*INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA*

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# Polarisation of the third harmonic generated by the pump field caused by collisions of electrons and ions in a plasma produced upon ionisation of a gas of excited hydrogen-like atoms

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***Abstract*. The polarisation properties of the third harmonic of the pump field are considered in a plasma produced upon ionisation of excited hydrogen-like atoms, taking into account *l* degeneration. These properties depend on the degree of circular polarisation and intensity of the pump field. The threshold nature of the total circular polarisation of the third harmonic appearing in the case of partial circular polarisation of the pump is established. This effect represents the bifurcation of the total circular polarisation. The conditions required to confirm experimentally the predicted polarisation properties of radiation are discussed.**

***Keywords***: *generation of harmonics, plasma, hydrogen-like atoms, radiation polarisation.*

**1.** Interest in the generation of pump-field harmonics in the plasma produced upon ionisation of preliminary excited atoms was aroused because, according to [1], the intensity of the third harmonic generated in such a plasma considerably exceeds that in the plasma produced upon ionisation of ground-state atoms. We study the generation of harmonics by the elliptically polarised pump field. The efficiency of harmonic generation by the elliptically polarised pump field in the plasma produced upon ionisation of a gas of hydrogen-like atoms in the ground state was investigated in [2] and in the excited state characterised by the principal quantum number $n$ in papers [3, 4]. It was shown in these papers that, when the pump energy exceeds the threshold value $q_{\rm th}$, the harmonic-generation efficiency is maximal not for a linearly polarised pump but for the case of the finite circular polarisation $A$ of the pump radiation. This effect was earlier experimentally observed in [5]. For a preliminary prepared plasma with the Maxwell velocity distribution of electrons, it was described in [6].

However, the model used in [6] cannot give the answer to the question about the role of excited states of atoms. This answer was obtained in papers [2 – 4], where it was shown that the maximum harmonic-generation efficiency for the specified pump intensity is achieved for a certain degree of circular polarisation of the pump, which depends on the ratio of the oscillation energy of electrons in the pump field to the excited-state energy of an atom before its ionisation.

The experimental effect observed in [5], which is consistent with the results obtained in [6, 2 – 4], is inherent in the bremsstrahlung of an electron experiencing oscillations near an ion in the electromagnetic pump-wave field. Let us emphasise that in papers [6] and [2 – 4] different plasma models were used, which, however, explained the generation of harmonics by the same reason, namely, by bremsstrahlung. In [6], the polarisation of generated harmonics was also studied. In this paper, unlike [6], where the electrons in a preliminary prepared plasma had the Maxwell distribution in the coordinate system oscillating together with electrons, we consider the plasma produced by a strong pump field. According to [7], we assume that the absolute value of the electric field strength $E$ satisfies the Bethe condition

$$|E| > \frac{I_{Zn}^2}{4Z|e^3|} = \frac{1}{4Z|e^3|}\left(\frac{Z^2 m_{\rm e} e^4}{2n^2\hbar^2}\right)^2. \quad (1)$$

Here, $e$ and $m_{\rm e}$ are the electron charge and mass, respectively; $Z$ is the atomic nucleus charge; $n$ is the principal quantum number of the electronic level from which ionisation of an atom occurs; and $I_{Zn}$ is the corresponding ionisation potential. Note that, when condition (1) is satisfied, no bound electronic states of a hydrogen-like atom with the energy characterised by the principal quantum number $n$ exist in a constant field.

Expression (1) can be written as an inequality for the radiation energy flux density $q$

$$q > q_{\rm B} = 1.37 \times 10^{14}\,\frac{Z^6}{n^8}\ \text{W cm}^{-2}.$$

We call the Bethe ionisation regime the regime in which the pump energy density satisfies inequality (2), while the switching time of the pump field proves to be insufficient for the rearrangement of electronic levels, so that the electrons are 'extracted' from atoms by the field in the states in which they were before ionisation. This means that the electrons extracted from the atoms have in the coordinate system oscillating together with them in the pump field the same velocity distribution as they had in the atom before ionisation. Therefore, the switching time $2\pi\tau_{\rm on}$ of a strong field should be smaller than the value

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$$\frac{n^2\hbar^2 2\pi}{Z^2 m_e e^4} \approx 1.5\times10^{-16}\,\frac{n^2}{Z^2}\ \ \text{s}, \tag{3}$$

which corresponds, in the quasi-classical case, to the rotation time of an excited electron in its orbit, and can be a few femtoseconds for excited states. By using the results [8, 9] for the electronic velocity distribution function, we obtain the expression

$$f_n(V) = \frac{8}{\pi^2 (V_Z/n)^3\left[1+(nV/V_Z)^2\right]^4}, \tag{4}$$

where $V_Z = Ze^2/\hbar$ is the Coulomb velocity unit [10].

Thus, the aim of our paper is to formulate the theory for studying the polarisation of pump-field harmonics generated in the plasma due to the retardation of electrons oscillating in the pump field by virtually immobile ions. The theory is applied to the third harmonic generation.

**2.** We will represent the electric field strength of the pump field in the form $\boldsymbol{E} = (E_x, E_y, 0)$, where

$$E_x = E\,\varepsilon_x \cos(\omega t - kz - \varphi),$$
$$E_y = -E\,\varepsilon_y \sin(\omega t - kz - \varphi). \tag{5}$$

Here, $\varepsilon_x$ and $\varepsilon_y$ are the components of the unit polarisation vector ($\varepsilon_x^2+\varepsilon_y^2=1$, $\varepsilon_x^2 \geqslant \varepsilon_y^2 \geqslant 0$) and the frequency $\omega$ and the wave vector $\boldsymbol{k}$ are related by the expression $\omega^2 = \omega_{\text{Le}}^2 + c^2k^2$, where $\omega_{\text{Le}}^2 = 4\pi e^2 N_e/m_e$ is the electron Langmuir frequency; and $N_e$ is the electron concentration.

By assuming, according to [11], that

$$\boldsymbol{E}(t,z) = \boldsymbol{E}\exp(-\mathrm{i}\omega t + \mathrm{i}kz + \mathrm{i}\varphi) + \boldsymbol{E}^*\exp(\mathrm{i}\omega t - \mathrm{i}kz - \mathrm{i}\varphi)$$

where $E$ is the constant amplitude, we write the expression for the polarisation tensor of the pump field:

$$\rho_{\alpha\beta} = \frac{J_{\alpha\beta}}{J},\quad J_{\alpha\beta} = \overline{E_\alpha E_\beta^*},\quad J_{\alpha\beta} = J, \tag{6}$$

where the first subscript corresponds to the line number and the second one the column number. The bar means averaging over the period of field variation in time. In our case, the expression for the polarisation tensor has the form

$$\rho_{\alpha\beta} = \begin{vmatrix} \varepsilon_x^2 & \mathrm{i}\varepsilon_x\varepsilon_y \\ -\mathrm{i}\varepsilon_x\varepsilon_y & \varepsilon_y^2 \end{vmatrix}.$$

The polarisation tensor can be also expressed in terms of the Stokes parameters $\xi_1$, $\xi_2$, and $\xi_3$ [11]

$$\rho_{\alpha\beta} = \frac{1}{2}\begin{vmatrix} 1+\xi_3 & \xi_1 - \mathrm{i}\xi_2 \\ \xi_1+\mathrm{i}\xi_2 & 1-\xi_3 \end{vmatrix}. \tag{7}$$

By comparing the two last expressions for the polarisation tensor, we obtain for the pump field (5) $\xi_1 = 0$, $\xi_2 = -2\varepsilon_x\varepsilon_y$, and $\xi_3 = \varepsilon_x^2 - \varepsilon_y^2$. In this case, $\xi_2 = -2\varepsilon_x\varepsilon_y = A(1) \equiv A$ is the degree of circular polarisation of the pump field ($-1 < A < 1$) [11] and $(\xi_3^2)^{1/2} = l = \rho^2$ is the degree of maximum linear polarisation.

**3.** Consider the problem of determining the influence of the degree of circular polarisation of harmonics on generation depending on the pump field. The harmonics are generated by the rapidly switched pump with the duration longer than its period when the plasma is produced from excited atoms with electrons. We will find the dependence of the degree of circular polarisation $A(2N+1, nV_E/V_Z, A)$ for the odd $2N+1$ harmonic, where $V_E = |eE|/(m_e\omega)$ is the amplitude of the oscillation velocity $\boldsymbol{u}_E = (u_{E_x}, u_{E_y}, 0)$ of an electron in the pump field;

$$u_{E_x} = -V_E\sin(\omega t - kz - \varphi);\ u_{E_y} = -V_E\cos(\omega t - kz - \varphi).$$

By using the Maxwell and Boltzmann equations, we can represent the electric field of harmonics in the form

$$\boldsymbol{E}^{(2N+1)} = \left(E_x^{(2N+1)}(t,z), E_y^{(2N+1)}(t,z), 0\right),$$

where

$$E_x^{(2N+1)}(t,z) = E_x^{(2N+1)}\sin[(2N+1)(\omega t - kz - \varphi)],$$
$$E_y^{(2N+1)}(t,z) = E_y^{(2N+1)}\cos[(2N+1)(\omega t - kz - \varphi)]. \tag{8}$$

In this case (cf. [3]), the constant amplitudes $E_x^{(2N+1)}$ and $E_y^{(2N+1)}$ are determined by the equations

$$\left[-(\omega^2 - c^2k^2)(2N+1)^2 + \omega_{\text{Le}}^2\right]E_x^{(2N+1)} = \varepsilon_x\,\frac{4\pi e^2 N_e}{m_e\omega^2}\,\nu_{xx}^{(2N+1)}(n,E,\rho)\omega(2N+1)E,$$
$$\left[-(\omega^2 - c^2k^2)(2N+1)^2 + \omega_{\text{Le}}^2\right]E_y^{(2N+1)} = \varepsilon_y\,\frac{4\pi e^2 N_e}{m_e\omega^2}\,\nu_{yy}^{(2N+1)}(n,E,\rho)\omega(2N+1)E, \tag{9}$$

where

$$\nu_{xx}^{(2N+1)} = \frac{16e^4 Z N_e \Lambda}{\rho^3 m_e^2 V_E^3}\,D\alpha^{(+)}(2N+1,\alpha,\rho)\Big|_{b=1},$$
$$\nu_{yy}^{(2N+1)} = \frac{16e^4 Z N_e \Lambda}{\rho^3 m_e^2 V_E^3}\,D\alpha^{(-)}(2N+1,\alpha,\rho)\Big|_{b=1} \tag{10}$$

are the effective frequencies of collisions between electrons and ions, determining harmonic generation. These frequencies can be found by using the kinetic Boltzmann equation with the Landau collision integral. In (10), $\Lambda$ is the Coulomb logarithm (we can assume for estimates that $\Lambda \approx 5$); $\alpha = b/x$; $x = nV_E/V_Z$; and

$$D = 1 - \frac{\mathrm{d}}{\mathrm{d}b} + \frac{1}{3}\frac{\mathrm{d}^2}{\mathrm{d}b^2}$$

is the differential operator; after applying this operator, we assume that $b = 1$.

The functions $\alpha^{(+)}$ and $\alpha^{(-)}$ are described by expressions

$$\alpha^{(+)}(2N+1,\alpha,\rho) = \frac{2\alpha\rho}{\pi(1+\rho^2)}$$
$$\times\int_{-1}^{+1}\mathrm{d}t'\,\Theta_{2N+1}^{(+)}(t')\left\{\frac{\rho^2\arctan\left[(1-\rho^2t')/(2\alpha^2)\right]^{1/2}}{\left[2\alpha^2(1-\rho^2t')\right]^{1/2}} - \right.$$

$$-\frac{\rho}{2\left[\left(1+t'\right)\left(1+\rho^2+2\alpha^2\right)\right]^{1/2}}$$

$$\times\ln\frac{\left(1+\rho^2+2\alpha^2\right)^{1/2}+\rho\left(1+t'\right)^{1/2}}{\left(1+\rho^2+2\alpha^2\right)^{1/2}-\rho\left(1+t'\right)^{1/2}}\Bigg\},\qquad(11)$$

$$\alpha^{(-)}(2N+1,\alpha,\rho)=\frac{2\alpha\rho}{\pi\left(1-\rho^2\right)}$$

$$\times\int_{-1}^{+1}\mathrm{d}t'\,\Theta^{(-)}_{2N+1}(t')\Bigg\{\frac{\rho^2\arctan\left[\left(1-\rho^2t'\right)/\left(2\alpha^2\right)\right]^{1/2}}{\left[2\alpha^2\left(1-\rho^2t'\right)\right]^{1/2}}$$

$$-\frac{\rho}{2\left[\left(1-t'\right)\left(1-\rho^2+2\alpha^2\right)\right]^{1/2}}\arctan\frac{\rho\left(1-t'\right)^{1/2}}{\left(1-\rho^2+2\alpha^2\right)^{1/2}}\Bigg\}.\qquad(12)$$

The quantities $\Theta^{(\pm)}_{2N+1}$ can be found from relations

$$Q_{(2N+1)/2}(z)\pm Q_{(2N-1)/2}(z)$$

$$=\pm\frac{1}{\sqrt{2}}\int_{-1}^{+1}\frac{\mathrm{d}t'}{\left(z-t'\right)^{1/2}}\,\Theta^{(\pm)}_{2N+1}\left(t'\right),\qquad(13)$$

where $Q_\nu(z)$ are the Legendre functions.

By using the condition $\omega^2=\omega^2_{\mathrm{Le}}+c^2k^2$, we obtain from Eqns (9) and (10)

$$E_x^{(2N+1)}=-\frac{4e^4ZN_{\mathrm{e}}\Lambda}{\rho^3m_{\mathrm{e}}^2V_E^3\omega}\frac{2N+1}{N(N+1)}\,\varepsilon_xD\alpha^{(+)}(2N+1,\alpha,\rho)\Big|_{b=1},\qquad(14)$$

$$E_y^{(2N+1)}=-\frac{4e^4ZN_{\mathrm{e}}\Lambda}{\rho^3m_{\mathrm{e}}^2V_E^3\omega}\frac{2N+1}{N(N+1)}\,\varepsilon_yD\alpha^{(-)}(2N+1,\alpha,\rho)\Big|_{b=1}.$$

Let us emphasise that we consider the transparent plasma with the positive dielectric constant because the pump field frequency exceeds the electron Langmuir frequency. In the case of the twofold excess, the dielectric constant of the plasma for the pump radiation is 0.75. For the third harmonic, it is approximately equal to 0.972, i.e., almost coincide with the dielectric constant of vacuum. For this reason, the third harmonic radiation in fact completely leaves the plasma.

By using expressions (14) and the definition of the polarisation tensor $\rho^{(2N+1)}_{\alpha\beta}$ for the $2N+1$ harmonic, we can write the general expression for the degree of circular polarisation of odd harmonics

$$A(2N+1,x,A)$$

$$=\frac{2AG(2N+1,x,\rho)H(2N+1,x,\rho)}{|A|\left[G^2(2N+1,x,\rho)+H^2(2N+1,x,\rho)\right.},\qquad(15)$$

where $\rho=(1-A^2)^{1/4}$; $|A|=(1-\rho^4)^{1/2}$;

$$H(2N+1,x,\rho)=\left(1+\rho^2\right)^{1/2}D\alpha^{(+)}(2N+1,\alpha,\rho)\big|_{b=1};$$

$$G(2N+1,x,\rho)=\left(1-\rho^2\right)^{1/2}D\alpha^{(-)}(2N+1,\alpha,\rho)\big|_{b=1}.\qquad(16)$$

In this case, according to (15),

$$A(2N+1,x,-A)=-A(2N+1,x,A).\qquad(17)$$

**4.** The general theoretical concepts formulated above can be applied to the third harmonic generation, when we have [2]

$$\alpha^{(+)}\left(3,\frac{b}{x},\rho\right)=\frac{\sqrt{2}\rho}{\left(1+\rho^2\right)^{1/2}}\left[\left(-\frac{1}{3}+\frac{4}{3\rho^2}\right)\right.$$

$$\times F\left(\arctan\frac{x\left(1+\rho^2\right)^{1/2}}{b\sqrt{2}},\frac{\rho\sqrt{2}}{\left(1+\rho^2\right)^{1/2}}\right)-\left(1+\frac{4}{3\rho^2}\right)$$

$$\left.\times E\left(\arctan\frac{x\left(1+\rho^2\right)^{1/2}}{b\sqrt{2}},\frac{\rho\sqrt{2}}{\left(1+\rho^2\right)^{1/2}}\right)\right]$$

$$+\frac{8b}{3x}\left[1-\left(\frac{x^2+2b^2-\rho^2x^2}{x^2+2b^2+\rho^2x^2}\right)^{1/2}\right],\qquad(18)$$

$$\alpha^{(-)}\left(3,\frac{b}{x},\rho\right)=-\frac{\sqrt{2}\rho}{\left(1+\rho^2\right)^{1/2}}\left\{\left(\frac{1}{3}+\frac{4}{3\rho^2}\right)\right.$$

$$\times F\left(\arctan\frac{x\left(1+\rho^2\right)^{1/2}}{b\sqrt{2}},\frac{\rho\sqrt{2}}{\left(1+\rho^2\right)^{1/2}}\right)-\left(-1+\frac{4}{3\rho^2}\right)$$

$$\times\frac{1+\rho^2}{1-\rho^2}\left[E\left(\arctan\frac{x\left(1+\rho^2\right)^{1/2}}{b\sqrt{2}},\frac{\rho\sqrt{2}}{\left(1+\rho^2\right)^{1/2}}\right)\right.$$

$$\left.\left.-\frac{b2\sqrt{2}}{\left(1+\rho^2\right)^{1/2}}\frac{\rho^2x}{\left[\left(x^2+2b^2\right)^2-\rho^2x^4\right]^{1/2}}\right]\right\}$$

$$-\frac{8b}{3x}\left[1-\left(\frac{x^2+2b^2+\rho^2x^2}{x^2+2b^2-\rho^2x^2}\right)^{1/2}\right],\qquad(19)$$

where $F$ and $E$ are the elliptic functions. For small values of $x/b$, these expressions have the approximate form

$$\alpha^{(+)}\left(3,\frac{b}{x},\rho\right)\approx\frac{\rho^5x^5}{30\,b^5}-\frac{\rho^5\left(4+\rho^2\right)x^7}{112\,b^7}$$

$$+\frac{\rho^5\left(4+2\rho^2+\rho^4\right)x^9}{144\,b^9}+O\left(x^{11}\right),\qquad(20)$$

$$\alpha^{(-)}\left(3,\frac{b}{x},\rho\right)\approx\frac{\rho^5x^5}{30\,b^5}+\frac{\rho^5\left(-4+\rho^2\right)x^7}{112\,b^7}$$

$$+\frac{\rho^5\left(4-2\rho^2+\rho^4\right)x^9}{144\,b^9}+O\left(x^{11}\right).\qquad(21)$$

Accordingly, we obtain

$$H(3,x,\rho)\approx\frac{8}{15}\,\rho^5x^5\left(1+\rho^2\right)^{1/2}\times$$

$$\times\left[1-\frac{25\left(4+\rho^2\right)x^2}{56}+\frac{25\left(4+2\rho^2+\rho^4\right)x^4}{48}+\ldots\right],$$

$$G(3,x,\rho)\approx\frac{8}{15}\,\rho^5x^5\left(1-\rho^2\right)^{1/2}\qquad(22)$$

$$\times\left[1+\frac{25\left(-4+\rho^2\right)x^2}{56}-\frac{25\left(4-2\rho^2+\rho^4\right)x^4}{48}+\ldots\right].$$

With the same accuracy, we have

$$A(3,x,A) \simeq A\left[1+\frac{25}{28}(1-A^2)x^2 - \frac{4175(1-A^2)^2-3750(1-A^2)^4}{4704}x^4\right]. \quad (23)$$

This approximate expression for the degree of circular polarisation of the third harmonic for small values of $x$ monotonically decreases from the maximum value $A(3,x,A)=+1$ for $A=-1$ down to zero for $A=0$ and down to the minimum value $A(3,x,A)=-1$ for $A=+1$. Note that for $|A|=1$, the third harmonic intensity vanishes.

For large values of $x$, i.e., for high pump field strengths, the degree of circular polarisation of the third harmonic nonmonotonically depends on $x$ and $A$. This is shown in Fig. 1, where the function $A(3,x,A)$ is presented in the interval $0<x<20$. In particular, the situation appears when, for the specified value of $x$, the degree of circular polarisation of the third harmonic first increases with increasing the degree of circular polarisation $A$ of the pump from its minimum value $A=-1$ and then begins to decrease. After achieving the minimum, the function $A(3,x,A)$ again increases, passes through zero and then exhibits a nonmonotonic behaviour, in accordance with the antisymmetry condition (17).

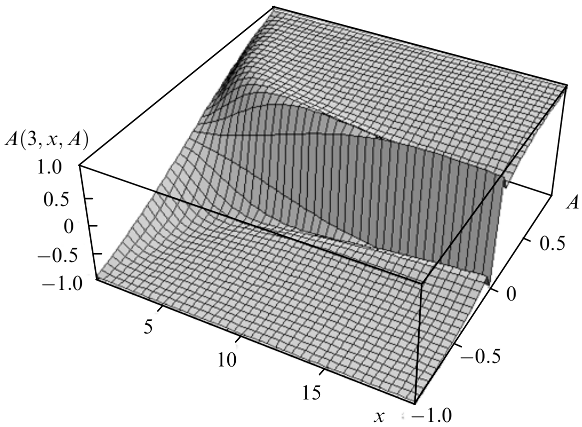


**Figure 1.**

The behaviour of the degree of circular polarisation of the third harmonic described above is better demonstrated in figures showing the sections of the three-dimensional function of Fig. 1 for different pump field strengths. Five such sections are shown in Fig. 2. Here, curve (*1*) ($x=1$) describes the monotonic dependence of the degree of circular polarisation of the third harmonic of the degree of circular polarisation of the pump field, while curve (*2*) ($x=1.5$) demonstrates an increase in the steepness of the monotonic dependence with decreasing the degree of circular polarisation of the pump. Curves (*3*) ($x=3$), (*4*) ($x=5$) and (*5*) ($x=7$) demonstrate, first, a drastic increase in the steepness of the decrease in $A(3,x,A)$ in the region of comparatively small values of $A$ and, second, the appearance of a nonmonotonic behaviour: for $-1<A<0$, the extremum $A(3,x,A)=-1$ is achieved. The degree of circular polarisation of the third harmonic tends to the same value for $A$ tending to $-1$. The curves exhibit a similar

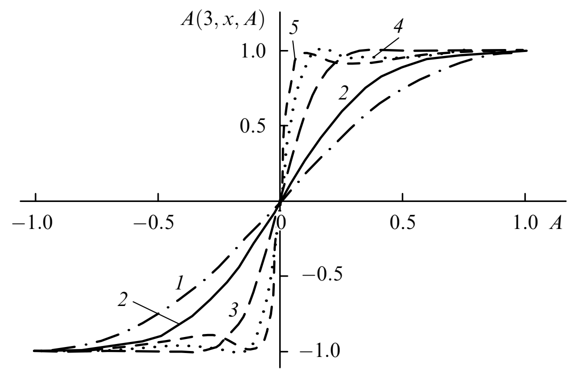


**Figure 2.**

behaviour for $A>0$, when $A(3,x,A)=+1$ for $A\neq 1$ and $A=1$.

The behaviour of the function $A(3,x,A)$ described by curves (*3–5*) in Fig. 2 is preserved as the pump field is further increased (Fig. 3). In the region of small degrees of circular polarisation of the pump, the steepness of the curves increases so that the curves corresponding to different $x>10$ cannot be distinguished at a linear scale for the variable $x$. For this reason, we plot the common logarithm of $A$ on the abscissa in Fig. 3. Curves (*1*), (*2*), (*3*), (*4*), and (*5*) correspond to the parameter $x=10$, 15, 30, 35, and 40, respectively.

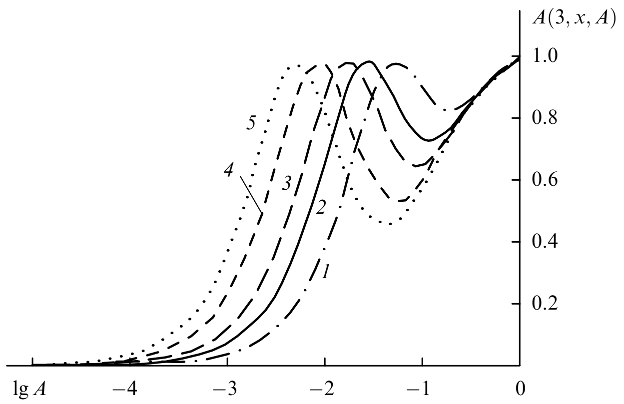


**Figure 3.**

It follows from Fig. 3 that the degree of circular polarisation of the third harmonic achieves unity, i.e., the circular polarisation is complete not only for $A=1$, when the third harmonic vanishes, but also for small values of $A$ (of the order of 0.01). In this case, the value of $A$ at which $A(3,x,A)=1$ decreases with increasing the pump field.

According to Figs 2 and 3, the assumed experimental studies of the degree of circular polarisation of the third harmonic can involve, first, comparatively rough measurements at moderate pump intensities, when $A(3,x,A)=\pm 1$ for the value of $A$ sufficiently distant from $A=0$ in the case $A(3,x,A)=0$; second, they can also involve comparatively accurate measurements, when the total circular polarisation of the third harmonic appear upon almost linearly polarised pumping, i.e., for $A\ll 1$, which naturally requires more accurate measurements of polarisation characteristics.

The behaviour of the curves presented in Fig. 2 shows that the nonmonotonic dependence and the corresponding possibility of the complete circular polarisation of the third harmonic for $A < 1$ (or $A > -1$) appear at moderate values of $x$ (or at moderate pump field strengths). In other words, the effect under study has a threshold. This is demonstrated, in particular, in Fig. 4, where the dependences of the degree of polarisation $A \equiv A(1)$ on $x$ are presented, for which the third harmonic radiation proves to be completely circularly polarised. Curve (*1*) in Fig. 4 corresponds to $A(3, x, A) = 1$, while curve (*2*) corresponds to $A(3, x, A) = -1$. It follows from Fig. 4 that the third harmonic becomes completely circularly polarised when the threshold $x_{\rm th} \approx 1.84456$ is exceeded near $A(1) = \pm 1$, and then the degree of circular polarisation of the third harmonic rapidly decreases in the absolute value with increasing pump power. It should be emphasised that this effect is in principle possible upon pumping by radiation with almost linear polarisation.

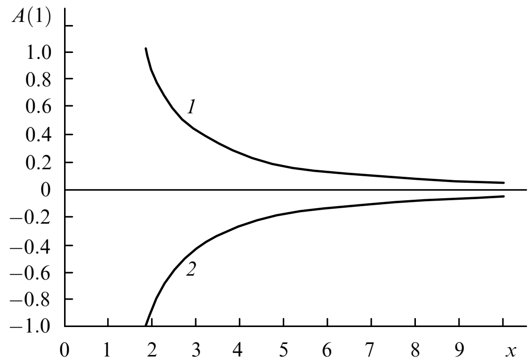


**Figure 4.**

**5.** The above discussion leads to the conclusion that some polarisation properties of the third harmonic generated by the pump field in the plasma produced due to ionisation of a gas of excited hydrogen-like atoms are inherent in the coherent bremsstrahlung harmonic generation in the plasma irrespective of the shape of the electron distribution function. Among these properties is the nonmonotonic dependence of the degree of circular polarisation of the third harmonic on the degree of circular polarisation of the pump, which was found earlier both in the asymptotic approximation [12] and numerically [4] for the case of preliminary prepared plasma with the Maxwell electron distribution. Another property is the generation of the completely circularly polarised third harmonic upon pumping by elliptically polarised radiation, which was also established asymptotically [12] and numerically [4]. The threshold nature of the effects under study established in our paper is a new one. We can assume that this property is also inherent in the bremsstrahlung generation of harmonics by the elliptically polarised pump field.

The important result concerning the influence of the excited states of atoms before their ionisation on the polarisation of the third harmonic is, first, qualitative and manifested in the dependence of $A(3, x, A)$ on the scaling argument $x = nV_E/V_Z$, which characterises the influence of the excited electronic state (in terms of the principal quantum number of this state) and, second, is revealed in the explicit dependence of the degree of circular polarisation of the third harmonic on $x$, which is illustrated in Figs 1–3.

Let us present here some expressions that allow one to understand better the experimental conditions under which the theoretical effects found above can be manifested. First of all, we write the expression for the energy flux density in the form

$$q = \frac{cI_{Zn}}{1024}\, x^2 \left(\frac{8\hbar\omega}{I_{Zn}}\right)^2 \frac{Z}{n^4 a_Z^3}$$

$$= 4.8 \times 10^{13} \frac{Z^2}{n^2} \frac{nV_E}{V_Z} \left(\frac{\hbar\omega}{1\ \text{eV}}\right)^2 \ \text{W cm}^{-2}, \qquad (24)$$

where $a_Z = \hbar^2/(m_{\rm e} Z e^2)$ is the Coulomb length unit [10]. Because the threshold for the appearance of nonmonotonic dependence of the degree of circular polarisation of the third harmonic found above is $x_{\rm th} \approx 1.84456 \approx 2$, we have for $\hbar\omega \approx 1$ eV the threshold density of the pump energy flux

$$q_{\rm th} \simeq 1.68 \times 10^{14} \frac{Z^2}{n^2} \ \text{W cm}^{-2}. \qquad (25)$$

According to (2), this expression can be rewritten in the form

$$q_{\rm th} = 1.2 \frac{n^6}{Z^4} q_{\rm B}. \qquad (26)$$

For example, for $Z = 2$ and $n = 7$, we obtain from (25) $q_{\rm th} = 1.37 \times 10^{13}$ W cm$^{-2}$ and $q_{\rm B} = 1.5 \times 10^9$ W cm$^{-2}$. The values of $q_{\rm B}$ and $q_{\rm th}$ for higher excited states will be even smaller.

Thus, we see that completely circularly polarised harmonics can be generated by using moderate pump intensities, when the ionisation of plasma corresponds to the Bethe regime. The Bethe condition itself (2) can be fulfilled quite simply, whereas it is more difficult to achieve condition (3), which for $Z = 2$ and $n = 6$ gives $\tau_{\rm on} < 1.35 \times 10^{-15}$ s.

In addition, electron–electron collisions should be considered, which can change the shape of the electron distribution, making it different from the distribution function (4) we used. To avoid this, the pump pulse duration should be small compared to the electron–electron collision time

$$\tau_{\rm ee} = \frac{m_{\rm e}^2 (V_Z/n)^3}{4\pi \Lambda e^4 N_{\rm e}} \simeq 2.8 \times 10^6 \frac{Z^3}{n^3} \left(\frac{1\ \text{cm}^{-3}}{N_{\rm e}}\right) \text{s}.$$

In particular, for $Z = 2$, $n = 7$, and $N_{\rm e} \approx 10^{17}$ cm$^{-3}$, we have $\tau_{\rm ee} = 6.5 \times 10^{-13}$ s. This estimate shows that it is possible to employ a radiation pulse whose duration greatly exceeds its switching time and is much shorter than the electron–electron collision time, which in turn allows one to use the quantitative theoretical results obtained above for the interpretation of possible experiments.

**6.** The results presented above lead to the following conclusions. First, a completely circularly polarised harmonic can be generated not only upon pumping by completely circularly polarised radiation but also by radiation with some degree of circular polarisation, which can be caused by the bifurcation of circular polarisation.

Second, this bifurcation has a threshold and appears when the energy of electron oscillations is comparable with the ionisation potential of the electronic level involved in the plasma production upon ionisation by the pump radiation. Third, the degree of circular polarisation of the third harmonic is the function of the ratio of the oscillation energy of the electron in the pump field to the energy of the *n*th electronic level from which ionisation occurs, which is related to the dependence of the bifurcation process on excitation of gas atoms before ionisation. Finally, the degree of circular polarisation of the pump required for the third harmonic generation with the 100 % circular polarisation decreases with increasing the ratio of these two energies.

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## References

1. Fedotov A.V., Naumov A.N., Silin V.P., Uryupin S.A., Zheltikov A.M., Tarasevich A.P., von der Linde D. *Phys. Lett. A*, **271**, 407 (2000).
2. Vagin K.Yu., Ovchinnikov K.N., Silin V.P. *Kvantovaya Elektron.*, **34**, 223 (2004) [*Quantum Electron.*, **34**, 223 (2004)].
3. Silin V.P., Silin P.V. *Kvantovaya Elektron.*, **35**, 157 (2005) [*Quantum Electron.*, **35**, 157 (2005)].
4. Silin V.P., Silin P.V. *Kratk. Soobshch. Fiz. FIAN* (in print).
5. Barnett N.H., Kan C., Corkum P.B. *Phys. Rev. A*, **51**, R3418 (1995).
6. Ovchinnikov K.N., Silin P.V. *Kvantovaya Elektron.*, **29**, 145 (1999) [*Quantum Electron.*, **29**, 983 (1999)].
7. Bethe H. *Quantunmechanik der Ein-und Zwei-Electronenprobleme, Handbuch der Physik* (Zweite Auflage, XXIV, Erste Teil, 1933).
8. Podolsky B., Pauling L. *Phys. Rev.*, **34**, 109 (1929).
9. Silin V.P., Silin P.V. *Kvantovaya Elektron.*, **33**, 897 (2003) [*Quantum Electron.*, **33**, 897 (2003)].
10. Landau L.D., Lifshits E.M. *Quantum Mechanics: Non-Relativistic Theory* (New York: Pergamon Press, 1980; Moscow: GIFML, 1963).
11. Landau L.D., Lifshits E.M. *The Theory of Field* (New York: Pergamon Press, 1980; Moscow: Nauka, 1973).
12. SilinV.P. *Zh. Eksp. Teor. Fiz.*, **114**, 864 (1998).