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Features of the phase dynamics in a ring solid-state laser

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***Abstract*. The peculiarities of the phase dynamics are studied in a ring solid-state laser operating in transient quasi-sinusoidal oscillation regimes of the first and second kinds (QS-1 and QS-2) appearing upon periodic modulation of the pump power. It is shown that recording of a change in the phase difference of counterpropagating waves in the QS-2 regime under certain conditions makes it possible to determine directly the mutual nonreciprocity of the laser resonator.**

Keywords: *ring solid-state laser, phase dynamics, quasi-periodic lasing, frequency nonreciprocity.*

1. Introduction

Most of the papers on the study of the nonlinear dynamics of radiation of ring solid-state lasers have been devoted to investigations of the temporal and spectral parameters of the intensity of counterpropagating waves (see, for example, [1 – 5]). However, the dynamics of optical phases of counterpropagating waves (the phase dynamics of radiation) has not been adequately studied so far. This is explained by the fact that the direct measurement of the optical phases of counterpropagating waves and their difference is a rather complicated problem. Nevertheless, the methods for obtaining phase information by analysing the signal of optically mixed (heterodyned) radiation from two lasers [6, 7] or signals obtained by mixing the optical fields of counterpropagating waves [8] open up the possibilities for the experimental study of the phase dynamics.

The phase dynamics of radiation from a solid-state ring laser in the regime of high-frequency switching of the lasing direction (the second-order self-modulation regime) was considered theoretically in [9]. Similar studies performed in [10] showed that solid-state ring lasers operating in the synchronous dynamic chaos regime exhibit abrupt variations in the phase difference of counterpropagating waves by π in the intervals between adjacent chaotic radiation pulses. This conclusion was experimentally confirmed by the analysis of optical mixing of counterpropagating waves in a monolithic ring chip Nd : YAG laser [8, 11].

It is known that in ring solid-state lasers, numerous transient oscillations regimes can exist, which appear both in autonomous ring lasers [12, 13] and non-autonomous lasers upon periodic modulation of their parameters [5]. One of the most widespread transient oscillation regimes of ring solid-state lasers with periodically modulated parameters are quasi-sinusoidal (QS) regimes [5],which can be of the first (QS-1) and second (QS-2) kinds [5, 14]. In these regimes, the emission spectrum of counterpropagating waves exhibits components at the self-modulation oscillation frequency $\omega_a/2\pi$, the modulating signal frequency $\omega_p/2\pi$, and frequencies corresponding to their linear combinations. The difference between the regimes is that in the QS-1 regime the most intense is the spectral component at the frequency $\omega_a/2\pi$, while in the QS-2 regime the most intense is the component at the frequency $\omega_p/2\pi$. These regimes are bistable in a certain region of laser parameters (depending on the initial conditions, either the QS-1 or QS-2 regime appears).

The aim of this paper is to study the features of the phase dynamics of solid-state ring lasers in the oscillation regimes. Of special interest is analysis of the phase dynamics in the presence of frequency nonreciprocity in the resonator because the results of these studies can be used in laser gyros.

2. Formulation of the problem

We analysed the phase dynamics by using the standard model of a ring solid-state laser [5] based on the system of equations for the complex amplitudes of counterpropagating waves $\tilde{E}_{1,2} = E_{1,2}\exp \mathrm{i}\varphi_{1,2}$ and spatial harmonics of the inversion populations N_0 and $N_\pm$:

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{1,2} + \frac{\sigma L_{\mathrm{ac}}}{2T}\left(N_0\tilde{E}_{1,2} + N_\mp\tilde{E}_{2,1}\right),$$

$$T_1\frac{\mathrm{d}N_0}{\mathrm{d}t} = N_{\mathrm{th}}(1+\eta)(1+\eta) - N_0\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right] - N_+ aE_1E_2^* + N_- aE_2E_1^*, \quad (1)$$

$$T_1\frac{\mathrm{d}N_+}{\mathrm{d}t} = -N_+\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right] - N_0 aE_2E_1^*.$$

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Here, ω/Q is the resonator bandwidth (losses for counterpropagating waves are assumed identical); $T = L/c$ is the resonator round-trip transit time; T_1 is the longitudinal relaxation time; L_{ac} is the active element length; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter: σ is the laser transition cross section (the laser frequency detuning from the gain line centre is assumed zero); $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the resonator; ω_1 and ω_2 are the resonator eigenfrequencies for counterpropagating waves; $\tilde{m}_{1,2} = m_{1,2}\exp(\pm \mathrm{i}\theta_{1,2})$ are the complex coefficients of coupling of counterpropagating waves via backscattering; and $m_{1,2}$ and $\theta_{1,2}$ are the moduli and phases of coupling coefficients. The pump rate is represented in the form $N_{th}(1+\eta)/T_1$, where N_{th} is the threshold inverse population and η is the pump power excess over the threshold.

Consider a ring laser with periodically modulated pump power. In this case, the pump power excess over the threshold is

$$\eta = \eta_0 + h\cos\omega_p t, \tag{2}$$

where η_0 is the threshold excess in the absence of modulation and h is the pump modulation depth.

The phase dynamics of the ring laser is studied by solving numerically the system of equations (1) in the following way. The complex amplitudes of counterpropagating waves are written in the form

$$\tilde{E}_1 = E_{r1} + \mathrm{i}E_{i1} \text{ and } \tilde{E}_2 = E_{r2} + \mathrm{i}E_{i2},$$

where E_{rj} and E_{ij} are the real and imaginary parts of the complex amplitudes of counterpropagating waves ($j = 1$, 2). The numerical solution of the system of equations (1) gives the time dependence of the fields of counterpropagating waves and then, by using the expressions

$$\cos\Phi = \frac{E_{r1}E_{r2} + E_{i1} + E_{i2}}{E_1E_2}, \quad \sin\Phi = \frac{E_{i1}E_{r2} - E_{i2}E_{r1}}{E_1E_2} \tag{3}$$

we study the time dependence of the phase difference $\Phi = \varphi_1 - \varphi_2$ for counterpropagating waves.

3. Results of numerical simulations

We used in numerical simulations the parameters of a monolithic ring chip laser studied in [3, 14]. The monolithic ring Nd:YAG laser investigated in these papers had the resonator with a perimeter 2.7 cm and the non-planarity angle 80°. The total round-trip transit losses in the resonator were 3.2 %, which for $\eta_0 = 0.21$ gave the relaxation oscillation frequency $\omega_r/2\pi = 65$ kHz. The self-modulation oscillation frequency in the absence of the frequency nonreciprocity of the resonator was 170 kHz, corresponding to the modulus of the coupling coefficient $m = m_1 = m_2 = 170$ kHz and the phase difference of coupling coefficients $\vartheta_1 - \vartheta_2 = 0.1$. The pump modulation frequency $\omega_p/2\pi$ was 50 kHz.

Our experimental study showed that quasi-periodic lasing regimes QS-1 and QS-2 can be observed under the same conditions (i.e., bistability takes place) at small modulation depths ($0.01 < h < 0.1$).

Consider first the phase radiation dynamics in the absence of pump modulation. In this case, a solid-state ring laser operates in the self-modulation oscillation regime of the first kind. Figure 1 shows the time dependences of the intensities and the phase difference of counterpropagating waves for $\Omega = 0$. One can see that the intensities I_1 and I_2 of counterpropagating waves oscillate out of phase at the frequency $\omega_a/2\pi$. The phase difference of counterpropagating waves is also a periodic function of time and varies from zero to π.

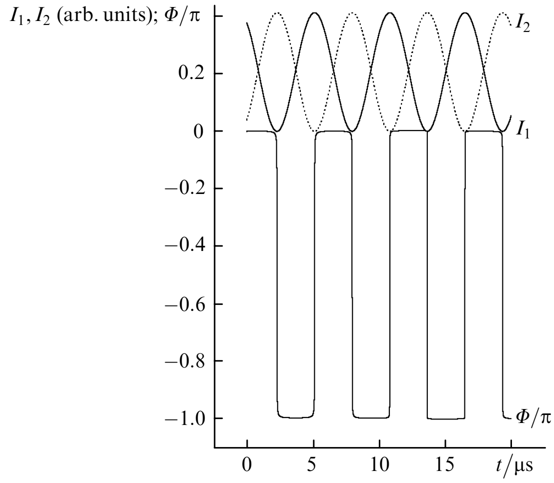

Figure 1. Time dependences of the intensity and phase difference of counterpropagating waves in the self-modulation lasing regime of the first kind for $\Omega = 0$.

Figure 2 shows the typical time dependences of the intensity and phase difference of counterpropagating waves in the QS-1 regime for $\Omega = 0$ and the pump modulation depth $h = 1$ %. In this case, the intensities I_1 and I_2 of counterpropagating waves also oscillate out of phase at the frequency $\omega_a/2\pi$ and, in addition, are modulated in phase at the frequency $\omega_p/2\pi$. The phase difference of counterpropagating waves for $\Omega = 0$ changes in finite limits, oscillating at the frequency $\omega_a/2\pi$. In this case, phase locking of quasi-periodic oscillations in counter directions

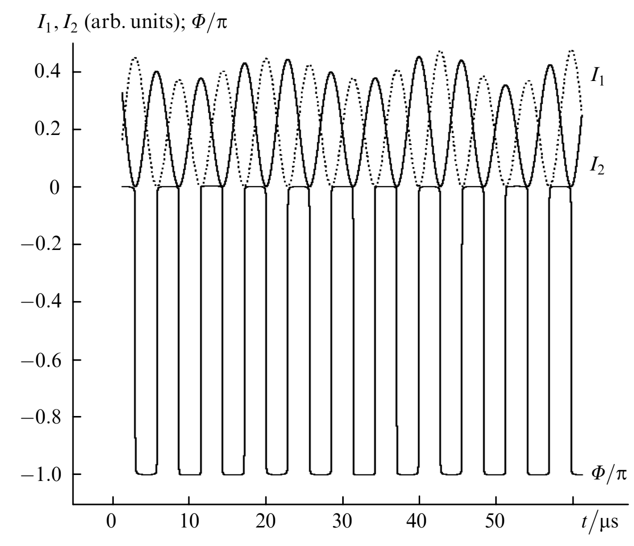

Figure 2. Time dependences of the intensity and phase difference of counterpropagating waves in the QS-1 regime for $\Omega = 0$ and the pump modulation depth $h = 1$ %.

takes place. Phase locking exists in a finite region of the values of optical nonreciprocity $|\Omega| < \Omega_{cr}$. For the laser parameters used, the value of $\Omega_{cr}/2\pi$ was ~ 100 kHz. As the nonreciprocity frequency increased ($|\Omega| < \Omega_{cr}$), phase locking changed to beats. Figure 3 shows the typical time dependences of the phase difference of counterpropagating waves and the intensity of one of the waves in the presence of frequency nonreciprocity. One can see that the phase difference still oscillates at the self-modulation frequency; however, it increases linearly in time so that the change in the phase difference during the period of self-modulation oscillations is 2π.

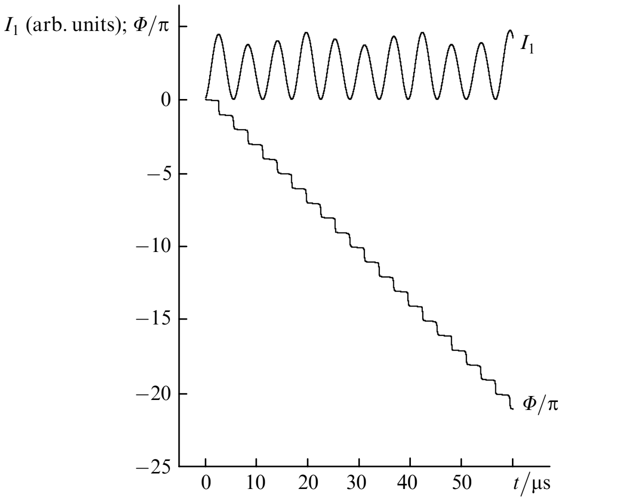

Figure 3. Time dependences of the intensity of one of the waves and phase difference of counterpropagating waves in the QS-1 regime for $\Omega/2\pi = -800$ Hz and the pump modulation depth $h = 1\,\%$.

Thus, the instant frequency difference of counterpropagating waves is not constant but oscillates at the frequency $\omega_a/2\pi$ with respect to the average frequency difference $\langle\dot{\Phi}\rangle/2\pi$. The average circular frequency difference $\langle\dot{\Phi}\rangle$ is equal to $\text{sign}(\Omega)\omega_a$. Based on these studies, we can conclude that the pump modulation in the QS-1 regime substantially affects not the phase but amplitude dynamics. The phase dynamics in the QS-1 regime is similar to that in the self-modulation regime and agrees with the theoretical predictions of [15].

Consider now the radiation dynamics in the QS-2 regime. The time dependence of the intensity of counterpropagating waves in this regime for $h = 2\,\%$ and $\Omega = 0$ is shown in Fig. 4. The phase dynamics is presented in Figs 5a, b. In Fig. 5a, it is shown in a narrow time interval, while the time interval in Fig. 5b is two orders of magnitude larger. One can see from Fig. 5a that in the QS-2 regime the phase difference oscillates at the self-modulation frequency and also increases linearly in time. It follows from Fig. 5b that the phase difference in the QS-2 regime changes by the saw-tooth law with a period τ which considerable exceeds the period of self-modulation oscillations. The average change in the phase difference of counterpropagating waves during the period is zero for $\Omega = 0$. The period of saw-tooth oscillations of the phase difference for the laser parameters used is ~ 3 ms.

If the optical nonreciprocity ($\Omega \neq 0$) takes place, the beat regime appears without the capture region. The phase

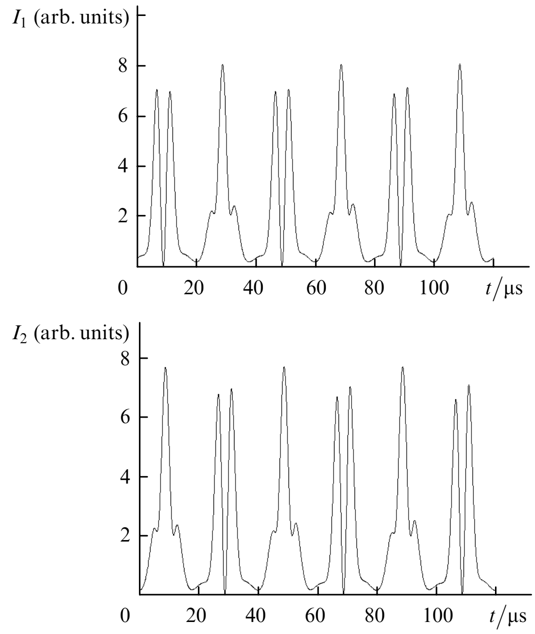

Figure 4. Time dependences of the intensity of counterpropagating waves in the QS-2 regime for $h = 2\,\%$ and $\Omega = 0$.

dynamics in this regime is shown in Fig. 6 for $h = 2\,\%$ and $\Omega/2\pi = 2$ kHz. One can see that the phase difference of counterpropagating waves reveals saw-tooth oscillations, which, unlike the phase-locking regime, are not periodic: the maxima and minima of saw-tooth oscillations linearly

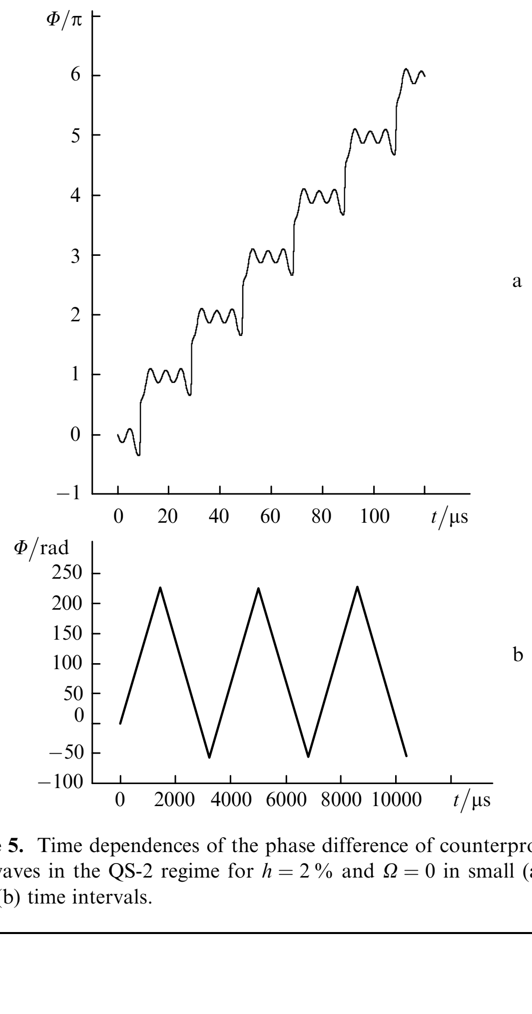

Figure 5. Time dependences of the phase difference of counterpropagating waves in the QS-2 regime for $h = 2\,\%$ and $\Omega = 0$ in small (a) and large (b) time intervals.

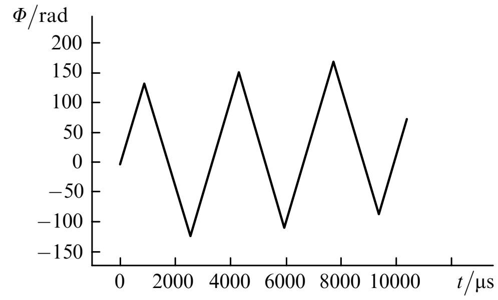

Figure 6. Time dependences of the phase difference of counterpropagating waves in the QS-2 regime for $h = 2\,\%$ and $\Omega/2\pi = 2$ kHz in a large time interval.

increase or decrease (in the case of the opposite sign of Ω) in time. The average change in the phase difference of counterpropagating waves during the saw-tooth modulation period τ is $\Omega\tau$. It is obvious that, by measuring the phase difference during the time interval $T_{\rm m}$ multiple of the period of saw-tooth oscillations, we can directly measure a change in the phase difference caused by the frequency nonreciprocity of the ring resonator: $\Delta\Phi = \Omega T_{\rm m}$. Therefore, in the case of a rotating ring laser in the QS-2 regime, its rotational velocity $\omega_{\rm rot}$ can be measured from the expression $\omega_{\rm rot} = \Delta\Phi/M$, where M is the scaling coefficient.

The optical nonreciprocity can be also measured in the QS-1 regime. In this case, the average frequency difference of counterpropagating waves proves to be equal to the frequency of self-modulation oscillations, which depends on the optical nonreciprocity according to the expression $\omega_{\rm a}(\Omega) = [(\omega_{\rm a}^2(0) + \Omega^2]^{1/2}$. Note that the measurement of small values of Ω is complicated because the frequency of self-modulation oscillations is unstable.

4. Conclusions

We have studied the phase dynamics of a solid-state ring laser in transient quasi-sinusoidal QS-1 and QS-2 regimes. It is shown that the phase dynamics in the QS-1 regime almost does not differ from dynamics in the regime of self-modulation oscillations. The phase dynamics in the QS-2 regime has a number of differences, namely, the frequency difference of counterpropagating waves appears, which changes by the saw-tooth law; the change in the phase difference of counterpropagating waves during a period τ of saw-tooth oscillations is equal to $\Omega\tau$. It follows from our study that the QS-2 regime can be of interest for measuring frequency nonreciprocity, in particular, in laser gyros.

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