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Dynamics of a photorefractive response and competition of nonlinear processes in self-pumping double phase-conjugate mirrors

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***Abstract*. The dynamics of formation of a nonlinear response of a double phase-conjugate (PC) $BaTiO_3$ mirror is calculated. It is shown that because of competition between processes of different types (related to the presence of several PC channels, the local and nonlocal components of the photorefractive nonlinearity), the transient and dynamic lasing regimes for this mirror can be substantially different. It is found that the development of lasing begins with the successive formation and phasing of dynamic holograms of two different types (two PC channels). It is shown that even under optimal conditions, the lasing regime is not stationary due to competition between processes of different types, and the parameters of output fields fluctuate in time in a nontrivial way (due to the presence of the in-phase and out-of-phase components). Several scenarios of transition to the dynamic chaos are described.**

Keywords: *double PC mirror, photorefractive nonlinearity, nonlinear response dynamics, competition between nonlinear processes, stable and unstable lasing regimes.*

1. Introduction

Phase distortions caused by the propagation of laser radiation in inhomogeneous paths can be compensated with the help of phase-conjugate (PC) mirrors based on photorefractive crystals (PRCs) [1], which can operate at radiation intensities up to a few mW cm^{-2} [2]. Some of these mirrors (the so-called self-pumping PC mirrors) require no additional pump sources [3] and can be used for conjugation of even noninterfering (incoherent or orthogonally polarised) light waves [4]. By varying the experimental parameters and geometry, one can either change the settling time of a nonlinear response in such mirrors from several tens of seconds to milliseconds and less [5] or obtain complicated self-oscillatory regimes with characteristic times up to several hours or days [2].

The authors of paper [6] showed that in 'one-crystal' double PC mirrors [1] based on a $BaTiO_3$ PRC, apart from a dynamic hologram formed in the self-intersection region of input beams, additional refractive-index grating also appear in the interaction geometry typical for 'two-crystal' PC mirrors [3]. The competition between the two resulting PC channels leads to the complex spatiotemporal dynamics of generated waves. The efficient generation of two PC waves is possible (at the maximum nonlinear reflectivity R_{max} of the PC mirror up to ~ 0.7) or of some resemblance of dynamic chaos in a system of thin soliton-like filaments. It was shown in paper [7] that in loop PC mirrors based on the same PRC, only a dynamic hologram 'operates', which is formed in the self-intersection region of the forward and backward beams. Therefore, such mirrors can provide the values $R_{max} = 0.8 - 0.9$ for the maximum overlap integral $H_{max} > 0.9$. The transition to unstable lasing regimes occurs in this case only due to self-action processes.

Below, by using calculation methods described in [7], we show that the formation dynamics of a nonlinear response of a double PC mirror is substantially more complicated than that of a loop PC mirror. Here, due to the competition between nonlinear processes of three types at once (competition of the first and second PC channels with self-action processes), the type of transient and dynamic lasing regimes can be substantially different depending on the experimental conditions.

2. Model of a double PC mirror

The geometry of the model problem is shown in Fig. 1. As in [6], we assume that the forward and backward light

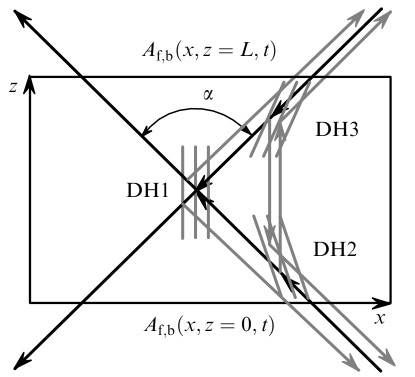

Figure 1. Interaction geometry of the waves $A_{f,b}(x, z, t)$ in a double PC mirror: DH1, DH2, and DH3 are dynamic holograms for the first (DH1, the self-intersection region of the waves) and second (DH2 and DH3) lasing channels.

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waves with the amplitudes $A_{f,b}$ and wave vectors $\boldsymbol{k}_{f,b} = \{k_x, \pm k_z\}$ propagate from the opposite faces of a PRC (planes $z = 0$, L) at a small ($k_z \gg k_x$) angle $\alpha/2$ to the positive (negative) direction of the z axis, respectively The wave-front tilts were taken into account in the expressions for $A_{f,b}$ by the phase factors of the type $\exp[ikx\sin(\alpha/2)]$.

The PRC response was calculated from the system of two-dimensional microscopic equations [8] taking into account transmission dynamic holograms (the grating vector is directed along the x axis) and neglecting the photovoltaic effect [1]:

$$\frac{\partial n}{\partial t} = \frac{\partial N_d^+}{\partial t} - \frac{1}{e}\frac{\partial j}{\partial x},$$

$$\frac{\partial N_d^+}{\partial t} = s(I + I_0)(N_d - N_d^+) - \gamma_r n N_d^+, \qquad (1)$$

$$j = e\mu n(E_0 + E_{sc}) - \mu\Theta\frac{\partial n}{\partial x},$$

$$\frac{\partial E_{sc}}{\partial x} = \frac{4\pi e}{\varepsilon}(n + N_a - N_d^+).$$

Here, n, N_a, N_d, and N_d^+ are the concentrations of free carriers, acceptors, neutral and ionised donors, respectively; s is the photoionisation cross section; $I(x,t)$ is the light intensity; the parameter I_0 describes the intrinsic conductivity of the PRC, defining the dark photoionisation rate as sI_0; γ_r is the recombination constant; e and μ are the charge and mobility of free carriers taking their signs into account ('$-$' for electrons and '$+$' for holes); $E_{sc}(x,t)$ is the electrostatic field of the crystal; ε is the quasi-static dielectric constant; Θ is the temperature in the energy units. We assume that the external electric field E_0 is applied to the PRC along the x axis, which allows one to consider the drift and diffusion components of the current density vector $\boldsymbol{j}$ only in one direction.

The problem was transferred to the class of self-consistent problems taking into account that $I(x,z,t) = |A_f(x,z,t)|^2 + |A_b(x,z,t)|^2$ (for incoherent or orthogonally polarised counterpropagating waves) and $E_{sc}(x,t)$ is specified by two standard truncated wave equations for the complex amplitudes $A_{f,b}(x,z,t)$ of the interacting light fields

$$\pm i\frac{\partial A_{f,b}}{\partial z} = \frac{1}{2k}\frac{\partial^2 A_{f,b}}{\partial x^2} + k\frac{\delta\eta}{\eta}A_{f,b} \mp i\alpha_a A_{f,b}. \qquad (2)$$

Here, $k = 2\pi\eta/\lambda$ is the wave number; η is the refractive index of the PRC; λ is the wavelength; $\delta\eta = -\frac{1}{2}r_{eff}\eta^3 E_{sc}(x,t)$ is the nonlinear addition to η; r_{eff} is the effective electro-optical constant; and α_a is the absorption coefficient. In (2), the spatially homogeneous addition to η caused by the field E_0 is omitted and the paraxial approximation is used. The two-dimensional model describes the interaction of the so-called slit beams in a PRC, which are often used in experiments with PRCs because of a strong anisotropy of the nonlinear response of the latter [9].

We assumed, as in [7], that the spatiotemporal spectrum of the field $E_{sc}(x,t)$ is specified by the distribution spectrum $I(x,z,t)|_{z=\text{const}}$ and the PRC plays the role of a spatiotemporal filter with the transfer function

$$T(\kappa,\Omega) = -\frac{E_0}{I_0}\frac{1 + i\kappa\Theta/(eE_0)}{1 - i\kappa aE_0(\chi+1) + \kappa^2 a\Theta(\chi+1)/e - i\Omega\tau_{di}}. \qquad (3)$$

Here, Ω is the temporal frequency of the κ component of the spatial intensity spectrum $I(\kappa,z,t)|_{z=\text{const}}$; $\tau_{di} = \varepsilon\gamma_r\chi/(4\pi e\mu sI_0)$ is the time of dielectric relaxation [1], and the following notations are used [10–12]

$$a = \frac{\varepsilon}{4\pi eN_a}, \quad \chi = \frac{N_a}{N_d - N_a}. \qquad (4)$$

3. Scheme of the numerical calculation

The self-consistent problem was solved by calculating numerically the evolution of distributions $A_{f,b}(x,z,t)$ and $\delta\eta(x,z,t)$ in time. All the variables were calculated using the grid with the number of nodes on the aperture ($X = 4$ mm) and length ($L = 4$ mm) of a PRC equal to 8192 and 512, respectively. The initial conditions corresponded to the 'switching on' of a PC mirror at the instant $t = 0$. Then (for $t \geqslant 0$), the input fields $A_f(x, z = 0, t)$ and $A_b(x, z = L, t)$ were assumed specified by superpositions of stationary useful signals $A_f^{(0)}(x, z = 0) = A_b^{(0)}(x, z = L)$ and delta-correlated (white, taking into account steps over x and t) noise signals $A_{f,b}^{n}(x,z,t)|_{z=0,L}$, whose average intensity $\langle I_n\rangle = \langle |A_{f,b}^{n}(x,z,t)|^2|_{z=0,L}\rangle$ was the same and varied within $10^{-3} - 10^{-6}$ of the maximum intensity I_{max} of useful signals.

The simulation was performed by using the approach described in [7], i.e., $A_{f,b}(x,z,t)$ was calculated assuming that the distribution $\delta\eta(x,z,t)$ is specified (the instant reaction of the light field to variations in the PRC). Each ith step in time (at the instant t_i) began (the arrow WE in Fig. 2) with the calculation of the instant distributions $A_{f,b}(x,z,t_i)$ and $I(x,z,t_i)$ by substituting distributions $\delta\eta(x,z,t_{i-1})$ for the instant t_{i-1} into (2). The spatial inhomogeneity η was taken into account by transmitting successively both light waves through infinitely thin phase screens, in which phase shifts were assumed specified by the nonlinear addition $\delta\eta(x, z = z_j, t_{i-1})$ (where j is the screen number) and the grid step over z. Diffraction effects were considered during the propagation of a signal between screens (the method of

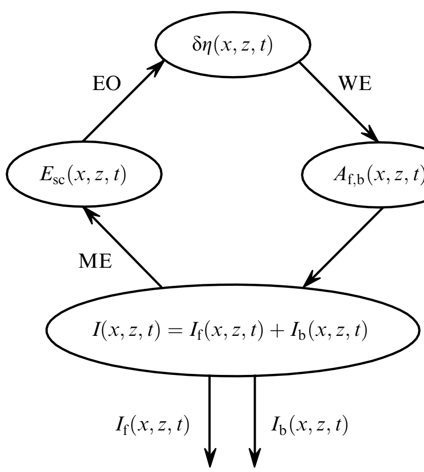

Figure 2. Solution of the self-consistent problem at each calculation step: WE is the calculation of $A_{f,b}(x,z,t)$ and $I(x,z,t) = I_f(x,z,t) + I_b(x,z,t)$ for the specified $\delta\eta(x,z,t)$ (wave equation); ME is the calculation of $E_{sc}(x,z,t)$ (constitutive equations); EO is the calculation of the nonlinear addition $\delta\eta(x,z,t)$ caused by the electrooptical effect.

separation over physical factors [11, 12]) by using the fast Fourier transform method. Then, system (1) was solved for the found distribution $I(x, z, t_i)$, and $E_{sc}(x, z, t_i)$ and $\delta\eta(x, z, t_i)$ required for the passage to the next step were calculated (the arrows ME and EO in Fig. 2) (the instant $t_{i+1} = t_i + \Delta t$).

As in [7], the time step $\Delta t = 0.1$ s was taken to be much shorter than the evolution time of the PRC state, and most of the parameters of the problem were not varied and corresponded to a $BaTiO_3$ PRC [5]:

sI_0/s^{-1}	4.0×10^{-5}
N_a/cm^{-3}	2.0×10^{17}
N_d/cm^{-3}	2.0×10^{18}
$s/cm^2\ W^{-1}\ s^{-1}$	0.67
$\mu/cm^2\ V^{-1}\ s^{-1}$	5.0×10^{-1}
$r_{eff}/cm\ V^{-1}$	9.7×10^{-7}
$\gamma_r/cm^3\ s^{-1}$	1.0×10^{-9}
ε	135
η	2.4
α_a/cm^{-1}	0.1

The intensity distributions $|A^{(0)}_{f,b}(x, z)|^2_{z=0,L}$ for both input beams were assumed Gaussian (the beam widths were 100 μm) for $\lambda = 0.514$ μm and $I_{max} = 2 - 200$ mW cm^{-2}. The period of dynamic gratings written in the PRC was changed by varying the intersection angle α of the beams within $10° - 15°$. The external electrostatic field E_0 was varied from 1 to 1000 V cm^{-1}.

4. Results of calculations

The lasing dynamics in a double self-pumping PC mirror under conditions close to optimum (in the energy efficiency) is illustrated in Fig. 3, where the time evolution of the nonlinear reflectivities (Fig. 3a)

$$R_{f,b}(t) = \left. \frac{\int_0^{X/2} |A_{b,f}(x, z, t)|^2 dx}{\int_0^{X/2} |A_{f,b}(x, z, t)|^2 dx} \right|_{z=0,L} \tag{5}$$

and the overlap integrals (Fig. 3b)

$$H_{f,b}(t) = \left. \frac{\left| \int_0^{X/2} A_{f,b}(x, z, t) A^*_{b,f}(x, z, t) dx \right|^2}{\int_0^{X/2} |A_{f,b}(x, z, t)|^2 dx \int_0^{X/2} |A_{b,f}(x, z, t)|^2 dx} \right|_{z=0,L} \tag{6}$$

is shown for the forward and backward waves $A_{f,b}$. The gradual transformation of the 'map' of the intensity distribution $I_f(x, z)$ in the PRC is presented in Figs 3c–e at the linear grey gradation scale (the darker regions corresponding to the lower intensities). Note that in the numerical calculation of $R_{f,b}(t)$ and $H_{f,b}(t)$, both output waves $A_{b,f}(x, z, t)|_{z=0,L}$ were transmitted through spatial filters separating halves the PRC apertures corresponding to the positions of the intensity maxima of the input beams $A^{(0)}_{f,b}$ in planes $z = 0, L$ [see (5), (6)].

The calculation showed (Fig. 3) that lasing in the PRC always begins with the formation of a dynamic hologram DH1 in the self-intersection region of the beams (the first lasing channel) and only then pairs of additional refraction-index gratings (DH2 and DH3) of the second PC channel are gradually formed (see Fig. 1). First the relation between the phases of the waves generated in these two PC channels

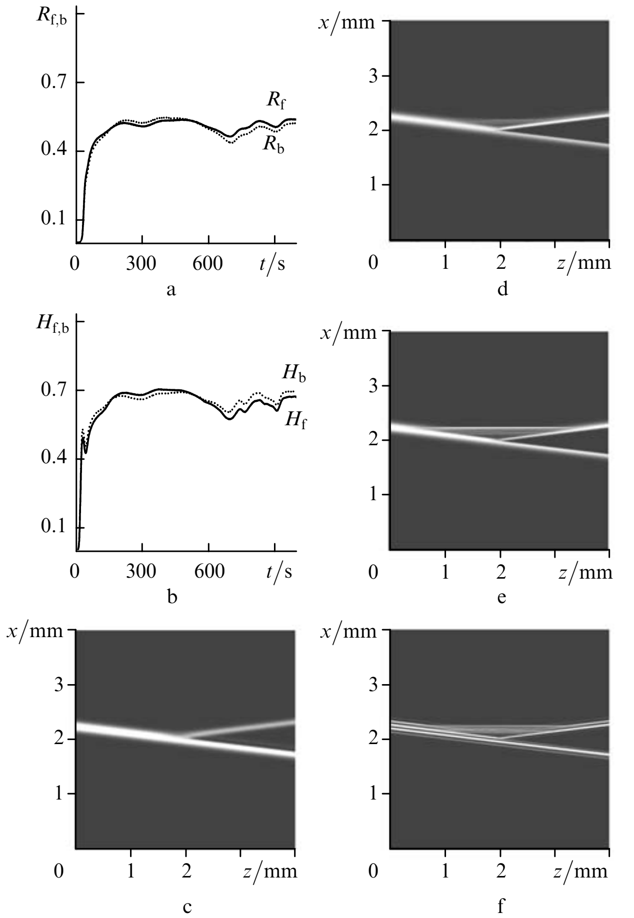

Figure 3. Development of lasing in a double PC mirror: dependences $R_{f,b}(t)$ (a) and $H_{f,b}(t)$ (b) and the 'maps' (linear grey gradation scale) of the distribution $I_f(x, z)$ at the instants $t = 41$ (c), 87 (d), and 1000 s (e) for $\alpha = 14°$, $I_{max} = 90$ mW cm^{-2}, $\langle I_n \rangle / I_{max} = 10^{-4}$, and $E_0 = 1$ V cm^{-1}, and the distribution $I_f(x, z)$ for the harmonic modulation of the input-beam intensity, $I_{max} = 180$ mW cm^{-2} and $t = 1000$ s (f).

is not optimal (the $H_{f,b}$ decreases in the short-time interval $t = 30 - 50$ s), but then the DH1, DH2, and DH3 gratings gradually become phased. And only then $H_{f,b}$ and $R_{f,b}$ achieve their maximum values ($H_{max} \simeq 0.6 - 0.7$ and $R_{max} \simeq 0.5 - 0.6$). Note that lasing is not nevertheless strictly stationary in this case as well, and the dependences $R_{f,b}(t)$ and $H_{f,b}(t)$ exhibit distinct slow in-phase oscillations. In addition, the forward and backward directions for a double PC mirror at each fixed instant prove to be non-equivalent ($R_f \neq R_b$ and $H_f \neq H_b$), the fluctuations of the differences $\Delta R(t) = R_f(t) - R_b(t)$ and $\Delta H(t) = H_f(t) - H_b(t)$ occurring out of phase [7]. Under optimal conditions, the PC mirror also well transfers the regular spatial intensity modulation to the output planes ($z = 0, L$), which was preliminary introduced to the distributions of the useful signals $A^{(0)}_{f,b}(x, z)|_{z=0,L}$ (Fig. 3f). However, to obtain the same values of H_{max} and R_{max} in this case, the maximum intensity I_{max} of the input signals should be doubled (the intensity averaged over x should be preserved).

The time during which the phase transition occurs (i.e., the conjugate components of the input field appear at the outputs of a double PC mirror) depends both on the average intensity of the interacting fields [5] and on the intersection angle α of the beams and the relative noise level $\langle I_n \rangle / I_{max}$ in the input radiation (Fig. 4). As α (Figs 4a, b) and $\langle I_n \rangle / I_{max}$

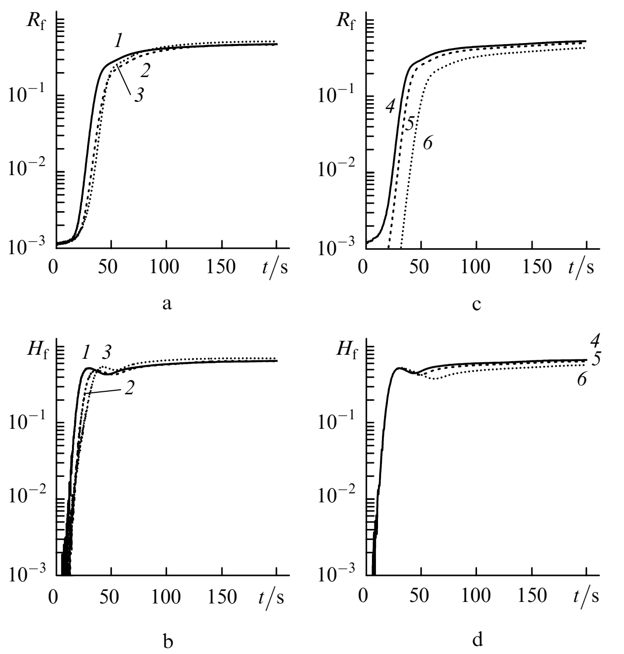

Figure 4. Slowing of the PC mirror 'start' with decreasing the intersection angle α of the beams and reduction of the relative noise level $\langle I_n \rangle / I_{max}$: dependences $R_f(t)$ (a, c) and $H_f(t)$ (b, d) for $\langle I_n \rangle / I_{max} = 10^{-4}$, $\alpha = 15°$ (*1*), 14° (*2*), and 13° (*3*) (a, b); $\alpha = 14°$, $\langle I_n \rangle / I_{max} = 10^{-4}$ (*4*), 5×10^{-5} (*5*) and 10^{-5} (*6*) (c, d), $I_{max} = 90$ mW cm^{-2}, $E_0 = 1$ V cm^{-1}.

(Figs 4c, d) increase, the conjugated component is formed faster. In this case, the character of changes in $R_{f,b}$ and $H_{f,b}$ in time upon varying α and $\langle I_n \rangle / I_{max}$ is qualitatively different. As α increases, the phase transition does begin earlier, i.e., the overlap integrals $H_{f,b}$ (Fig. 4b) and reflectivities $R_{f,b}$ (Fig. 4a) more rapidly achieve their maximum values H_{max} and R_{max}, respectively. At the same time, the increase in $\langle I_n \rangle / I_{max}$ almost does not affect the dependences $H_{f,b}(t)$ (Fig. 4d). This means, that, other conditions being the same, the dependences $R_{f,b}(t)$ more rapidly achieve the maximum exclusively due to the increase in the amplitude of initial noise seeds (Fig. 4c).

As in a loop PC mirror [7], the increase in the intensity I_{max} of the input beams (the increase in the nonlocal nonlinear component in the PRC [1]) leads to a drastic decrease in $H_{f,b}$ for a double PC mirror [curves (*2*) and (*3*) in Figs 5a, b]. If the intensities I_{max} exceed the optimal level, then after the formation of both lasing channels (Figs 6c, d), complex irregular systems of thin soliton-like filaments begin to appear in the forward and backward beams due to self-action processes [7] (Fig. 6e). It is in these systems that some resemblance of dynamic chaos appears then (Fig. 6f). This results in the almost complete destruction of the refractive-index gratings DH1, DH2, and DH3 in the PRC. As the strength E_0 of a static electric field applied to the PRC is increased (with increasing the local nonlinearity component in the PRC [1]), the generation efficiency of conjugate waves in a double PC mirror also decreases [curves (*5*) and (*6*) in Figs 5c, d].

However, the type of variations observed in this case is substantially different (Fig. 7). The dependences $H_{f,b}(t)$ become strongly irregular and, although the overlap integral

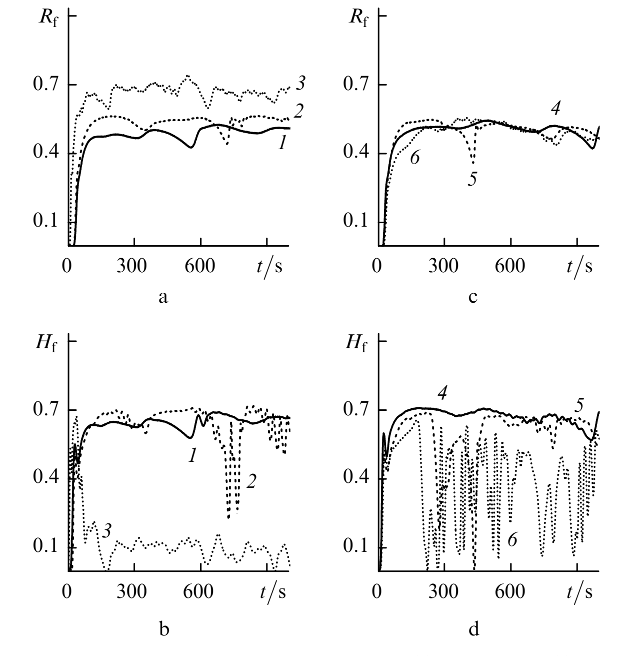

Figure 5. Transformation of the dependences $R_f(t)$ (a, c) and $H_f(t)$ (b, d) upon variations in the intensity I_{max} of the input beams and in the static electric field strength E_0: $E_0 = 1$ V cm^{-1}, $I_{max} = 90$ (*1*), 110 (*2*), and 140 mW cm^{-2} (*3*) (a, b), $I_{max} = 90$ mW cm^{-2}, $E_0 = 1$ (*4*), 50 (*5*), and 100 V cm^{-1} (*6*) (c, d), $\alpha = 14°$, $\langle I_n \rangle / I_{max} = 10^{-4}$.

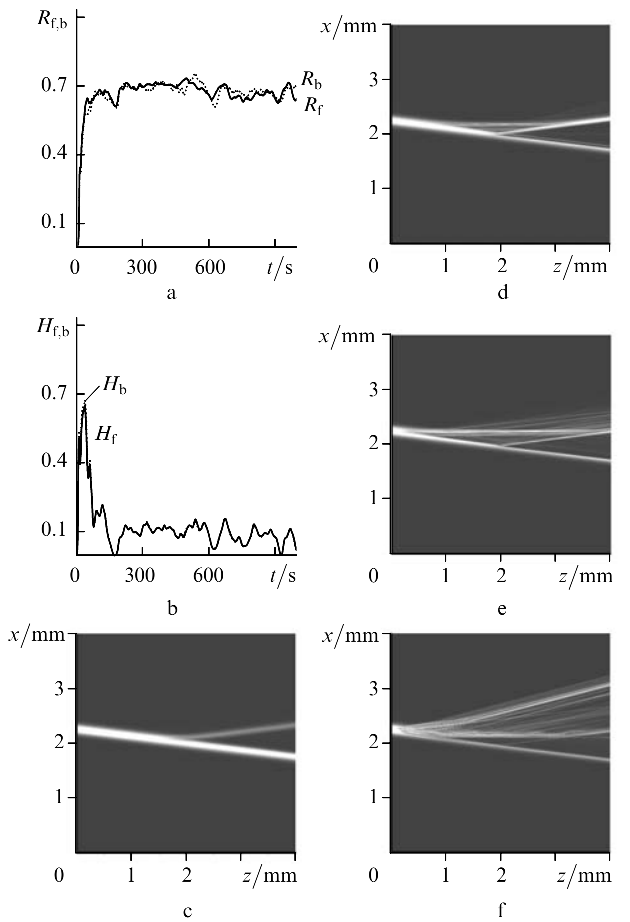

Figure 6. Development of the instability in a double PC mirror: dependences $R_{f,b}(t)$ (a) and $H_{f,b}(t)$ (b) and 'maps' of $I_f(x, z)$ (c–f) for $\alpha = 14°$, $I_{max} = 140$ mW cm^{-2}, $\langle I_n \rangle / I_{max} = 10^{-4}$, $E_0 = 1$ V cm^{-1} at the instants $t = 13$ (c), 35 (d), 64 (e), and 350 s (f).

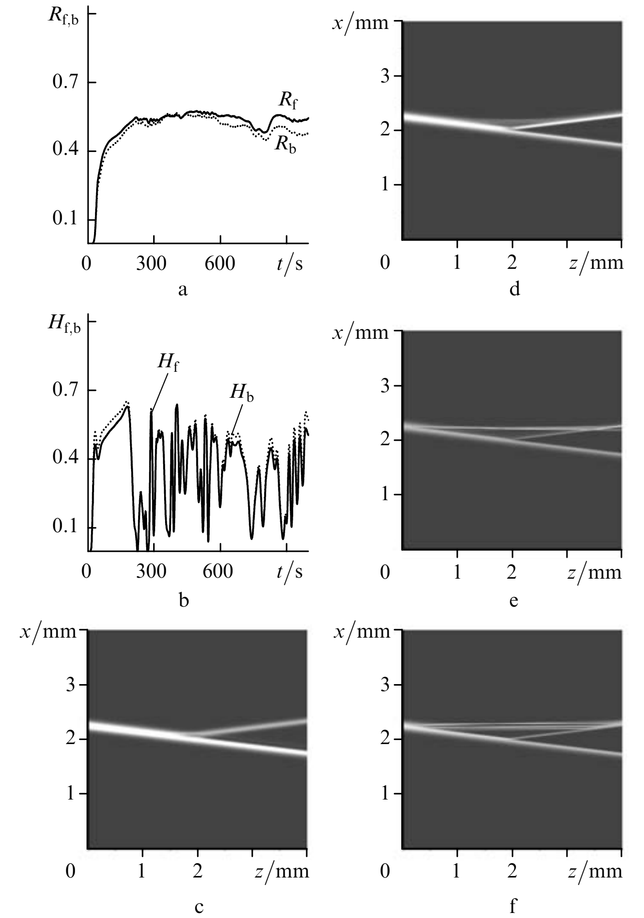

Figure 7. Development of the instability in a double PC mirror: dependences $R_{f,b}(t)$ (a) and $H_{f,b}(t)$ (b) and 'maps' of $I_f(x,z)$ (c–f) for $\alpha = 14^\circ$, $I_{max} = 90$ mW cm^{-2}, $\langle I_n \rangle / I_{max} = 10^{-4}$, $E_0 = 100$ V cm^{-1} at the instants $t = 44$ (c), 100 (d), 163 (e), and 1000 s (f).

still achieves approximately the same values ($H_{max} \simeq 0.6 - 0.7$) at certain instants, the depth of the irregular modulation of $H_{f,b}(t)$ is virtually 100 % (Fig. 7b). In this case, unlike the situation considered above (Fig. 6), the dynamic holograms DH1, DH2, and DH3 formed in the PRC are not destroyed (Figs 7c–f). In our opinion, this means that, as the nonlocal nonlinearity component of the PRC increases, phase relations between the components of the input fields generated in the two channels described above begin to fluctuate in time. These fluctuations give rise to sharp irregular 'spikes' in the dependences $H_{f,b}(t)$.

5. Conclusions

Our simulations have shown that the nonlinear reflectivities $R_{f,b}$ of a double $BaTiO_3$ PC mirror can achieve the values 0.5–0.6 for the overlap integrals $H_{f,b}$ up to 0.6–0.7. We have found that the generation of conjugate waves begins with the formation of a dynamic hologram in the self-intersection region of the forward and backward beams. Then, additional refractive-index gratings appear in the PRC (the second PC channel). Under optimal conditions, the lasing regime is nonstationary, and $R_{f,b}(t)$ and $H_{f,b}(t)$ fluctuate in phase. Moreover, the forward and backward directions for a double PC mirror at each instant prove to be non-equivalent, so that fluctuations of $R_{f,b}$ and $H_{f,b}$ for the forward and backward beams occur out of phase.

In the case of very intense input fields, the formation of both lasing channels is completed by the decay of the beams interacting in a PRC into complex systems of thin soliton-like filaments destroying regular refractive-index gratings. An increase in the strength of the electric field applied to the PRC leads to almost 100 % irregular modulation of dependences $H_{f,b}(t)$, although the overlap integral at the maxima of these dependences achieves approximately the same values as under optimal conditions. In this case, fluctuations of phase relations are observed between the components of the output fields generated in the two channels described above. Such phase fluctuations give rise to abrupt irregular 'spikes' in the dependences $H_{f,b}(t)$, while dynamic holograms in the PRC are not destructed.

The data presented above are in qualitative agreement with all the known experiments and, in our opinion, explain the characteristic features of the formation kinetics of a photorefractive response in self-pumping double PC mirrors [2, 5, 9, 13, 14].

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