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# Enhancement of the measurement sensitivity in small lateral shearing holographic interferometry

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***Abstract.*** **To increase the measurement sensitivity in lateral shearing holographic interferometry, it is proposed to record, without the use of nonlinear effects, a pair of interference patterns tuned to closely spaced fringes and to specify small lateral shears between interfering beams having the same magnitude but opposite directions. The measurement sensitivity is enhanced at the stage of optical processing of such a pair of holographic interference patterns upon their exact coincidence or when they are located in optically conjugate planes. According to the results of experimental verification of the method, the sensitivity of the interference patterns increased twofold and fourfold.**

***Keywords***: *holographic interferometry, small lateral shear, enhancement of measurement sensitivity, optical processing.*

## 1. Introduction

An analysis of the wavefront shape of optical beams or phase objects and of the quality of various optical systems shows that in some cases the lateral shearing interferometry is not only inferior to the two-beam interferometry with the reference wave, but also has a number of indisputable advantages, such as the simplicity of the optical scheme and a low vibrational sensitivity [1, 2]. Currently, shearing interferometry is also used for measuring small wavefront distortions [3, 4].

The holographic version of lateral shearing interferometry considerably extanded the range of applications of the method due to the possibility of compensating aberrations and an arbitrary tuning of fringes in interference patterns, as well as increasing the measurement sensitivity [5–8]. A higher measurement sensitivity of small lateral shearing holographic interferometry ($\Delta s \ll r_0$, where $\Delta s$ is the lateral shear and $r_0$ is the mean size of an inhomogeneity of the phase object under study) was achieved by nonlinear recording of holographic interference patterns without changing $\Delta s$ [7] or by rerecording the initial linear interference patterns into secondary interference patterns under nonlinear conditions [8].

In this work, we show the possibility of increasing the measurement sensitivity of small lateral shearing holographic interferometry without nonlinear recording of holographic interference patterns. It is proposed to record a pair of holographic interference patterns with small lateral shear of the same magnitude but opposite signs. The optical processing of these interference patterns leads to possible versions of patterns whose sensitivity is higher by a factor of two and four.

Earlier, the aberrational characteristics of optical systems were studied by three-beam interferometry using opposite shear of wavefronts [9].

## 2. Recording of lateral shearing holographic interference patterns

Figure 1 shows the optical scheme for obtaining pairs of small lateral shearing holographic interference patterns. The optical beam under study is split by beamsplitter (*1*) and directed into lateral shearing interferometers (*2*, *4*) along two separate channels. The holographic interference patterns with small lateral shears $+\Delta s$ and $-\Delta s$ are recorded in planes (*3*) and (*5*). Figure 1 also shows the position of the coordinate systems $x$, $y$, $z$ and $x'$, $y'$, $z'$, the directions of

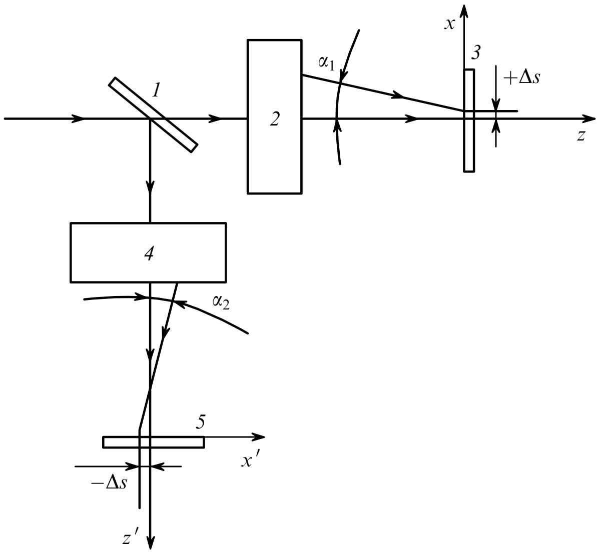

**Figure 1.** Optical scheme for recording lateral shearing holographic interference patterns (*1*) beamsplitter; (*2*, *4*) lateral shearing interferometers; (*3*, *5*) planes for recording holographic interference patterns.

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shears $+\Delta s$ and $-\Delta s$, and angles $\alpha_1$ and $\alpha_2$ between the interfering beams in each channel. An important feature of the method used here is that the formation of lateral shear holographic interference patterns in planes (*3*) and (*5*) requires their tuning to finite-width interference fringes [10] oriented, for example, parallel to $y$ and $y'$ axes.

Note that to provide spatial filtering of the corresponding diffraction orders during optical processing of small lateral shearing holographic interference patterns, the spatial frequencies $\xi_1$ and $\xi_2$ determined by the angles $\alpha_1$ and $\alpha_2$ should be at least 5 mm$^{-1}$ [11].

The difference between the spatial frequencies $\xi_1$ and $\xi_2$ is chosen from the condition of fringe tuning in the interference pattern observed during optical processing of coincident small lateral shearing holographic interference patterns. When the spatial frequencies are equal ($\xi_1 = \xi_2$), the interference pattern will display tuning to an infinitely wide fringe, while tuning to a finite-width fringe will be observed when these frequencies are slightly different ($|\xi_1 - \xi_2| \ll \xi_1$). The period of tuning fringes will be determined by the quantity $|\xi_1 - \xi_2|^{-1}$ [11].

To provide the desired tuning of interference fringes obtained during optical processing of coincident or optically conjugate lateral shearing holographic interference patterns, the fringe density of these interference patterns can be monitored with the help of reference diffraction gratings mounted in planes (*3*) and (*5*). The spatial frequencies of lateral shearing holographic interference patterns coincide with those of the reference diffraction grating when a moire pattern tuned to an infinitely wide fringe is observed in the grating plane. Thus, the required tuning of frequencies $\xi_1$ and $\xi_2$ of lateral shearing holographic interference patterns can be performed by using a set of reference diffraction gratings.

After recording the lateral shearing holographic interference patterns on a photographic emulsion and passing to the general coordinate system $xy$, the amplitude transmissions of the holographic interference patterns can be represented in the form

$$\tau_1(x,y) \sim 1 + \cos\left[2\pi\xi_1 x + \Delta s\frac{\partial\Phi(x,y)}{\partial x}\right], \tag{1}$$

$$\tau_2(x,y) \sim 1 + \cos\left[2\pi\xi_2 x - \Delta s\frac{\partial\Phi(x,y)}{\partial x}\right], \tag{2}$$

where $\Phi(x,y)$ is the phase describing the shape of the wavefront of the optical beam under study.

## 3. Recording of interference patterns of enhanced sensitivity

Upon illumination of exactly coincident lateral shearing holographic interference patterns (1), (2) and separation of the first-order diffraction by the known method [11, 12], the intensity distribution in the interference pattern is

$$I'(x,y) \sim 1 + \cos\left[2\pi(\xi_1 - \xi_2)x + 2\Delta s\frac{\partial\Phi(x,y)}{\partial x}\right]. \tag{3}$$

For $\xi_1 = \xi_2$, the interference pattern is tuned to an infinitely broad fringe, while for $\xi_1 \neq \xi_2$, it is tuned to finite-width fringes. In the latter case, the period of fringes will be determined by the quantity $|\xi_1 - \xi_2|^{-1}$. It follows from (3) that the location of fringes in the interference pattern reflects the behaviour of the phase derivative $\partial\Phi(x,y)/\partial y$, with a measurement sensitivity twice as high as for lateral shearing interference patterns (1) or (2).

A higher measurement sensitivity can be obtained by optical processing the pair of lateral shearing holographic interference patterns (1), (2) with the help of the scheme shown in Fig. 2, which is used for optical processing the holograms of phase objects [11]. In this case, the lateral shearing holographic interference patterns are formed in optically conjugate planes (*1*, *5*). When the first holographic interference pattern, for example, of type (1), is illuminated by a coherent collimated beam, waves of the type

$$A(x,y) = a\exp\left\{\mathrm{i}\left[2\pi\xi_1 x + \Delta s\frac{\partial\Phi(x,y)}{\partial x}\right]\right\} \tag{4}$$

and

$$A^*(x,y) = a\exp\left\{-\mathrm{i}\left[2\pi\xi_1 x + \Delta s\frac{\partial\Phi(x,y)}{\partial x}\right]\right\} \tag{5}$$

are diffracted from it to the ±1st orders, where $a$ is the real amplitude.

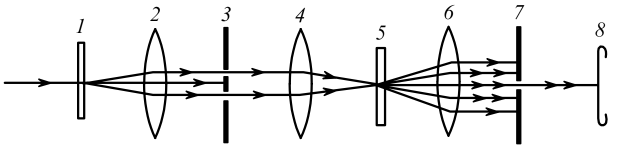

**Figure 2.** Optical diagram for processing conjugate lateral shearing holographic interference patterns: (*1*, *5*) holographic interference patterns; (*2*, *4*, *6*) objectives; (*3*, *7*) filtering diaphragms; (*8*) plane for observation of holographic interference patterns.

Waves of type (4) and (5) in the rear focal plane of objective (*2*) are separated by two apertures in the diaphragm (*3*) and illuminate the second lateral shearing holographic interference pattern (*5*) through objective (*4*). The distribution of complex amplitudes at the output of the second holographic interference pattern has the form $[A(x,y) + A^*(x,y)]\tau_2(x,y)$.

It can be shown that two waves of the type

$$B(x,y) = b\exp\left\{\mathrm{i}\left[2\pi(\xi_1 - \xi_2)x + 2\Delta s\frac{\partial\Phi(x,y)}{\partial x}\right]\right\}, \tag{6}$$

and

$$B^*(x,y) = b\exp\left\{-\mathrm{i}\left[2\pi(\xi_1 - \xi_2)x + 2\Delta s\frac{\partial\Phi(x,y)}{\partial x}\right]\right\}, \tag{7}$$

where $b$ is the real amplitude, will propagate near the normal to the holographic interference pattern (*5*). In the rear focal plane of objective (*6*), waves of the type (*6*) and (*7*) are separated by an aperture in diaphragm (*7*) and form an interference pattern

$$I''(x,y) \sim 1 + \cos\left[4\pi(\xi_1 - \xi_2)x + 4\Delta s\frac{\partial\Phi(x,y)}{\partial x}\right], \tag{8}$$

in plane (*8*), the period of fringes in the pattern being determined by the parameter $|2(\xi_1 - \xi_2)|^{-1}$. The measurement sensitivity aimed at determining the phase derivative $\partial\Phi(x, y)/\partial x$ increases fourfold in this case as compared to the scheme described by expression (1) or (2). Compared the scheme described by expression (3), the measurement sensitivity is twice as high.

## 4. Experimental results

The technique of increasing the measurement sensitivity of in lateral shearing holographic interferometry was tested experimentally during visualisation of the melting zones of polymethyl methacrylate crystals during their thermal processing.

A broadened light beam from a helium – neon laser passed through a sample and was directed to the experimental setup shown schematically in Fig. 1. Four-mirror lateral shearing interferometers (*2*) and (*4*) were used to vary arbitrarily the lateral shear and fringe density of shearing interference patterns. The fringe densities $\xi_1$ and $\xi_2$ in the initial holographic interference patterns were equalised by using an amplitude reference diffraction grating with a fringe density of 10 mm$^{-1}$. Such a grating was prepared photographically on a Micrat LOI-2 photographic plate by projecting on it a high-quality interference pattern obtained in a Mach–Zehnder interferometer. While recording holographic interference patterns, the lateral shear was set at 0.5 mm for a characteristic size $\sim$ 10 mm of the inhomogeneity region of the investigated sample. During tuning of the fringe density in the initial pair of lateral shearing holographic interference patterns, the reference diffraction grating was mounted in both cases in such a way that its lines had the same orientation. A peculiar feature of the experiment was the requirement to determine the first phase derivative $\partial\Phi(x, y)/\partial y$. For this purpose, the relative lateral shears were specified between interfering beams relative to the $y$ axis. The lateral shearing holographic interference patterns were recorded on a photographic plate FG-690 under ‘linear conditions’. Higher-order diffractions were not observed visually in the diffraction spectra.

Figure 3 shows the interference patterns obtained during optical processing of the pair of coincident lateral shearing holographic interference patterns with identical ($\xi_1 = \xi_2$) (Fig. 3a) and different ($\xi_1 \neq \xi_2$, where $|\xi_1 - \xi_2| \ll \xi_1$) frequencies (Fig. 3b). The interference pattern shown in Fig. 3c was obtained upon optical processing of the pair of optically conjugate holographic lateral shearing interference patterns that were used for obtaining the interference patterns shown in Fig. 3b. The ratio of the inflection (bending) of an interference fringe to the period for the interference patterns shown in Fig. 3c is twice that for the preceding interference pattern (Fig. 3b).

In conclusion, note that for an insufficient shearing of the interference patterns, the sensitivity can be enhanced by rerecording the initial lateral shearing holographic interference patterns using the method described in [13].

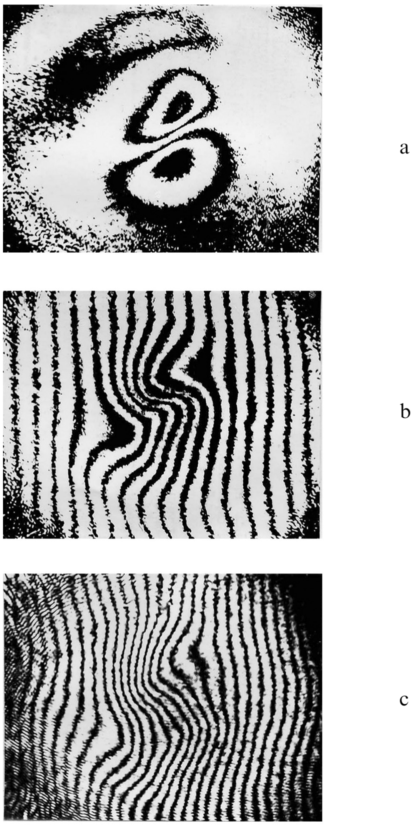

**Figure 3.** Interference patterns obtained during optical processing of coincident (a, b) and optically conjugate (c) lateral shearing holographic interference patterns.

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