 - (1 - 2\delta) = 2\delta = 2(1-K)$, the absorption cross section is described in this case by the expression

$$\sigma \approx \sigma_0 \frac{\pi}{3}\frac{x}{x^2+y^2}(1-K).$$

Finally, if a cylindrical particle is a dielectric ($K = 1$), its absorption cross section is zero because the energy of an external electromagnetic field does not dissipate in such particles.

4. Discussion of the results

The dimensionless absorption cross section F depends on a combination of three dimensionless quantities x, y, and K. The presence of a dielectric nucleus in a prolate cylindrical particle (recall that $L \gg R_2$) naturally leads to the results that differ from those obtained for a homogeneous cylindrical metal particle. This is explained by the fact that, except reflection of electrons from the external surface of the particle, their scattering from the cylindrical nucleus also takes place; in addition, the energy of the external electromagnetic field does not dissipate in the nucleus.

Figure 1 shows the dependences of the dimensionless absorption cross section F on the dimensionless frequency y of the external field for the fixed dimensionless inverse mean free path x of electrons. It follows from analysis of these curves that for small values of y ($y < 3$), the particles with the metal shell containing a greater amount of the metal (which means that the parameter K for such particles is minimal) have the maximum dimensionless absorption cross section. For high dimensionless frequencies of the external field ($y > 3$), all the three dependences merge because the macroscopic asymptotics takes place.

Figure 2 shows the dependences of the dimensionless cross section F on the dimensionless inverse mean free path x of electrons. If the constant dimensionless frequency of the external field is y, then for any x the cross section F is greater than unity for particles for which the linear size of the nucleus minimal (we consider particles with the same radius and length).

In this paper, we studied along with the dependence of the dimensionless absorption cross section F on x and y also the dependence of F on the ratio K of radii of the nucleus and particle. We analysed the latter dependence by using Fig. 3, which shows the dimensionless specific absorption cross section for a cylindrical metal particle with a dielectric nucleus

$$G(K) = \frac{F(K)}{1-K^2}.$$

One can see from Fig. 3 that the specific absorption cross section (for any K) is higher for particles in the external electric field with the lower frequency. This is explained by the fact that in high-frequency fields, the electrons subjected to the action of the external field with a relatively longer oscillation period experience a greater acceleration.

Figure 4 and 5 show the dimensionless absorption cross sections calculated for an inhomogeneous cylindrical particle by expression (20) obtained in this paper and by the Drude formula for an inhomogeneous cylinder [which can be easily obtained from (27)]. These dependences demonstrate a noticeable difference between the exact kinetic calculation and classical Drude calculation.

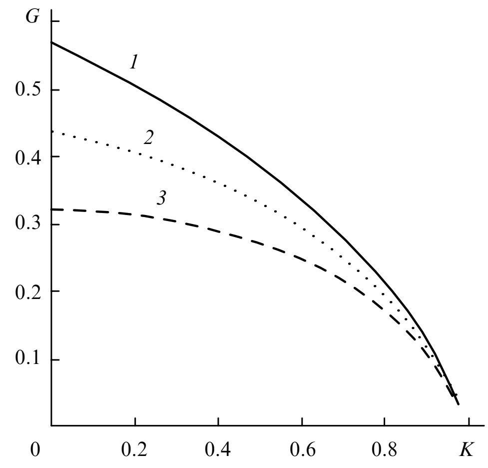

Figure 3. Dependences of the dimensionless specific absorption cross section G on the ratio K of radii of the nucleus and particle for $x = 0.3$, $y = 0.3$ (1), 0.6 (2), and 0.9 (3).

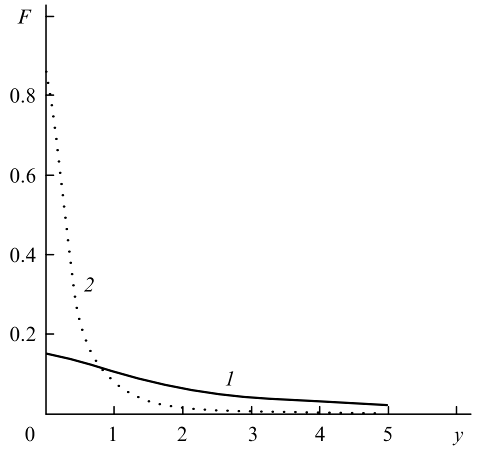

Figure 4. Dependences of the dimensionless absorption cross section F on the dimensionless frequency $y = R_2\omega/v_F$ obtained by exact kinetic calculation (*1*) and by using the Drude formula ($x = 0.3$, $K = 0.7$) (*2*).

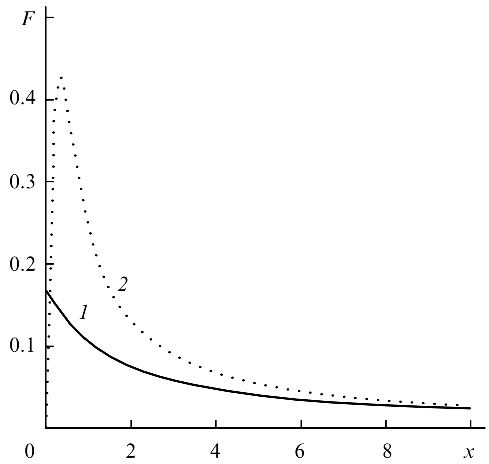

Figure 5. Dependences of the dimensionless absorption cross section F on the dimensionless inverse mean free path of electrons $x = R_2/(\tau v_F)$ obtained by exact kinetic calculation (*1*) and by using the Drude formula ($x = 0.3$, $K = 0.7$) (*2*).

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