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RADIATION SCATTERING

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On radiation scattering by a particle located in a regularly inhomogeneous medium

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Abstract. **The scattering characteristics of a spherical particle placed into a regularly inhomogeneous medium are studied. It is shown that the inhomogeneity of the medium surrounding the particle substantially changes the scattering pattern of a light wave compared to scattering of light by a particle in a homogeneous medium considered in the classical theory. The expressions for the scattering amplitude are obtained for the conditions when an inhomogeneous medium can be represented as a defocusing or focusing channel. It is shown that the defocusing action of the medium is manifested in a significant broadening of the scattering amplitude and its shift compared to the classical case. The focusing of scattered radiation by the medium can cause both broadening and narrowing of the scattering amplitude, as well as periodic variations in its position. The influence of regular refraction on the propagation of a radiation beam in the scattering medium is estimated.**

Keywords: *radiation scattering, scattering amplitude, defocusing medium, focusing medium, thermal self-action of laser radiation, nuclear-pumped laser.*

1. Introduction

The propagation of electromagnetic waves in real media is always accompanied by scattering. Depending on specific conditions, scattering can be caused, for example, by microscopic fluctuations or turbulent pulsations of the medium density, by dispersed hydrosols and aerosols or defects and disseminations. One of the divisions of the theory of radiation transfer in media with local inhomogeneities is the theory of scattering by a single inhomogeneity. The classical theory describing the scattering of a wave by a particle located in a homogeneous medium (see, for example, [1 – 4]) can explain a variety of natural phenomena caused by wave scattering (see, for example, [5, 6]). However, a number of problems of laser physics and nonlinear optics, which require the consideration of the regular inhomogeneity of the medium surrounding scatterers, cannot be described within the framework of the classical scattering theory.

Thus, the study of the thermal self-action of laser radiation in aerodispersion media [7] and a turbulent atmosphere [8] should take into account that scattering occurs from inhomogeneities in the presence of a thermal lens induced by the laser beam itself. The regular inhomogeneities produced in the medium in this case substantially affect the scattering characteristics of local inhomogeneities (aerosols and turbulent vortices) on the routes under study. Scattering of radiation also occurs in the regularly inhomogeneous active media of lasers pumped by a hard ioniser, for example, by fission fragments. In this case, the average inhomogeneity of the energy deposition to the active medium results in the formation of an extended gas lens [9] determining the mode structure of the laser beam produced in the resonator. The scattering of the beam by the ioniser tracks or pulsations of the active medium density induced by the beam additionally reduced the radiation intensity and affects the coherent and energy characteristics of radiation. This field of study also includes the problem of laser radiation propagation in graded-index fibres, where scattering is caused by random disseminations and defects in fibres.

From the point of view of the wave theory, these quite important problems of quantum electronics have common features, first, because both the incident and scattered waves propagate in a regularly inhomogeneous medium and, second, the observation point of the scattered field is also located in the inhomogeneous medium in the general case. Although the phenomenon of wave scattering by an isolated particle in media with lens properties appearing during the propagation of radiation in a regularly inhomogeneous medium with scatterers is widespread, it has not been adequately studied so far and only a few papers have been devoted to this problem (see, for example, [10] where the propagation of radio waves in the ionosphere was investigated in the geometric optics approximation). In this connection, we suppose that a more detailed study of this problem is interesting and important both from the cognitive and practical points of view.

In this paper, we investigate the scattering characteristics of a large, optically soft spherical particle in a regularly inhomogeneous medium. We obtained the expression for the scattering amplitude in the case when the medium surrounding the particle can be treated as a refraction channel. This expression was analysed for two typical scattering situations: in the defocusing refraction channel formed during the thermal self-action of a laser beam in the turbulent atmosphere and in the focusing active medium of a nuclear-pumped laser.

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2. Formulation and general solution of the problem

Consider the scattering of an electromagnetic wave by a particle in a medium with the permittivity $\varepsilon_m(\boldsymbol{R})$ (where $\boldsymbol{R} = \{x, y, z\}$ is the three-dimensional radius vector) dependent on coordinates. Without loss of generality, we will assume that the medium surrounding the particle is transparent and the medium and particle are nonmagnetic.

Let us assume that the inhomogeneous medium in which the particle is placed (its centre is located at the point $\boldsymbol{R}_s$) occupies the half-space $z \geqslant 0$. We will direct the Z axis of the Cartesian coordinate system along the propagation direction of a wave incident on the medium and denote the electric field vector of the wave by $\boldsymbol{E}_i(\boldsymbol{R})$ (Fig. 1).

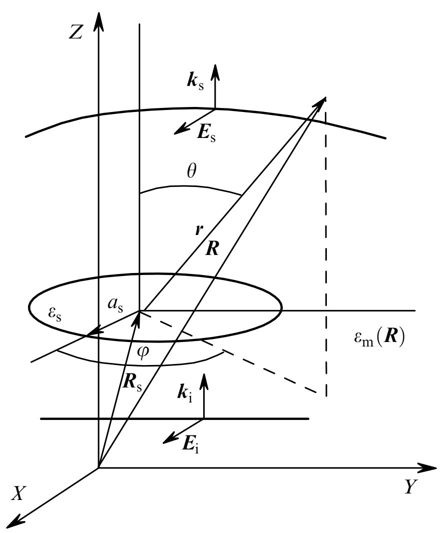

Figure 1. Illustration to the scattering problem.

Assuming that the spatial inhomogeneity of the permittivity of the surrounding medium can be neglected at the scale of the order of the linear size a_s of the particle [i.e., the scale l_m of variation of the field $\varepsilon_m(\boldsymbol{R})$ greatly exceeds the particle size, $l_m \gg a_s$], we represent the permittivity of the medium-particle system in the form

$$\varepsilon(\boldsymbol{R}) = \varepsilon_m(\boldsymbol{R}_s)[1 + \Delta\varepsilon_m(\boldsymbol{R})] + \Theta(\boldsymbol{R} - \boldsymbol{R}_s)\Delta\varepsilon_s. \quad (1)$$

Here, $\Delta\varepsilon_m(\boldsymbol{R}) = [\varepsilon_m(\boldsymbol{R}) - \varepsilon_m(\boldsymbol{R}_s)]/\varepsilon_m(\boldsymbol{R}_s)$ is the relative variation in the medium permittivity [so that $\Delta\varepsilon_m(\boldsymbol{R}_s) = 0$]; ε_s is the permittivity of the particle; $\Delta\varepsilon_s = \varepsilon_s - \varepsilon_m(\boldsymbol{R}_s)$;

$$\Theta(\boldsymbol{R}) = \begin{cases} 1 \text{ for } \boldsymbol{R} \in V_s, \\ 0 \text{ for } \boldsymbol{R} \notin V_s; \end{cases} \quad (2)$$

and V_s is the scattering particle volume ($a_s \sim V_s^{1/3}$).

Consider the case when the spatial inhomogeneity of the medium permittivity satisfies the conditions of smallness, $\max|\Delta\varepsilon_m(\boldsymbol{R})| \ll 1$, and continuous inhomogeneity, $kl_m \gg 1$ (where $k = 2\pi[\varepsilon_m(\boldsymbol{R}_s)]^{1/2}/\lambda$ is the wave number, and λ is the wavelength). These conditions allow one to neglect the depolarisation of the scattered wave propagating in the inhomogeneous medium and to determine the electric field vector $\boldsymbol{E}_s(\boldsymbol{R})$ of the scattered wave from the expression (see, for example, [4])

$$\boldsymbol{E}_s(\boldsymbol{R}) = -\mathrm{rot\,rot}\iiint_{V_s} \mathrm{d}^3R'\Delta\varepsilon_s\boldsymbol{E}(\boldsymbol{R}' + \boldsymbol{R}_s)\bar{G}(\boldsymbol{R}|\boldsymbol{R}' + \boldsymbol{R}_s), \quad (3)$$

in which $\boldsymbol{E}(\boldsymbol{R})$ is the electric field vector inside the particle; $\bar{G}(\boldsymbol{R}|\boldsymbol{R}')$ is the Green function of the Helmholtz equation

$$\Delta\bar{G}(\boldsymbol{R}|\boldsymbol{R}') + k^2[1 + \Delta\varepsilon_m(\boldsymbol{R})]\bar{G}(\boldsymbol{R}|\boldsymbol{R}') = \delta(\boldsymbol{R} - \boldsymbol{R}'); \quad (4)$$

and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplace operator.

According to (3), if the solution of the inner problem for $\boldsymbol{E}(\boldsymbol{R})$ is known from some additional considerations, then for the above assumptions about the field of the permittivity $\varepsilon_m(\boldsymbol{R})$, the solution of Maxwell's equations for the vector field of the scattered wave is finally reduced to the solution of one scalar equation (4) for the Green function. In the case under study, due to the condition $l_m \gg a_s$, the solution of the inner problem can be found by the standard methods of the scattering theory in a homogeneous medium, by using them for a spatially homogeneous subregion in the vicinity of the particle.

Let us now analyse Eqn (4). We will study the scattering of waves by a large, optically soft particle, such that $ka_s \gg 1$ and $k|\Delta\varepsilon_s|a_s \ll 1$. It is known [2] that upon scattering of a wave by such a particle in a homogeneous medium, the main part of energy of the scattered wave is concentrated within a narrow angular cone near the propagation direction of the incident wave. Because this result remains also valid in a continuously inhomogeneous medium (for $kl_m \gg 1$), the energy-carrying part of the scattered wave can be represented in the form of a wave beam, whose effective size is determined by the width of the central maximum of the scattering amplitude of the particle. Due to the condition $\max|\Delta\varepsilon_m(\boldsymbol{R})| \ll 1$, the propagation direction of this beam will not differ significantly from that of the incident wave. (Note that the propagation directions of the energy-carrying parts of the scattered and incident waves in a homogeneous medium coincide.) These circumstances enable the simplification of Eqn (4) by using the quasi-optical approximation.

We will seek the solution of Eqn (4) for $z > z'$ in the form

$$\bar{G}(\boldsymbol{R}|\boldsymbol{R}') = -\frac{\mathrm{i}}{2k}G(\boldsymbol{R}|\boldsymbol{R}')\exp[\mathrm{i}k(z - z')]. \quad (5)$$

By substituting this relation into (4) and neglecting the second derivative with respect to z, we obtain the parabolic quasi-optical equation for the function $G(\boldsymbol{R}|\boldsymbol{R}')$

$$2\mathrm{i}k\frac{\partial}{\partial z}G + \Delta_\perp G + k^2\Delta\varepsilon_m(\boldsymbol{R})G = 0, \quad (6)$$

$$G(\boldsymbol{R}|\boldsymbol{R}')|_{z=z'} = \delta(\boldsymbol{\rho} - \boldsymbol{\rho}'), \quad (7)$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y$ is the Laplace operator in the transverse coordinates and $\boldsymbol{\rho} = \{x, y\}$.

In the considered case of light scattering by an optically soft particle, the internal field in (3) can be replaced by the incident-wave field by using the so-called Rayleigh–Hans approximation (see, for example, [2–4]):

$$\mathbf{E}(\mathbf{R})|_{\mathbf{R}\in V_s} = \mathbf{E}_i(\mathbf{R}).$$

Then, taking (5) into account, we obtain for the scattered-wave field

$$\mathbf{E}_s(\mathbf{R}) = \frac{i}{2k}\Delta\varepsilon_s \operatorname{rot}\operatorname{rot}\iiint\limits_{V_s} d^3R'\mathbf{E}_i(\mathbf{R}' + \mathbf{R}_s) \times G(\mathbf{R}|\mathbf{R}' + \mathbf{R}_s)\exp[ik(z - z' - z_s)]. \quad (8)$$

We will assume for definiteness that the incident wave is linearly polarised and represent its only nonzero component E_{ix} in the form

$$E_{ix}(\mathbf{R}) = U_i(\mathbf{R})\exp(ikz). \quad (9)$$

Here, $U_i(\mathbf{R})$ is the complex amplitude of the wave incident on a particle, which for $z > 0$ satisfies the parabolic equation

$$2ik\frac{\partial}{\partial z}U_i + \Delta_\perp U_i + k^2\Delta\varepsilon_m(\mathbf{R})U_i = 0 \quad (10)$$

with the boundary condition

$$U_i(\mathbf{R})|_{z=0} = U_0(\boldsymbol{\rho}), \quad (11)$$

where $U_0(\boldsymbol{\rho})$ is the amplitude of the wave incident on an inhomogeneous medium.

Within the framework of the quasi-optical approximation, the differential operator rot rot in (8) acts only on the factor $\exp(ikz)$. Taking this into account, we obtain the complex amplitude $U_s(\mathbf{R})$ of the scattered-wave field, which is determined from the relation $\mathbf{E}_s(\mathbf{R}) = \mathbf{e}_x U_s(\mathbf{R})\exp(ikz)$, where $\mathbf{e}_x$ is the unit vector in the direction X, in the form

$$U_s(\mathbf{R}) = -\frac{ik\Delta\varepsilon_s}{2}\iiint\limits_{V_s} d^3R' U_i(\mathbf{R}' + \mathbf{R}_s)G(\mathbf{R}|\mathbf{R}' + \mathbf{R}_s). \quad (12)$$

It follows from (12) that the complex amplitude of the scattered-wave field is determined by the convolution of the Green function $G(\mathbf{R}|\mathbf{R}')$ and the complex amplitude $U_i(\mathbf{R})$ of the incident wave, which depend on the spatial distribution of the permittivity in the parts of the beam path from the input to the medium to a scattering particle and from the particle to the observation point.

3. Amplitude of the wave scattering by a particle in the refraction channel

To obtain a comparatively simple expression for the scattered wave amplitude, which will nevertheless reflect the basic features of scattering caused by the regular inhomogeneity of the medium, we consider the case when the wave incident on a particle is a wave beam of radius a_0, this radius and the characteristic transverse scale l_s of variations in the scattered-wave amplitude (the effective radius of the scattered wave) being considerably smaller than the minimum scale of variation $\Delta\varepsilon_m(\mathbf{R})$. In this case, the variation in the permittivity of the medium in the region $|\boldsymbol{\rho}| \ll l_m$ can be expanded into a Taylor series in coordinates transverse to the predominant propagation direction of the waves. By retaining only the expansion terms quadratic in x and y, we obtain

$$\Delta\varepsilon_m(\mathbf{R}) = \Delta\varepsilon_m(z, 0) + \boldsymbol{\varepsilon}_1(z)\boldsymbol{\rho} + \beta^2(z)\rho^2, \quad (13)$$

where $\boldsymbol{\varepsilon}_1(z) = \{\varepsilon_{1x}(z), 0\}$ is the two-dimensional vector with the components $\varepsilon_{1x}(z) = (\partial/\partial x)\Delta\varepsilon_m(z,\boldsymbol{\rho})|_{\boldsymbol{\rho}=0}$ and $\varepsilon_{1y} = (\partial/\partial y)\Delta\varepsilon_m(z,\boldsymbol{\rho})|_{\boldsymbol{\rho}=0} = 0$; and

$$\beta^2(z) = \frac{1}{2}\frac{\partial^2}{\partial x^2}\Delta\varepsilon_m(z,\boldsymbol{\rho})|_{\boldsymbol{\rho}=0} = \frac{1}{2}\frac{\partial^2}{\partial y^2}\Delta\varepsilon_m(z,\boldsymbol{\rho})|_{\boldsymbol{\rho}=0}.$$

Representation (13) corresponds to the aberration-free description of the refractive properties of the medium. It is successfully used for studying the propagation of waves in nonlinear optics [7], atmospheric optics [8], and ocean acoustics [11].

The Green function $G(\mathbf{R}|\mathbf{R}')$, which is the solution of the problem (6), (7), for the permittivity variation of the type (13), is determined by the expression [12]

$$G(\mathbf{R}|\mathbf{R}') = \frac{k}{2\pi i h_2(z|z')}\exp\left\{\frac{ik}{2h_2(z|z')}\left[\rho^2 h_2'(z|z') + \rho'^2 h_1(z|z') - 2\boldsymbol{\rho}\boldsymbol{\rho}' + 2\boldsymbol{\rho}\boldsymbol{\rho}_r(z|z') + 2\boldsymbol{\rho}'\boldsymbol{\rho}_r'(z|z')\right] + i\Psi(z|z')\right\}, \quad (14)$$

where

$$\boldsymbol{\rho}_r(z|z') = \frac{1}{2}\int_{z'}^{z} d\eta\boldsymbol{\varepsilon}_1(\eta)h_2(\eta|z'); \qquad \boldsymbol{\rho}_r'(z|z') = \frac{1}{2}\int_{z'}^{z} d\eta\boldsymbol{\varepsilon}_1(\eta h_2(z|\eta); \quad (15)$$

$\Psi(z|z')$ is the phase factor, which is not important for the problem under study. In (14) and (15), $h_j(z|z')$ $(j = 1, 2)$ are the solutions of the differential equation

$$h_j''(z|z') - \beta^2(z)h_j(z|z') = 0 \quad (16)$$

with the boundary conditions

$$h_1(z'|z') = h_2'(z|z')|_{z=z'} = 1, \qquad h_2(z'|z') = h_1'(z|z')|_{z=z'} = 0. \quad (17)$$

The prime in the functions $h_j(z|z')$ means differentiation with respect to z.

We assume that a beam with a plane wave front is incident on an inhomogeneous medium. The boundary condition (11) for this beam has the form $U_i(\mathbf{R})|_{z=0} = A_0$. The solution of Eqn (10) with this boundary condition has the form

$$U_i(\mathbf{R}) = \frac{A_0}{h_1(z|0)} \times \exp\left\{\frac{ik}{2h_1(z|0)}\left[\rho^2 h_1'(z|0) + 2\boldsymbol{\rho}\tilde{\boldsymbol{\rho}}_r(z|0)\right] + i\tilde{\Psi}(z)\right\}, \quad (18)$$

where

$$\tilde{\boldsymbol{\rho}}_{\rm r}(z|0) = \frac{1}{2}\int_0^z {\rm d}\eta\varepsilon_1(\eta)h_1(\eta|0);$$

and $\tilde{\Psi}$ is the part of the phase shift independent of $\boldsymbol{\rho}$.

By substituting expression (14) for the Green function and expression (18) for the complex amplitude of the incident wave and neglecting a change in the function $h_j(z|z')$ at the $a_{\rm s}$ scale, we obtain the complex amplitude of the scattered wave

$$U_{\rm s}(\boldsymbol{R}) = -\frac{{\rm i}k}{2}\,\Delta\varepsilon_{\rm s}V_{\rm s}U_{\rm i}(\boldsymbol{R}_{\rm s})\,\frac{h_2(z|z_{\rm s})}{z-z_{\rm s}}\,G(\boldsymbol{R}|\boldsymbol{R}_{\rm s})F(\boldsymbol{R}|\boldsymbol{R}_{\rm s}). \quad (19)$$

Here, $V_{\rm s}$ is the scatterer volume;

$$F(\boldsymbol{R}|\boldsymbol{R}_{\rm s}) = \frac{1}{V_{\rm s}}\,\frac{z-z_{\rm s}}{h_2(z|z_{\rm s})}\iiint\limits_{V_{\rm s}} {\rm d}^3R' \times \exp\left\{\frac{{\rm i}k\rho'^2}{2L_{\rm f}(z|z_{\rm s})} - {\rm i}\boldsymbol{k}_{\rm s}(\boldsymbol{R}|\boldsymbol{R}_{\rm s})\boldsymbol{\rho}'\right\} \quad (20)$$

is the amplitude of scattering in the inhomogeneous medium;

$$L_{\rm f}(z|z_{\rm s}) = \left[\frac{h_1(z|z_{\rm s})}{h_2(z|z_{\rm s})} + \frac{h_1'(z_{\rm s}|0)}{h_1(z_{\rm s}|0)}\right]^{-1} \quad (21)$$

is the effective refraction length;

$$\boldsymbol{k}_{\rm s}(\boldsymbol{R}|\boldsymbol{R}_{\rm s}) = k\,\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}_{\rm s}) - \boldsymbol{\rho}_{\rm s}^{\rm (m)}(z|z_{\rm s})}{h_2(z|z_{\rm s})} \quad (22)$$

is the scattering vector; and

$$\boldsymbol{\rho}_{\rm s}^{\rm (m)}(z|z_{\rm s}) = \boldsymbol{\rho}_{\rm r}'(z|z_{\rm s}) + \boldsymbol{\rho}_{\rm s}\left[\frac{h_2(z|z_{\rm s})}{L_{\rm f}(z|z_{\rm s})} - 1\right] + \tilde{\boldsymbol{\rho}}_{\rm r}(z_{\rm s}|0)\,\frac{h_2(z_{\rm s}|0)}{h_1(z_{\rm s}|0)} \quad (23)$$

is the displacement vector of the scattering amplitude maximum.

Before proceeding to analysis of the features of light scattering in a regularly inhomogeneous medium, note that, aside from the form of the function $L_{\rm f}$ and vector $\boldsymbol{k}_{\rm s}$, expression (20) for the scattering amplitude also differs from the conventional expression of the classical theory (see, for example, [2]) by the factor $(z-z_{\rm s})/h_2(z|z_{\rm s})$. This factor appears for the following reasons. The energy flux of scattered radiation from a scatterer through a sphere concentric in the quasi-optical approximation reduces to the flux through the plane $z = {\rm const}$. Because the scattered radiation power is determined by the integral from the function $|F|^2$ over this plane, the additional factor appears from the condition of independence of this integral of the state of the medium inhomogeneity. Indeed, it follows from simple calculations of the integral

$$\iint {\rm d}^2\rho|F(z,\boldsymbol{\rho})|^2 = \left[\frac{2\pi(z-z_{\rm s})}{kV_{\rm s}}\right]^2 \times \iint {\rm d}z_1{\rm d}z_2\iint {\rm d}^2\rho\Theta(z_1,\boldsymbol{\rho})\Theta(z_2,\boldsymbol{\rho}) \sim (z-z_{\rm s})^2 \quad (24)$$

that the inhomogeneity of the medium does not affect the result, which is manifested in its proportionality to the square of the distance from the observation point to the scatterer. The classical scattering theory gives the same result in the quasi-optical approximation.

Let us find the basic features of the wave scattering caused by the inhomogeneity of the scatterer environment by the example of light scattering by a sphere of radius $a_{\rm s}$. We consider the case when the observation point lies in the far-field diffraction zone. Note that the condition of Fraunhofer diffraction in a regularly inhomogeneous medium has the form $|L_{\rm f}(z|z_{\rm s})| \gg ka_{\rm s}^2$, and because the effective refraction length $L_{\rm f}$ does not coincide with $z-z_{\rm s}$ due to the refraction of scattered radiation [see (21)], it considerably differs from its value in a homogeneous medium. By integrating (20) over the sphere volume under the condition $|L_{\rm f}| \gg ka_{\rm s}^2$ (see, for example, [2]), we obtain the scattering amplitude

$$F(\boldsymbol{R}|\boldsymbol{R}_{\rm s}) = 3\,\frac{z-z_{\rm s}}{h_2(z|z_{\rm s})}\,\frac{\sin q_{\rm s} - q_{\rm s}\cos q_{\rm s}}{q_{\rm s}^3}, \quad (25)$$

where $q_{\rm s} = q_{\rm s}(\boldsymbol{R}|\boldsymbol{R}_{\rm s}) = a_{\rm s}|\boldsymbol{k}_{\rm s}(\boldsymbol{R}|\boldsymbol{R}_{\rm s})|$. It follows from expression (25) that the main energy of the scattered wave is concentrated within the solid angle, where $q_{\rm s} \leqslant q_0$ ($q_0 = 4.493$ is the first zero of the function F). In this case, the maximum of scattered radiation will be observed in the direction for which $q_{\rm s} = 0$.

One can see from expression (22) for the scattering vector that the condition $q_{\rm s} = 0$ is fulfilled for $\boldsymbol{\rho} - \rho_{\rm s} = \boldsymbol{\rho}_{\rm s}^{\rm (m)}(z|z_{\rm s})$, i.e., the vector $\boldsymbol{\rho}_{\rm s}^{\rm (m)}$ determines the value and direction of the shift of the 'centre of gravity' of scattered radiation in the plane $z = {\rm const}$ with respect to the transverse coordinate $\boldsymbol{\rho} = \boldsymbol{\rho}_{\rm s}$ of the sphere position. Therefore, the first characteristic feature of the behaviour of $U_{\rm s}$ related to the inhomogeneity of the scattering sphere environment is the shift of the scattered radiation amplitude. It follows from (23) that the shift of the scattered wave amplitude is determined by two factors: the particle position in the plane $z = z_{\rm s}$ (vector $\boldsymbol{\rho}_{\rm s}$) and regular refraction in the inhomogeneous medium, which is described by the linear term in the expansion of $\Delta\varepsilon_{\rm m}$ in $\boldsymbol{\rho}$ [see (13)].

Let us now analyse the influence of the regular inhomogeneity of a medium on the angular size of the region where the main energy of the scattered wave is concentrated. We define the angular width of the scattering amplitude at a distance of $z-z_{\rm s}$ from the scattering particle by the expression

$$\Delta\theta(z|z_{\rm s}) = \frac{|\boldsymbol{\rho}_0 - \boldsymbol{\rho}_{\rm s} - \boldsymbol{\rho}_{\rm s}^{\rm (m)}(z|z_{\rm s})|}{z-z_{\rm s}}, \quad (26)$$

representing deviations from the direction $q_{\rm s} = 0$ at which the condition $q_{\rm s} = q_0$ is fulfilled. In (26), $\boldsymbol{\rho}_0$ is the radius vector of points in the plane $z = {\rm const}$ for which the condition $q_{\rm s} = q_0$ is fulfilled. From the definition of $q_{\rm s}$ and expression (22) for $\boldsymbol{k}_{\rm s}$, we obtain

$$\Delta\theta(z|z_{\rm s}) = \frac{q_0}{ka_{\rm s}}\,\frac{|h_2(z|z_{\rm s})|}{z-z_{\rm s}}. \quad (27)$$

It follows from (27) first of all that the angular width of the scattering amplitude in an inhomogeneous medium depends in the general case on the distance $z-z_{\rm s}$ and, depending on the refraction type and position of the observation plane, it can be either larger or smaller than the scattering amplitude in a homogeneous medium. Therefore, the second important feature of scattering in an inhomogeneous medium, which follows from expressions (22), (25) and (27) and is described within the framework of the model considered here, is the refraction change in the angular width of the scattering amplitude. Note also that expression (27) allows us to estimate qualitatively the value of integral (24) for the square of the scattering amplitude. Integral (24) is of the order of the product of the square of the scattering amplitude maximum $|F_{\rm m}|^2$ by the area ΔS of the region within which in the cross section $z = \text{const}$ the main energy of the scattered wave is concentrated. Because $\Delta S \sim (z - z_{\rm s})^2 \Delta\theta^2$, we can write approximately

$$\iint {\rm d}^2\rho |F(z,\boldsymbol{\rho})|^2 \sim \Delta S |F_{\rm m}|^2 \sim (z - z_{\rm s})^2,$$

which shows that the value of the integral is independent of the degree of inhomogeneity of a medium in which the scattered wave propagates.

4. Scattering by a particle upon thermal self-action and in the active medium of a laser

By using the general relations for the scattering amplitude obtained in section 2, we will study light scattering by a particle upon thermal self-action of a laser beam in the atmosphere and active medium of a nuclear-pumped laser. Consider the case when the observation point of scattered radiation is located in an inhomogeneous medium and the refraction channels are longitudinally homogeneous and axially symmetric.

Scattering in the thermal self-action channel. Upon thermal self-action of a laser beam in the atmosphere, the permittivity field becomes inhomogeneous in the self-action region due to the nonuniform heating of the air over the laser beam cross section. The self-action in an axially symmetric channel is $\varepsilon_{1x} = 0$. The parameter β^2 determining $\Delta\varepsilon_{\rm m}$ in (13) can be estimated from the expression

$$\beta^2(z) = \frac{1}{2}\frac{\partial\varepsilon}{\partial T}\frac{\partial^2}{\partial x^2}\,T(z,\boldsymbol{\rho})|_{\boldsymbol{\rho}=0} \sim \frac{1}{2}\left|\frac{\partial\varepsilon}{\partial T}\right|\frac{\Delta T}{a_0^2} > 0,$$

where a_0 is the laser beam radius and T is the air temperature in the self-action region. Because the temperature change ΔT in the self-action region lies between 0.001 and 0.1 °C [8] and the temperature variation in the air permittivity in the visible range is $|\partial\varepsilon/\partial T| \approx 2 \times 10^{-6}$, upon irradiation of a medium by a laser beam of radius $a_0 = 1 - 10$ cm, a defocusing thermal lens is formed, whose parameter β^2 varies between 10^{-11} and 10^{-7} cm^{-2}. The solutions of Eqn (16) for the constant values of this parameter have the form

$$h_1(z|z') = \cosh\beta(z - z'),\quad h_2(z|z') = \frac{\sinh\beta(z - z')}{\beta}.$$

By substituting these functions into (23) and (27), we obtain the expressions for displacement vector $\boldsymbol{\rho}_{\rm s}^{(\rm m)}$ of the scattering amplitude maximum and the angular width $\Delta\theta$ of the scattering amplitude:

$$\boldsymbol{\rho}_{\rm s}^{(\rm m)}(z|z_{\rm s}) = \boldsymbol{\rho}_{\rm s}\left(\frac{\cosh\beta z}{\cosh\beta z_{\rm s}} - 1\right),$$

$$\Delta\theta(z|z_{\rm s}) = \frac{q_0}{ka_{\rm s}}\,\frac{\sinh\beta(z - z_{\rm s})}{\beta(z - z_{\rm s})}.$$

The results obtained above show that the shift of the scattering amplitude and its width monotonically increase with distance from a scatterer in the thermal self-action region due to the defocusing action of the thermal lens. At distances exceeding the refraction length $L_{\rm r} = 1/\beta$, both the deviation from the direction along the Z axis of the 'centre of gravity' of scattered radiation and the broadening of the scattering amplitude, as follows from the expressions presented above, occur exponentially. The expression for $\boldsymbol{\rho}_{\rm s}^{(\rm m)}$ also shows that the scattering amplitude shifts in the direction of the particle position with respect to the axis of the refractive defocusing channel.

The main features of the behaviour of a wave scattered in a defocusing inhomogeneous medium are illustrated in Fig. 2, where the dependence of the square of the scattering amplitude in the XZ plane on the variable $g = 2ka_{\rm s}\sin\theta/2$ is presented; the numbers at the curves are the values of $\beta(z - z_{\rm s})$. The calculations were performed by expressions (21)–(23), (25) in the spherical coordinate system (r, θ, φ) (Fig. 1) with the origin located at the centre of a scattering sphere (with Cartesian coordinates $\boldsymbol{R}_{\rm s} = \{x_{\rm s}, 0, z_{\rm s}\}$. One can see from Fig. 2 that at distances from the scatterer exceeding the refraction length, the scattering amplitude noticeably broadens and its maximum shifts.

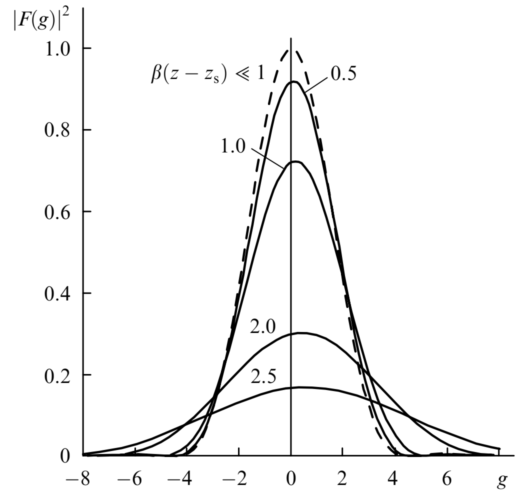

Figure 2. Square of the scattering amplitude as a function of the variable $g = 2ka_{\rm s}\sin\theta/2$ in a defocusing medium for $x_{\rm s} = 0.3$ cm, $k = 10^5$ cm^{-1}, $a_{\rm s} = 0.1$ cm, and $\beta = 10^{-4}$ cm^{-1}.

Scattering in the active medium of a nuclear-pumped laser. At present the most promising lasers of this type are lasers pumped by fission fragments [9]. A layer of a fissionable substance in these lasers is deposited on the side surface of the laser element, whose length is typically a few metres and the transverse size is a few centimetres (of the order of the fragment free path in the laser medium). A specific feature of the surface pumping is that upon deceleration of fragments in the laser medium, apart from excitation of active atoms, the gas density is also redistributed, resulting in the formation of a regular inhomogeneity of the permittivity field. The spatial distribution of this inhomogeneity can be approximately described by expression (13). Because the extended gas lens formed in this case is, as a rule, focusing (see, for example, [9]), the coefficient β^2 in (13) is negative. Its value depends on the specific pump energy E_0, pressure, and composition of the laser mixture. Special experimental studies performed on the Stand-B facility at the SSC RF-IPPE showed [13] that for $E_0 > 200$ mJ cm^{-3} and laser mixtures of interest (buffer gases He–Ar, Ar at a pressure of 0.5–1 atm), the coefficient $\beta^2 > 10^{-5}$ cm^{-2}. Therefore, refractive perturbations of the wave parameters at distances of a few metres from a source will be significant.

Scattering in the active medium of a laser can occur either from tracks of fission fragments or from medium density perturbations caused by the 'shot' type of pumping. Note that, while the influence of the track structure of the nuclear-excited plasma on kinetic processes in the active medium of a laser is known (see, for example, [14] for a He–Cd laser), no data are available at present about the effect of 'shot' pump fluctuations on random variations in the medium density. However, we can state definitely that the 'shot' pump effect causes density pulsations in the medium, and their influence on the lasing process and characteristics of the laser beam at high energy depositions can be considerable.

For constant negative values of the parameter β^2, the solutions of Eqn (16) are determined by the expressions

$$h_1(z|z_s') = \cos\beta(z - z'), \quad h_2(z|z') = \frac{\sin\beta(z - z')}{\beta}.$$

By substituting these expressions into (23) and (27), we obtain the shift vector and the width of the scattering amplitude in the form

$$\boldsymbol{\rho}_s^{(m)}(z|z_s) = \boldsymbol{\rho}_s\left(\frac{\cos\beta z}{\cos\beta z_s} - 1\right),$$

$$\Delta\theta(z|z_s) = \frac{q_0}{ka_s}\,\frac{|\sin\beta(z - z_s)|}{\beta(z - z_s)}.$$

These results show that the characteristics of the wave scattered in the active medium of a nuclear-pumped laser depend non-monotonically on the distance to a scatterer. Radiation focusing by a gas lens can result both in the broadening and narrowing of the scattering amplitude. In addition, it causes periodic variations in the position of the scattering amplitude maximum, which can approach the laser tube axis or remove from it.

Figure 3 shows the dependence of the square of the scattering amplitude in the XZ plane on the variable $g = 2ka_s \sin\theta/2$ in the focusing active medium of a nuclear-pumped laser (see Fig. 2). Calculations were performed by expressions (21)–(23), (25). One can see that at distances from a scatterer exceeding the refraction length, the scattering amplitude noticeably narrows and its maximum shifts.

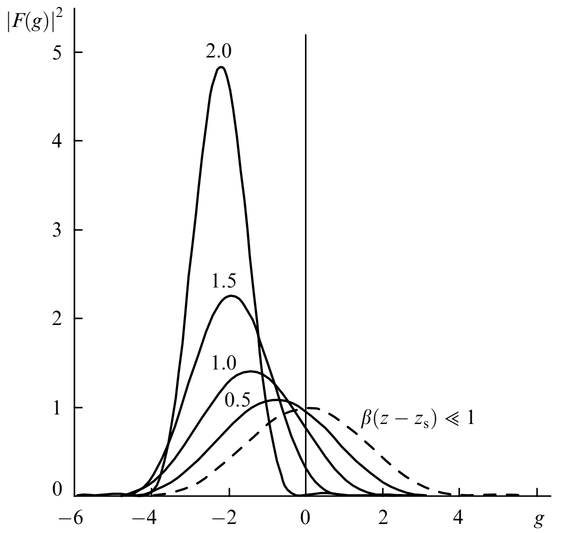

Figure 3. Square of the scattering amplitude as a function of the variable $g = 2ka_s \sin\theta/2$ in a focusing medium for $x_s = 0.1$ cm, $k = \pi \times 10^4$ cm^{-1}, $a_s = 0.1$ cm, and $\beta = 10^{-2}$ cm^{-1}.

5. Effect of regular refraction on the propagation of a radiation beam in a scattering medium

It follows from the results obtained in previous sections that the behaviour of such an important characteristics as the scattering amplitude is substantially different for scattering by a spherical particle in a homogeneous medium and in refraction channel. This difference is especially noticeable if the observation point is located at a distance from a scatterer comparable with the refraction length or exceeding it. It is obvious that these differences should preserve during the propagation of radiation in a medium containing many scatterers. Let us estimate the influence of the regular inhomogeneity of a medium on the characteristics of radiation propagating in a medium with randomly distributed scatterers. For this purpose, consider the propagation of a radiation beam in a refraction channel containing many uniformly distributed identical spherical particles.

It is known that the propagation of a radiation beam through a scattering medium is accompanied by the formation of secondary scattered waves. As a result, a part of the initial beam energy is 'carried away' by scattered radiation, and the beam amplitude decreases during the beam propagation. According to the general concepts of the wave propagation theory in scattering media (see, for example, [15]), the complex amplitude U of the wave in such media can be represented as a sum of the amplitudes of the unscattered ($\bar{U}$) and scattered ($\tilde{U}$) fields:

$$U = \bar{U} + \tilde{U}. \tag{28}$$

Let us discuss the dependence of the complex amplitudes of both waves on the regular and scattering optical characteristics of the medium.

The complex amplitude of the unscattered field is described by the expression [2]

$$\bar{U}(L, \boldsymbol{\rho}) = U_i(L, \boldsymbol{\rho}) \exp\left(-\frac{\alpha_{ext} L}{2} + ik\Delta n_s L\right), \tag{29}$$

where L is the path length; α_{ext} is the extinction coefficient of the medium; Δn_s is the change in the refractive index of the medium caused by scatterers; U_i is te complex amplitude of the wave incident on scatterers (taking the regular refraction into account), which obeys Eqn (10) with the initial condition (11). Expression (29) shows that the spatial structure of the unscattered beam $\bar{U}$ in the cross section $z = \text{const}$ is completely determined by the structure of the wave U_i incident on scatterers and depends both on the distribution of the complex amplitude U_0 at the input to the medium and the distribution of the regular inhomogeneity of the permittivity $\Delta\varepsilon_m$. The presence of scatterers in the medium giving rise to the amplitude reduction and a change in the phase of the unscattered field is taken into account by the exponential factor in (29). The extinction coefficient α_{ext} determining the rate of decreasing of the wave amplitude with increasing L in the case of a nonabsorbing medium, assuming the statistical independence of the positions of identical particles, is equal to the product of their average concentration $\bar{n}$ and the integrated scattering cross section σ_{sct}, [2]

$$\alpha_{ext} = \bar{n}\sigma_{sct}. \tag{30}$$

The change in the refractive index of a scattering medium, which determines the phase shift of the wave caused by scatterers in the case of uniformly distributed particles, has the form [2]

$$\Delta n_s = \frac{\bar{n}\Delta\varepsilon_s V_s}{2}, \tag{31}$$

where $\Delta\varepsilon_s$ are deviations of the scatterer permittivity from the characteristic value of the medium permittivity.

By using the definition of the scattering cross section

$$\sigma_{sct} = \frac{P_s}{I_i},$$

where

$$P_s = \frac{c}{8\pi}\iint d^2\rho |U_s(z,\boldsymbol{\rho})|^2$$

is the scattered radiation power,

$$I_i = \frac{c}{8\pi}|U_i|^2$$

is the intensity of the wave incident on a scatterer, as well as the expressions for the complex amplitude of the scattered wave (19) and integral (24), we obtain

$$\sigma_{sct} = \left(\frac{k\Delta\varepsilon_s}{2}\right)^2 \iint dz_1 dz_2 \iint d^2\rho\, \Theta(z_1,\boldsymbol{\rho})\Theta(z_2,\boldsymbol{\rho}). \tag{32}$$

Note that in a particular case of a scattering sphere of radius a_s, expression (32) is reduced to the known result (see, for example, [2]) for the integrated scattering cross section for a large, optically soft spherical particle:

$$\sigma_{sct} = \frac{\pi(k\Delta\varepsilon_s a_s^2)^2}{2}.$$

By substituting (32) into (30), we obtain the expression

$$\alpha_{ext} = \bar{n}\left(\frac{k\Delta\varepsilon_s}{2}\right)^2 \iint dz_1 dz_2 \iint d^2\rho\, \Theta(z_1,\boldsymbol{\rho})\Theta(z_2,\boldsymbol{\rho}). \tag{33}$$

for the extinction coefficient of the medium, from which follows, in particular, that the coefficient α_{ext} is independent of the presence of a regular inhomogeneity of the scatterer environment.

Thus, our analysis has shown that a continuously inhomogeneous medium, in which the scale of permittivity variation significantly exceeds the scatterer size (the approximation used in the paper), changes neither the integrated characteristic (33) of scatterers nor the refractive index (31) of the scattering medium compared to a homogeneous medium. Therefore, the regular optical inhomogeneity of the medium affects the unscattered field amplitude $\bar{U}$ in the same manner as the incident wave field U_i. This result is explained physically by the fact that the effect of refraction on the propagation of the incident wave in the vicinity of a scatterer is negligible.

Let us analyse now the influence of the regular inhomogeneity of a medium on the scattered wave field $\tilde{U}$. By using expression (19) for the complex amplitude U_s of the wave scattered by a particle in the single-scattering approximation and assuming that statistically independent identical scatterers are distributed uniformly, we obtain the average intensity $\langle \tilde{I} \rangle$ of scattered radiation

$$\langle \tilde{I}(L,\boldsymbol{\rho}) \rangle = \frac{c}{8\pi}\langle |\tilde{U}|^2 \rangle = \bar{n}\int_0^L dz_s \iint d^2\rho_s \frac{c}{8\pi}|U_s(L,\boldsymbol{\rho}|z_s,\boldsymbol{\rho}_s)|^2$$

$$= \bar{n}\left[\frac{k^2\Delta\varepsilon_s V_s}{4\pi(L-z_s)}\right]^2 \int_0^L dz_s \iint d^2\rho_s I_i(z_s,\boldsymbol{\rho}_s)|F(L,\boldsymbol{\rho}|z_s,\boldsymbol{\rho}_s)|^2. \tag{34}$$

It follows from (34) that the intensity distribution of the scattered field in the section $z = \text{const}$ is determined by the scattering amplitude of an individual particle. For clearness, consider the scattering of a narrow laser beam by a thin layer of scatterers. By neglecting in this case a change in the scattering amplitude within the region illuminated by the beam, we obtain from (34) the expression

$$\langle \tilde{I}(L,\boldsymbol{\rho}) \rangle = \bar{n}\Delta z P_i \left[\frac{k^2\Delta\varepsilon_s V_s}{4\pi(L-z_s)}\right]^2 |F(L,\boldsymbol{\rho}|z_s,0)|^2,$$

where $P_i = \iint d^2\rho_s I_i(z_z,\boldsymbol{\rho}_s)$ is the scattered beam power, and Δz and z_s are the thickness and the coordinate z of the layer, respectively.

The results presented in the previous section show that the behaviour of the scattering amplitude of a particle substantially depends on the type of a regular inhomogeneity of the medium. If the medium is defocusing on average, the combined action of an ensemble of scatterers due to a considerable broadening of the scattering amplitude and its shift gives rise to a considerable fluctuation broadening of the beam compared to the case of beam propagation in a homogeneous medium. Upon the thermal self-action of a laser beam in the atmosphere, the characteristic defocusing lens L_r changes from 10^4 to 10^5 cm depending on the beam power. Therefore, the increase in the fluctuation broadening of the beam on paths of lengths $z > L_r$ (due to a change in the type of scattering from a single inhomogeneity) will be substantial.

In a focusing medium, both the width of the scattering amplitude and the angular position of its maximum can change nonmonotonically. Therefore, the regular focusing inhomogeneity of a medium affects the field scattered by an ensemble of particles in a more varied way than the defocusing medium.

Note at the end of this section that the results obtained in this paper allow the explanation of some effects appearing during the propagation of waves in regularly inhomogeneous media with the permittivity fluctuations. Thus, the exponential broadening of the scattering amplitude, if interpreted as the effective decrease in the radius of scatterers, can qualitatively explain [15] a weaker rise of the dispersion of radiation intensity fluctuations in a defocusing refraction channel [16, 17]. The shift of the 'centre of gravity' and broadening of the scattering amplitude with distance from a particle explain qualitatively the exponential increase in the fluctuation change of the effective radius [18, 19] and jitter dispersion [20] of a laser beam propagating in the defocusing refraction channel in the turbulent atmosphere.

6. Conclusions

Let us summarise the results of the paper. The phenomenon of scattering of a wave by a particle located in an inhomogeneous medium is quite often encountered in practice, for example, during the generation and propagation of laser beams. We described this phenomenon by using the efficient methods of theoretical optics (the Green function method and quasi-optical approximation) to obtain the integrated expression for the complex amplitude of a scattered wave in terms of the Green function of the inhomogeneous medium. The amplitude of radiation scattered by a spherical particle in an inhomogeneous medium was obtained within the framework of the aberration-free approximation. The scattering amplitude was analysed quantitatively for the two scattering problems: in the thermal self-action defocusing channel of the laser beam in the atmosphere and in the focusing medium of a nuclear-pumped laser. It was shown that the inhomogeneity of the medium drastically changes the scattering pattern compared to scattering in a homogeneous medium, which is manifested in the refractive change in the angular width of the scattering amplitude and its shift. The analysis of the influence of the regular refraction on the propagation of a radiation beam in a scattering medium showed that the average inhomogeneity of the permittivity field in the medium considerably changes the scattered field intensity, which is determined by the scattering amplitude of individual particles.

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