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Peculiarities of the optimisation of optical resonators with regular small-scale inhomogeneities of an active medium

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***Abstract*. The peculiarities of parametric optimisation of resonators containing small-scale inhomogeneities are considered. The two most important practical cases are studied in detail: an unstable resonator with small-scale phase inhomogeneities in the active medium and a stable resonator with small-scale amplitude modulations caused by the use of a perforated output mirror.**

Keywords: *optical resonator, small-scale inhomogeneities, perforated mirrors, resonator optimisation, laser radiation divergence, intracavity losses.*

Intracavity aberrations, which most often appear in high-power lasers, can be conventionally divided into large-scale aberrations with the characteristic size comparable to the output aperture of a laser and small-scale aberrations comparable to the Fresnel zone size of the resonator [1–3]. Small-scale amplitude and phase inhomogeneities are caused by the presence of repeating design elements forming the active medium such as sequences of axially symmetric nozzles in the honeycomb nozzle unit in supersonic gas flow lasers [4], a sectional cathode in fast-flow electric-discharge lasers [5], flashlamps of a multitube pump unit of solid-state lasers [6], and perforated mirrors of a stable resonator to couple out radiation in CO_2 lasers [7–9].

In some cases, the small-scale inhomogeneities substantially affect radiation characteristics. The degree of the effect depends both on the type and parameters of the resonator and the structure of inhomogeneities themselves.

The two cases are most important from the practical point of view: small-scale phase inhomogeneities of the active medium in an unstable resonator and the amplitude modulation caused by the perforated output mirror used in a stable resonator. Consider the peculiarities of optimisation of parameters of such resonators for both types of inhomogeneities.

The laser radiation parameters can be divided into three main groups: energetic (power, energy, divergence, output energy density, etc.), spectral (wavelength, spectral width, etc.), and correlation (coherence and polarisation of radiation). Here, we are dealing with high-power lasers intended for the transfer of radiation energy to the far-field zone and (or) irradiation of an object in the focal plane and, therefore, the choice of the parameter for optimisation and possible functional and parametric restrictions will concern the first group of parameters. Table 1 lists the optimised parameters and parametric restrictions.

The laser radiation power can be estimated from the Rigrod formula [9]

$$P = \frac{I_s t S}{2}\frac{2k_0 L_0 + \ln(1-t-\beta)}{-\ln(1-t-\beta)}, \tag{1}$$

where L_0 is the active-medium length along the resonator axis; S is the aperture area; k_0 is the small-signal gain averaged over the medium volume; I_s is the saturation intensity; β is intracavity losses; t is the transmission coefficient of the output mirror, which for an unstable resonator can be expressed in terms of the magnification factor M as $t = 1 - 1/M^2$. The diffraction-limited divergence of the output radiation is

$$\theta = 2\frac{\lambda}{D}\frac{M}{M-1}. \tag{2}$$

Consider the peculiarities of optimisation of a stable resonator with a perforated mirror. The perforated mirror consists of n holes of diameter $d_0 \leqslant 2(\lambda L)^{1/2}$, where L is the resonator length. The transmission coefficient of the perforated mirror is $t = nS_0/S$, where $S_0 = \pi d_0^2/4$ is the area of one hole and S is the total area of the mirror. The construction of the perforated mirror is described in detail in [9].

The total losses in a resonator with a perforated mirror are $\beta = \beta_1 + \beta_2 + \beta_3$. Here, β_1 are losses on mirrors (due to absorption and scattering on the mirror surface), β_2 are diffraction losses at small 'apertures', which are formed between holes, and β_3 are absorption and scattering losses on edges of the holes [7].

It was shown in [7] that $\beta_3 = 4ht/d_0$, where h is the thickness of a technological facet on the hole edge (in real mirrors, $h = 0.05 - 0.1$ mm). The dependence of β_2 on the resonator parameters is more complicated. This component depends on the Fresnel number of the resonator $N_F = D^2(4\lambda L)^{-1}$, on the Fresnel number of the hole $N_0 = d_0^2/(4\lambda L)$ and the stability parameters of the resonator $g_1 = 1 - L/R_1$, and $g_2 = 1 - L/R_2$, where R_1, R_2 are the radii of curvature of mirrors. The nomogram from which losses β_2 can be determined is presented in [7]. The depend-

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Table 1. Optimised resonator parameters.

Optimised parameter	Possible restrictions	Resonator or amplifier type	Resonator or amplifier properties	Applications of the laser with the indicated resonator (amplifier)
P	$\theta \leqslant \theta_{\max}$	Stable resonator	$N_F > 10$	Adjustment of a laser (system for the active-medium preparation), radiation focusing
P/θ^2	$P \geqslant P_{\min}$	Stable resonator	$N_F < 10$	
P/θ^2	$P \geqslant P_{\min}$	Unstable resonator or single-pass amplifier	–	Radiation transfer to the far-field zone (radiation focusing on a target)
PSh	$P \geqslant P_{\min}$	Single-pass amplifier	–	

Note: P is the laser radiation power; θ is the radiation divergence; $N_F = a^2/(\lambda L)$ is the Fresnel number of the resonator; a is half the characteristic size of the aperture; L is the resonator length; λ is the radiation wavelength.

ence of the loss component β_2 on the basic parameters of the resonator can be writen in the general form

$$\beta_2 = \beta_2(t, N_0, N_F, g_1, g_2).$$

Figure 1 shows the calculated dependences of β_2 on the transmission coefficient t of the output mirror of the resonator for different N_F and $N_0 = 0.25$, $g_1 = 1$, and $g_2 = 0.75$. The calculation of the resonator in the diffraction approximation was performed by the method described in [10].

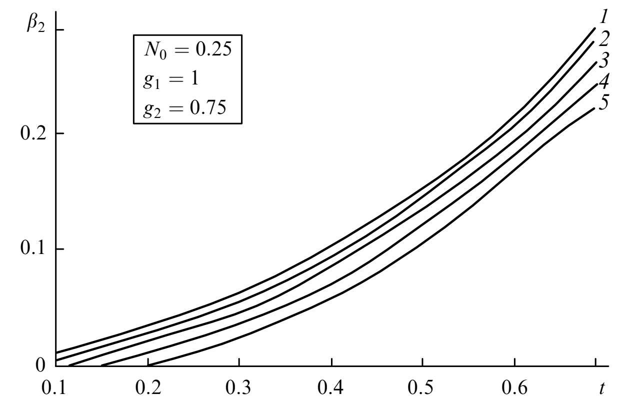


Figure 1. Dependences of β_2 on the transmission coefficient t of the output resonator mirror for $N_F = 20$ (*1*), 40 (*2*), 60 (*3*), 80 (*4*), and 100 (*5*).

Intracavity losses for a resonator with a perforated output mirror depend on the transmission coefficient of the output mirror in a complicated way:

$$P = \frac{I_s t S}{2}$$

$$\times \frac{2k_0 L_0 + \ln\{1 - t - [\beta_1 + 4ht/d_0 + \beta_3(t, N_0, N_F, g_1, g_2)]\}}{-\ln\{1 - t - [\beta + 4ht/d_0 + \beta_3(t, N_0, N_F, g_1, g_2)]\}}. \quad (3)$$

Figure 2 shows the dependences of the output power P of the laser with a stable resonator on the transmission coefficient t of the output mirror calculated by expression (3) for the parameters of the gas-dynamic CO_2 laser described in [7]: $k_0 = 0.5$ m^{-1}, $I_s = 2$ kW cm^{-2}, the active-medium length in the nozzle unit is 1 m, the distance between the resonator mirrors is 1.7 m, and the transverse size of the active medium is 0.12×0.12 m. The loss coefficient β_{10} at each perforated copper mirror was 0.015, while the total reflection losses were $\beta_1 = k\beta_{10}$ (k is the number of reflections during the round-trip transition of radiation in the resonator). We considered four possible schemes of the resonator presented in Table 2. The parameters of the resonator with the perforated mirror were compared with those of a standard resonator with the output mirror made of a zinc selenide crystal (hereafter, the crystal mirror).

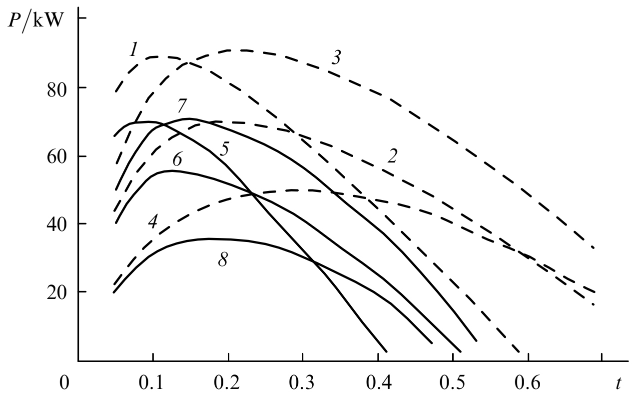
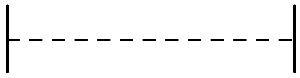
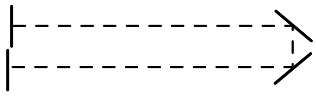
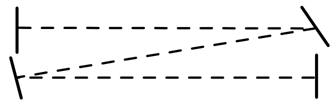
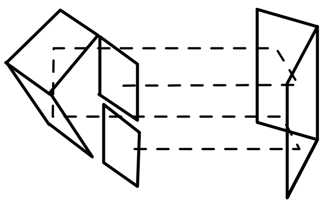


Figure 2. Dependences of the output power P of the gas-dynamic CO_2 laser with a stable resonator on the transmission coefficient t of the output crystal (*1–4*) and perforated (*5–8*) mirror. Curves (*1*) and (*5*) correspond to scheme No. 1, curves (*2*) and (*6*) to scheme No. 2, curves (*3*) and (*7*) to scheme No. 3, and curves (*4*) and (*8*) to scheme No. 4 (Table 2).

It follows from Fig. 2 that, both for the crystal and perforated mirrors, the maximum output power is achieved when the single-pass or three-pass resonator is used. The maximum output power for the double-pass and four-pass resonators is lower by 25 % and 50 %, respectively. This is explained by the fact that the number of reflections from mirrors and, hence, the loss component β_1 increase with increasing the number of passes; in addition, the area of the emitting aperture decreases. This in turn leads to the decrease in the output power, according to (1) and (2). On the other hand, the length of the active medium along the resonator axis increases, resulting in the increase in the output power. In the case of a three-pass resonator, the increased output power compensates for the enhanced losses and a decrease in the aperture. In the double-pass and four-pass resonators, the increase in losses and the decrease in the aperture dominate and the output power is reduced (losses in the four-pass resonator increase considerably, up to 21 %).

Table 2.

Scheme number	Type and scheme of a stable resonator	Aperture/m	$(L_0/\mathrm{m})/(L/\mathrm{m})$	N_F	k
1	Single-pass 	0.12×0.12	1/1.7	200	2
2	Double-pass (Π-shaped) 	0.12×0.06	2/3.4	56	6
3	Three-pass (Z-shaped) 	0.12×0.06	3/5	38	6
4	Four-pass 	0.06×0.06	4/6.8	12	14

It is also obvious that the maximum output power for the resonator with the perforated mirror is lower by 15 % – 25 % than that for the resonator with the crystal mirror, which is confirmed by experiments [7] and is explained by the increase in β_2 and β_3. In this case, the optimal transmission for resonators with the perforated mirror is always considerably lower (10 % – 20 %), which is also caused by increased losses.

Consider the influence of small-scale phase inhomogeneities of the active medium on the operation of an unstable resonator. In this case, three basic factors should be taken into account:

(i) The relation between the size of the first Fresnel zone of the resonator and the period of a spatial inhomogeneity;

(ii) the possibility of self-compensation of inhomogeneities during the propagation of oblique beams;

(iii) the increase in the divergence with decreasing the inhomogeneity scale.

Of special importance in the unstable resonator is the relation between the size of the first Fresnel zone and the period of a phase inhomogeneity in the active medium. As shown in [2], such inhomogeneities affect most strongly the divergence angle and output power, especially when the characteristic size of a small-scale inhomogeneity coincides with the size of the equivalent Fresnel zone of the unstable resonator. If the inhomogeneity is equivalent to a convergent lens, a local resonator with stability parameters substantially different from the calculated ones can appear in the axial region of the resonator. In this case, the diffraction coupling between the lasing region and the rest of the active medium is violated, resulting in the maximum increase in the radiation divergence. When the inhomogeneity size differs from the size of the Fresnel zone, the influence of perturbations on the radiation power and divergence is weak [2].

It is obvious that these properties determine the optimal parameters of the resonator. Consider an unstable resonator with the aperture 0.12×0.12 m, the distance between mirrors 3 m, and the active-medium length 2 m. The small-signal gain of the active medium is 0.5 m^{-1}, as before, and the saturation intensity is 2 kW cm^{-2}. The active medium contains a periodic inhomogeneity, which is similar to that simulated in [2] and has the characteristic spatial scale 2 cm. The diameter of the equivalent Fresnel zone of the unstable resonator can be determined from the expression

$$d_\mathrm{F} = 2\left(\frac{2\lambda L}{M-1}\right)^{1/2}.$$

For the parameters of the unstable resonator specified above and the magnification factor $M = 1.64$, the diameter of the Fresnel zone will be equal to the spatial scale of the inhomogeneity. In this case, according to [2], the increase in the radiation divergence and decrease in the output power will be greater than in an unperturbed active medium.

Figure 3 shows the dependences of the normalised divergence angle and normalised power on the magnification factor of the unstable resonator when the inhomogeneity is equivalent to a convergent or divergent lens. The divergence angle and power were normalised to their values obtained in the ideal active medium.

In both cases, the dependences of the radiation divergence have a pronounced maximum at a certain critical value of the magnification factor $M = 1.65$. This occurs when the inhomogeneity period coincides with the size of the Fresnel zone of the unstable resonator. In the axial region, which plays the role of a master oscillator, the local change in the stability parameters appears, which, as pointed out above, leads to the violation of the diffraction coupling between the axial region and the rest of the region of the unstable resonator, thereby breaking the matching of oscillation processes in the unstable resonator. All this distorts the mode structure, thereby increasing the radiation divergence.

When the magnification factor exceeds the critical value, less than a period of the inhomogeneity fits into the Fresnel

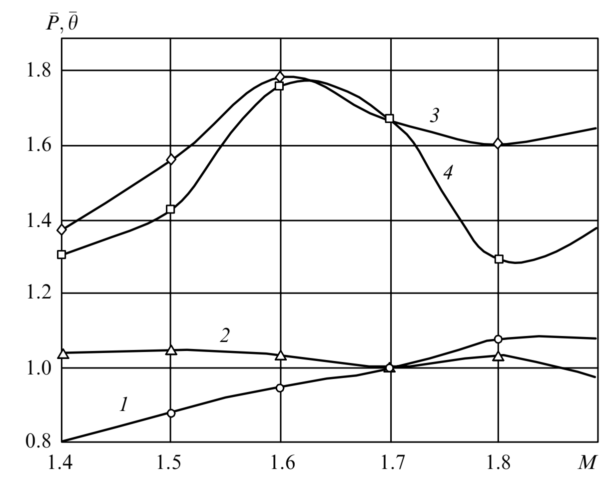


Figure 3. Dependences of the normalised power $\bar{P}$ (*1, 2*) and the normalised divergence $\bar{\theta}$ (*3, 4*) on M for an unstable resonator with an inhomogeneity on the resonator axis corresponding to a convergent (*1, 3*) and divergent (*2, 4*) lenses.

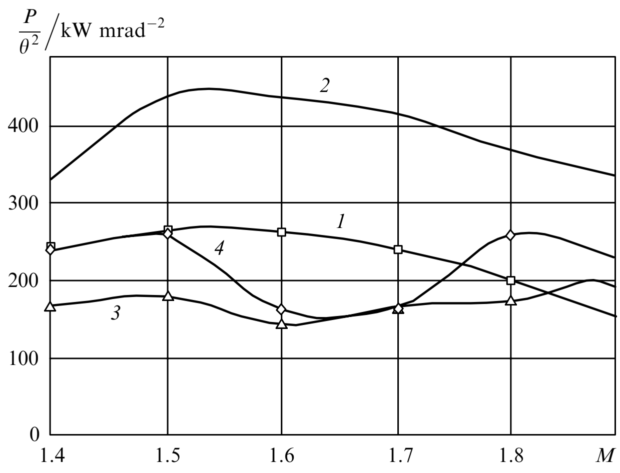


Figure 4. Dependences of the criterion P/θ^2 on the magnification factor M: (*1*) calculation by expressions (1) and (2); (*2–4*) calculation in the diffraction approximation; [(*2*) – ideal active medium, (*3*) – convergent lens on the resonator axis, (*4*) – divergent lens on the resonator axis].

zone and, hence, the phase dispersion in the region of the master oscillator becomes smaller than the dispersion over the total period of the spatial perturbation. When the magnification factor is lower than its critical value, the situation appears when the Fresnel zone contains more than one period of perturbations of the active medium. In this case, the diffraction 'spreading' of the inhomogeneity occurs in the axial region of the unstable resonator, i.e., its partial self-compensation.

In both cases, when the Fresnel zone is larger or smaller than the inhomogeneity period, a mode of the unstable resonator approaches in shape to the mode of a resonator with the ideal active medium, while the radiation divergence decreases compared to the case of the coincident inhomogeneity period and Fresnel zone size.

One can see from Fig. 3 that the relative radiation power in the case of a scattering inhomogeneity located on the resonator axis is almost independent of the magnification factor and, therefore, of the relation between the Fresnel zone size and inhomogeneity period. When the inhomogeneity on the resonator axis is equivalent to a convergent lens, the output power differs from the ideal case, but no more than by 20 %.

These properties lead to the corresponding differences in the dependence of the optimised parameter P/θ^2 of the unstable resonator on M. Figure 4 shows such dependences for four cases:

(i) The calculation by using expression (1) and (2);

(ii) the calculation of the resonator in the diffraction approximation for the ideal active medium;

(iii) the calculation of the resonator in the diffraction approximation when the axial inhomogeneity is equivalent to a divergent lens;

(iv) the calculation of the resonator in the diffraction approximation when the axial inhomogeneity is equivalent to a convergent lens.

One can see from Fig. 4 that the magnification factor $M \approx 1.6$ optimal for the ideal resonator, at which the maximum power output is achieved, proves to be the worst when the resonator contains small-scale inhomogeneities; for such a magnification factor, the Fresnel zone size coincides with the inhomogeneity period and the angular divergence of radiation becomes maximal. In this case, the optimal values are $M = 1.5$ or 1.8. In both cases, the far-field intensity, determined by the P/θ^2 criterion, is almost the same. However, depending on the inhomogeneity type, the output power for $M = 1.5$ can be higher by 15 % – 35 % than for $M = 1.8$. At the same time, for $M = 1.5$, the sensitivity of the unstable resonator to its possible inaccurate alignment and misalignment during operation (odd-order aberrations) increases.

Figure 5 shows similar dependences of the output power P on M. One can see that the optimal value of M for the ideal unstable resonator from the point of view of the maximum power calculated in the diffraction approximation is higher approximately by 20 % than that calculated by the Rigrod formula. This is explained, in particular, by the difference of the real transmission coefficient of the output mirror of the unstable resonator from that calculated in the geometrical approximation by the expression $t = 1 - 1/M^2$.

In summary, the following conclusions can be made:

(i) The decrease in the output power by 15 % – 25 % and the decrease in the optimal transmission coefficient of the output mirror by 10 % – 15 % in a stable resonator with the perforated output mirror compared to a similar resonator

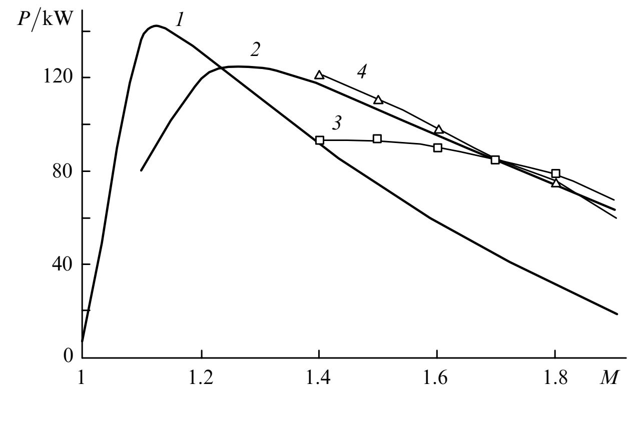


Figure 5. Dependences of the output power P on the magnification factor M (notation as in Fig. 4).

with the crystal output mirror are caused by a higher level of intracavity losses due to absorption and scattering of radiation at perforated mirrors. The output power in the stable resonator with the perforated mirror is determined to a great extent by the parameters of holes and the optical scheme of the resonator.

(ii) The output radiation of the unstable resonator with small-scale inhomogeneities in the active medium has the maximum divergence when the inhomogeneity period coincides with the size of the Fresnel zone of the resonator.

(iii) When the magnification factor corresponds to the size of the Fresnel zone of the resonator equal to the inhomogeneity period, the far-field radiation intensity decreases, which affects the choice of the optimal magnification factor during the optimisation of the unstable resonator.

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