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Fourth seminar to the memory of D.N. Klyshko

The fourth seminar to the memory of D.N. Klyshko was held at the M.V. Lomonosov Moscow State University on 17–19 May 2005. One can fully confirm that this seminar, in which almost all leading scientists in the field of quantum optics and quantum information participated, has become traditional. Seven papers based on the reports presented at the seminar have been prepared for publishing in this issue of Quantum Electronics. Although it is only a part of presented reports, we hope that the papers being published correctly reflect the research area and the spirit of the seminar.

The programme and abstracts of papers reported at the fourth seminar to the memory of D.N. Klyshko as well as information on previous seminars held in 2002, 2003 and 2004 can be found at the web-site: http://qopt.phys.msu.su.

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Formation of two-dimensional nonspreading atomic wave packets in the field of two standing light waves

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Abstract. **The formation of two-dimensional nonspreading atomic wave packets produced in the interaction of a beam of two-level atoms with two standing light waves polarised in the same plane is considered. The mechanism providing a dispersionless particle dynamics is the balance of two processes: a rapid decay of the atomic wave function away from the field nodes due to spontaneous transitions to nonresonance states and the quantum broadening of the wave packets formed in the close vicinity of field nodes. Coordinate-dependent amplitudes and phases of the two-dimensional wave packets were found for the $j_g = 0 \leftrightarrow j_e = 1$ transition.**

Keywords: *atomic optics, wave packets, lithography*.

1. Introduction

This work is a continuation of our investigation [1, 2] of nonspreading wave packets produced in the interaction between a plane atomic wave and a standing light wave. Recall that in a strictly resonance case, when the frequency of laser radiation coincides with the frequency of the working transition of two-level atoms, the particle dynamics is determined by a purely imaginary periodic potential which takes into account the irreversible spontaneous relaxation of the atoms from the upper level to other nonresonance levels. The escape of particles from the interaction with the field (due to transitions to nonresonance levels) leads to the disappearance (removal) of substantial portions of the atomic plane wave with time in all spatial regions except the vicinities of field nodes, where the population of the working upper level is small. Narrow wave packets are formed in the field nodes, which experience quantum broadening due to the Heisenberg uncertainty principle. The competition of two processes – the quantum broadening of the packet and the removal of its periphery due to irreversible spontaneous relaxation – results in a stationary value of the width of the wave packets formed near the nodes [1].

In [1], the analytic solution of the nonstationary Schrödinger equation with an imaginary potential linearised near the field node was found, which is in complete agreement with a recent unique experiment [2].

In the present work, we generalised the results to the two-dimensional case, when the atomic beam interacts with two perpendicular-polarised standing light waves. In this case, in the vicinity of the field nodes the problem reduces to the nonstationary two-dimensional Schrödinger equation with a purely imaginary effective potential representing a quadratic form in spatial variables. We also discuss the general formulation of the problem, when the polarisation vectors of the standing waves are not orthogonal.

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2. Basic equations and qualitative analysis

Consider a beam of two-level atoms propagating along the z axis and interacting with the 'light crystal' formed by two perpendicular-polarised standing light waves $\boldsymbol{E}_1 = \boldsymbol{e}_y E_1 \times \sin kx$ and $\boldsymbol{E}_2 = \boldsymbol{e}_x E_2 \sin ky$ (Fig. 1). We will assume below that the longitudinal particle velocity (along the z axis) is high and only weakly changes during the interaction. We also assume that the external field frequency coincides with the frequency of the $j_g = 0 \leftrightarrow j_e = 1$ atomic transition (Fig. 2), where j_g and j_e are the quantum numbers of the ground ($|g\rangle$) and excited ($|e\rangle$) states. Because the atoms experience a dipole interaction with the field, one of the states $|j_e = 1,\ m_e = \pm 1\rangle$ that gives rise to a nonzero matrix element of the dipole moment, can play the role the excited $|e\rangle$ state. The existence of a width γ of the excited level (Fig. 2) implies that, along with the induced $|e\rangle \leftrightarrow |g\rangle$ transitions, the atom can spontaneously emit a photon to find itself in some undetectable state $|i\rangle$, thereby escaping the resonance interaction with the field.

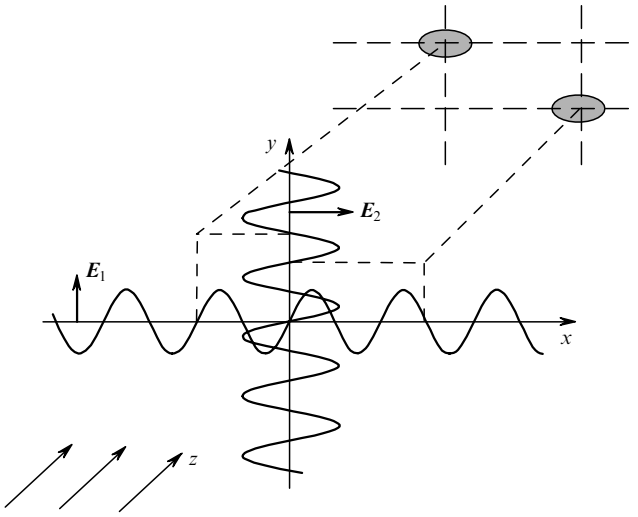

Figure 1. Scheme of the formation of nonspreading atomic wave packets near the nodes of a standing wave field.

The dynamics of a two-level atom is determined by the Schrödinger equation for the probability amplitudes of excited (Ψ_e) and ground (Ψ_g) states [1, 5]

$$i\frac{\partial \Psi_e}{\partial t} + i\frac{\gamma}{2}\Psi_e = \hat{T}\Psi_e - (\Omega_1^* \sin kx + \Omega_2^* \sin ky)\,\Psi_g, \qquad (1)$$

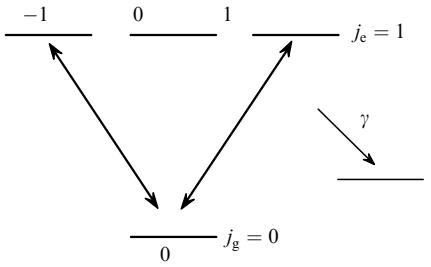

Figure 2. Scheme of atomic levels.

$$i\frac{\partial \Psi_g}{\partial t} = \hat{T}\Psi_g - (\Omega_1 \sin kx + \Omega_2 \sin ky)\,\Psi_e, \qquad (2)$$

where $\hat{T}$ is the kinetic energy operator; $\Omega_{1,2}$ are the Rabi frequencies of the $|e\rangle \leftrightarrow |g\rangle$ working transition;

$$\hat{T} = \frac{1}{2M}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right); \quad \Omega_{1,2} = \langle g|\hat{d}_{y,x}|e\rangle E_{1,2};$$

and $\hat{d}_{x,y}$ are the dipole transition operators.

We are interested in relatively strong fields, $|\Omega_{1,2}| \sim \gamma$, when the local saturation parameter

$$\frac{|\Omega_1 \sin kx + \Omega_2 \sin ky|}{\gamma} \sim 1$$

is far enough from the nodes $\{x = \pi n/k, y = \pi m/k\}$ of the standing waves. As a result, the upper level population is also ~ 1 and the density of particles resonantly interacting with the field decreases with the rate γ. For a sufficiently long interaction time, $t \gg 1/\gamma$, the atomic wave function $\Psi_{e,g}$ vanishes almost everywhere except the small vicinities of standing wave nodes, $|k\delta x|, |k\delta y| \ll 1$, where the upper level population is small,

$$\frac{|\Omega_1 \sin kx + \Omega_2 \sin ky|^2}{\gamma^2} \sim \frac{|\Omega_1 k\delta x + \Omega_2 k\delta y|^2}{\gamma^2} \ll 1,$$

and the particle density decreases with a substantially lower rate

$$\gamma\frac{|\Omega_1 k\delta x + \Omega_2 k\delta y|^2}{\gamma^2} \ll \gamma.$$

The characteristic sizes of the vicinities of the field nodes δx and δy, $\delta x \sim \delta y$, are determined from the condition

$$\frac{|\Omega_1 k\delta x + \Omega_2 k\delta y|^2}{\gamma} t \sim 1,$$

which finally gives

$$\delta x(t) \sim \delta y(t) \sim \frac{\gamma}{k\Omega_0}\frac{1}{(\gamma t)^{1/2}}, \quad \Omega_0 = |\Omega_1 + \Omega_2|.$$

Thus, the spontaneous relaxation of atoms to nonresonance states (see Fig. 2) results in the formation of narrow atomic wave packets with the width $\delta x \sim \delta y$, which decreases as $1/\sqrt{t}$ with time. The rate of wave-packet narrowing may be qualitatively defined as

$$v_{narr} \sim \frac{\delta x(t)}{t} \sim \frac{\delta y(t)}{t} \sim \frac{\sqrt{\gamma}}{k\Omega_0}\frac{1}{t^{3/2}}.$$

On the other hand, the wave-packet narrowing in the coordinate space results in the increase of the characteristic widths of the momentum distribution

$$\delta p_x(t) \sim \frac{1}{\delta x(t)} \sim \frac{k\Omega_0}{\gamma}\sqrt{\gamma t}, \quad \delta p_y(t) \sim \frac{1}{\delta y(t)} \sim \frac{k\Omega_0}{\gamma}\sqrt{\gamma t}$$

with time and, accordingly, of the wave-packet spreading rate

$$v_{\rm spr} \sim \frac{\delta p_x(t)}{M} \sim \frac{\delta p_y(t)}{M} \sim \frac{1}{M\delta x(t)} \sim \frac{1}{M\delta y(t)} \sim \frac{k\Omega_0\sqrt{t}}{M\sqrt{\gamma}}.$$

As t increases, the wave-packet spreading rate $v_{\rm spr}(t)$ increases and becomes equal at some instant t_0 to the narrowing rate $v_{\rm narr}(t)$. For $t > t_0$, no further narrowing of the wave packet occurs, because its rapid spreading restores its width attained at $t = t_0$. Therefore, by equating $v_{\rm spr}(t)$ and $v_{\rm narr}(t)$, we can qualitatively estimate both the narrowing time t_0 of the wave-packet and its minimal width $\delta x_{\rm min} \sim \delta y_{\rm min}$:

$$t_0 \sim \frac{1}{\omega_0},$$

where $\omega_0 = \Omega_0(\omega_r/\gamma)^{1/2}$; $\omega_{\rm r} = k^2/2M$ is the recoil frequency for an atom of mass M, and

$$\delta x_{\rm min} \sim \delta y_{\rm min} \sim \frac{1}{(M\omega_0)^{1/2}},$$

which coincides with the characteristic length in the effective oscillator potential formed in the vicinity of the node of the standing wave.

3. Oscillator approximation

Consider now the approximate solution of the system of equations (1), (2). From the above qualitative consideration it follows that nonspreading wave packets are formed for a long time $t \gg 1/\gamma$ for $|\Omega_{1,2}| \sim \gamma$. That is why the time derivative in Eqn (1) can be neglected. Furthermore, the characteristic kinetic energy is low compared to the width of the excited level, $\omega_{\rm r} \ll \gamma$, which allows us to omit the operator of differentiation with respect to the spatial variables in Eqn (1). Therefore, in accordance with the results of [1, 2], the wave function of the excited state is

$$\Psi_{\rm e} \approx -\frac{(\Omega_1^* \sin kx + \Omega_2^* \sin ky)}{{\rm i}\gamma/2}\Psi_{\rm g}. \qquad (3)$$

By substituting approximate expression (3) into Eqn (2) and taking into account that $\sin kx \approx kx$ and $\sin ky \approx ky$ near the field node, we obtain the Schrödinger equation for the wave function $\Psi_{\rm g}$, which describes the centre-of-mass motion of the atoms in the ground state,

$${\rm i}\frac{\partial\Psi_{\rm g}}{\partial t} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Psi_{\rm g} - {\rm i}\left(\frac{M\omega_x^2}{2}x^2 + \frac{M\omega_y^2}{2}y^2\right)\Psi_{\rm g},$$

where

$$\omega_{x,y} = 2|\Omega_{1,2}|\left(\frac{2\omega_r}{\gamma}\right)^{1/2}. \qquad (4)$$

This equation should be solved by using the initial condition $\Psi_{\rm g}(x, y, 0) = k/\pi$, which corresponds to a plane atomic wave with a zero transverse momentum. Equation (4) was derived taking into account the fact that the quantity $\Omega_1 \propto \langle {\rm g}|\hat{d}_y|{\rm e}\rangle$ assumes purely imaginary values and the quantity $\Omega_2 \propto \langle {\rm g}|\hat{d}_x|{\rm e}\rangle$ – purely real values, and therefore ${\rm Re}\{\Omega_1^*\Omega_2\} = 0$. Therefore, a special selection of the atomic transition enabled obtaining the equation with a separable potential, which splits into two independent equations for the amplitudes $\Psi_{x,y}$ by separating variables, $\Psi_{\rm g}(x, y, t) = \Psi_x(x, t)\,\Psi_y(y, t)$. The solution of the one-dimensional equation for the function $\Psi_\rho(\rho, t)$, where $\rho = x, y$, was obtained in Ref. [1]:

$$\Psi_\rho(\rho, t) = \left(\frac{k/\pi}{\cosh\beta t}\right)^{1/2} \exp\left(-\frac{1}{2}\alpha_\rho\rho^2\tanh\beta_\rho t\right), \qquad (5)$$

where $\alpha_\rho = M\omega_\rho\exp(-{\rm i}\pi/4)$; $\beta_\rho = \omega_\rho\exp({\rm i}\pi/4)$.

Therefore, the two-dimensional probability density $|\Psi_{\rm g}(x, y, t)|^2$ is a Gaussian surface with the time-dependent dimensions of localisation region $\delta x(t) = 1/\{{\rm Re}[\alpha_x \times \tanh(\beta_x t)]\}^{1/2}$, $\delta y(t) = 1/\{{\rm Re}[\alpha_y\tanh(\beta_y t)]\}^{1/2}$. Note that the quadratically coordinate-dependent phase of the wave function (5) can also be measured in a real experiment (see [2]). The steady-state shape of the nonspreading wave packet for $t > 1/\min\{\omega_x, \omega_y\}$ is an ellipse with the axial ratio $\delta x_0/\delta y_0 = (E_2/E_1)^{1/2}$.

Consider now the general case, when the standing waves forming a 'light crystal' are again polarised in the same plane as before, but the polarisation vectors are not orthogonal, making an angle $\pi/2 + \theta$ (Fig. 3), i.e.

$$\boldsymbol{E}_1 = (-E_1\sin\theta\,\boldsymbol{e}_x + E_1\cos\theta\,\boldsymbol{e}_y)\sin\boldsymbol{kr},$$

$$\boldsymbol{E}_2 = E_2\sin ky\;\boldsymbol{e}_x, \qquad (6)$$

where $\boldsymbol{kr} = kx\cos\theta + ky\sin\theta$.

Like before, the nonspreading wave packets will be formed near the nodes of the oblique lattice (Fig. 3) due to the establishment of equilibrium between the two processes: a rapid decrease of the atomic wave function away from the field nodes and the quantum broadening of the wave packets being formed.

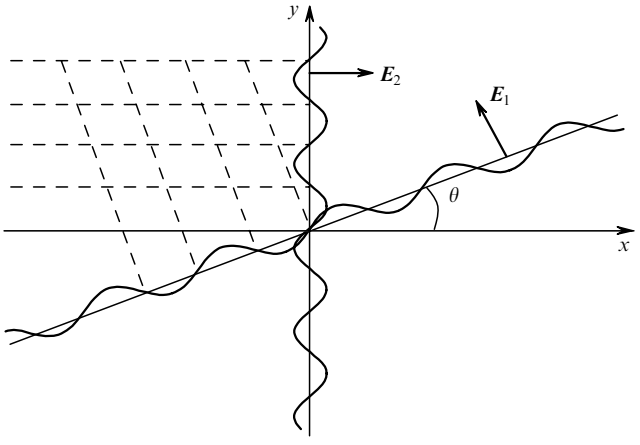

Figure 3. 'Light crystal' formed by the standing waves with non-orthogonal polarisation vectors.

By omitting the derivatives with respect to time and the spatial variables in the equation for the wave function of the excited state and taking into account that the quantity a ${\rm Re}\{d_x^* d_y\}$ vanishes for the $j_{\rm g} = 0 \leftrightarrow j_{\rm e} = 1$ transition, we obtain the equation for the probability amplitude $\Psi_{\rm g}(x, y, t)$ near the field node $x = y = 0$:

$$\mathrm{i}\frac{\partial \Psi_g}{\partial t} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi_g + \frac{(Ax^2+By^2+Cxy)}{\mathrm{i}\gamma/2}\Psi_g, \quad (7)$$

where

$$A = k^2 d^2 E_1^2 \cos^2\theta;$$

$$B = k^2 d^2 (E_2^2 - 2E_1E_2\sin^2\theta + E_1^2\sin^2\theta);$$

$$C = k^2 d^2 E_1 (E_1 - E_2)\sin 2\theta;$$

$$d \equiv |d_x| = |d_y|.$$

By rotating coordinate axes by an angle φ such that

$$\tan\varphi = \frac{C}{A-B} = \frac{E_1\sin 2\theta}{E_2 + E_1\cos 2\theta},$$

we will pass in (7) to new variables

$$x_1 = x\cos\varphi + y\sin\varphi, \quad x_2 = -x\sin\varphi + y\cos\varphi$$

to obtain the equation for the wave function of the $\Psi_g(x_1, x_2, t)$ ground state with a diagonalised potential energy

$$\mathrm{i}\frac{\partial \Psi_g}{\partial t} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)\Psi_g - \mathrm{i}\left(\frac{M\omega_1^2}{2}x_1^2 + \frac{M\omega_2^2}{2}x_2^2\right)\Psi_g, \quad (8)$$

where

$$\omega_{1,2} = d\left(\frac{2\omega_r}{\gamma}\right)^{1/2}|E_1 - E_2 \pm (E_1^2 + E_2^2 + 2E_1E_2\cos 2\theta)^{1/2}|.$$

The solution of Eqn (8) with the initial condition $\Psi_g(x_1, x_2, 0) = k/\pi$ for $t > 1/\min\{\omega_1, \omega_2\}$ is an ellipse with the axial ratio equal to $(\omega_1/\omega_2)^{1/2}$. In the case of equal amplitudes of the standing waves, $E_1 = E_2$ or $\omega_1 = \omega_2 \equiv \Omega$, the final shape of the nonspreading wave packet is a circle of diameter $(2\omega_r^2/\Omega^2)^{1/4}$.

4. Conclusions

We have considered the formation of two-dimensional nonspreading atomic wave packets produced in the interaction of two-level atoms with standing light waves polarised in the same plane. It was shown that a special choice of the atomic transition permits the analytic description of the system under study to be greatly simplified and the two-dimensional problem to be reduced to two one-dimensional Schrödinger equations with the known solution.

The interaction of two-level atoms with a two-dimensional light crystal was considered in Refs [3, 4], but the subject of the research was the intensity of the atomic wave transmitted through a light crystal. We have obtained both the amplitude and phase of the wave packets formed in the vicinity of the field nodes. The coherent properties of the set of wave packets may be employed, for instance, in atomic interferometry.

Note that the proposed analytic approach corresponds to a completely open two-level system, when all spontaneous transitions transform atoms to some nonresonance states, resulting in the escape of the atoms from the process of resonance interaction with the field. In a real experiment [2], 16 % of the atoms that spontaneously emit a photon return to the initial state.

To take this effect into account, it is necessary to solve the equations for the atomic density matrix (see Ref. [5]) describing the recoil effect due to a spontaneous transition. The solution of this problem both in the one-dimensional and the two-dimensional cases will be the subject of our future investigations.

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References

1. Fedorov M.V., Efremov M.A., Yakovlev V.P., Schleich W.P. *Zh. Eksp. Teor. Fiz.*, **124**, 578 (2003).
2. Stuetzle R., Goebel M.C., Hoerner T., Kierig E., Mourachko I., Oberthaler M.K., Efremov M.A., Fedorov M.V., Yakovlev V.P., van Leeuwen K.A.H., Schleich W.P. *e-print quant-ph/0412138*.
3. Petra S.J.H., van Leeuwen K.A.H., Feenstra L., Hogervorst W., Vassen W. *Appl. Phys. B*, **79**, 279 (2004).
4. Turlapov A., Tonyushkin A., Sleatoret T. *Phys. Rev. A*, **68**, 023408 (2003).
5. Kazantsev A.P., Surdutovich G.I., Yakovlev V.P. *Mekhanicheskoe deistvie sveta na atomy* (Mechanical Action of Light on Atoms) (Moscow: Nauka, 1991).